STRONOMY

OHN CHARLES DUNCAN

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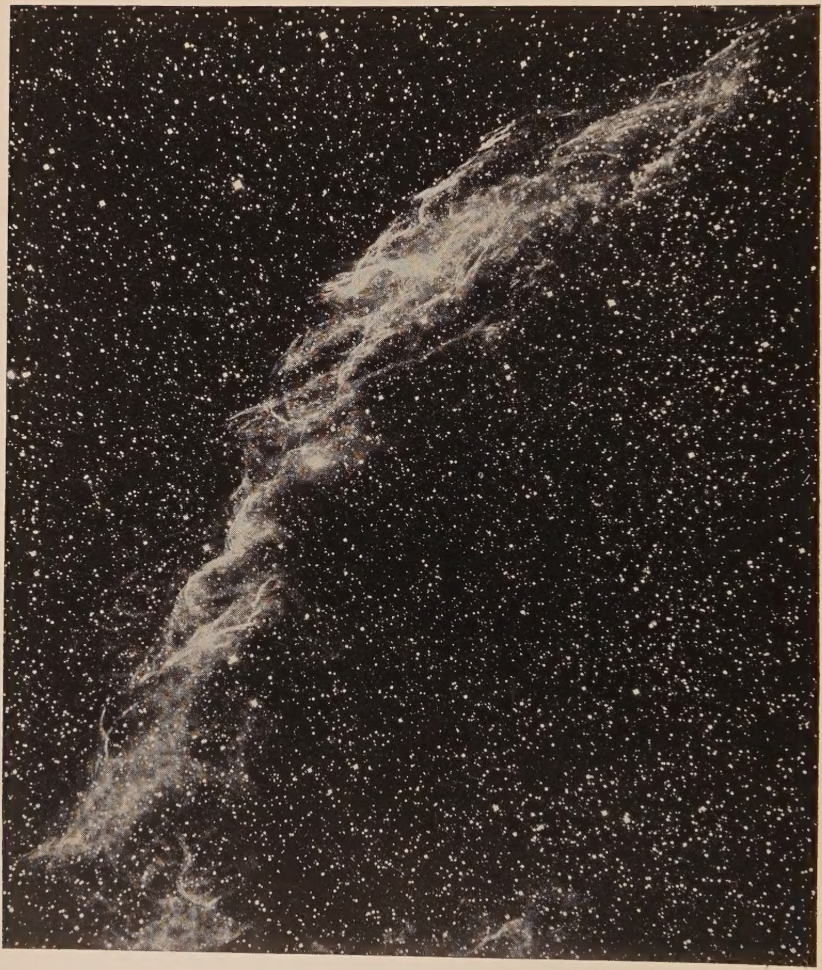

THE VEIL NEBULA N. G. C. 6992 CYGNI

Photographed by J. C. Duncan, 1928, August 15, with the 100-inch Hooker Telescope of the Mount Wilson Observatory. Exposure, 6 hours and 30 minutes

ASTRONOMY

A TEXT BOOK

BY

JOHN CHARLES DUNCAN, Ph.D.

Professor of Astronomy in Wellesley College

ILLUSTRATED

SECOND EDITION REVISED
1930

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ASTRONOMY

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SIR,

We are told, and, if I remember right, it is also your Opinion, that three of the finest sights in Nature, are a rising Sun at Sea, a verdant Landskip with a Rainbow, and a clear Star-light Evening.

THOMAS WRIGHT, *1750*

FROM THE PREFACE TO THE FIRST EDITION

This book is intended to present a general view of the science of the stars, and to be suitable for the use of beginning classes in Astronomy in colleges and universities. While assuming the degree of mental maturity commonly expected of college students and occasionally employing the principles and methods of Algebra, Geometry, and elementary Trigonometry, it avoids difficult mathematical discussions and requires no previous study of science.

The form of the text has been developed during sixteen years of teaching, first at Harvard and Radcliffe Colleges and later at Wellesley, the material having been presented in the form of illustrated lectures which were supplemented by numerous laboratory exercises and evening observations. The writing of the manuscript was begun at Wellesley in 1924, but the greater part was written at the offices of the Mount Wilson Observatory from February to June, 1926.

In the preparation of the book, I have received assistance from many sources. For the older, long-established facts of Astronomy I have made free use of existing texts, particularly those of Young and Moulton, while in the discussion of more recent work I have had the help of many friends. Director W. S. Adams of the Mount Wilson Observatory read all of the manuscript except Chapters XI and XII and made many helpful suggestions, particularly on the chapters relating to the Sun and the stars. Astronomers J. A. Anderson, E. P. Hubble, A. H. Joy, P. W. Merrill, S. B. Nicholson, C. E. St. John, and G. Strömberg of the Mount Wilson Observatory, R. H. Curtiss of Detroit Observatory, and C. O. Lampland of Lowell Observatory read parts of the manuscript dealing with subjects in which they were particularly interested, and gave me the benefit of their expert knowledge. The Detroit, Harvard, Lick, Lowell, Mount Wilson, Sproul, Van Vleck, and Yerkes observatories contributed photographic illustrations. Dr. E. G. Martin, Professor of Physiology in Stanford University, author of text books, made valuable suggestions on the form of the manuscript. In the preparation of the printer's copy and in reading the proof I have had the assistance of my wife, Katharine B. Duncan, and of my daughter Eunice. Professor Leah B. Allen of Wellesley College was at considerable pains to send material to me from Wellesley to Pasadena. Misses Margaret Holbrook and Helen Mitchell, Assistants in Astronomy at Wellesley College, contributed Figure 108 and Plate 8 respectively.

I am greatly indebted to Directors Hale and Adams of the Mount Wilson Observatory for the privilege which I have enjoyed during several years, both as a regular member of their staff and as a visitor, of using the unrivalled equipment of their Observatory in making many of the celestial photographs which are here reproduced as illustrations.

John C. Duncan

Pasadena, California
August, 1926

PREFACE TO THE SECOND EDITION REVISED

The material of the first edition, including both text and illustrations, has been carefully revised and rearranged and somewhat extended. My aim was to bring the work thoroughly up to date and to embody in it such new features as have been shown, by my own experience and by that of other teachers and of our students, to be desirable.

The exercises at the ends of chapters, which were added in compliance with requests from a number of users of the book, have been designed to serve not merely as review questions, but rather for the purpose of stimulating thought.

A new set of star maps has been provided. With the purpose of suggesting to the eye the actual appearance of the night sky, the stars are represented on a blue ground by white dots, the areas of which are proportional to the amount of light sent to the Earth by the respective stars, as indicated by their visual magnitudes; and the lettering and reference lines, in darker blue, are suppressed as much as can well be done without sacrificing legibility. On account of the necessarily small scale, only about four hundred stars are shown. These were chosen mainly for their importance in well-known constellation figures, and no rigorous limit of magnitude was set. Outlines of the constellations were drawn according to my own customary conception of them, but it is realized that after a little experience each observer will form his own pictures. The globular projection was adopted as affording a minimum of distortion. The equinox is that of 1950.0.

A number of users of the first edition made friendly suggestions for its improvement; especially valuable lists were sent by Professor Philip Fox, then of Northwestern University, and Professor Frank Schlesinger of Yale. The authors and publishers of Russell, Dugan, and Stewart's *Astronomy* kindly gave me permission to use material from that excellent and compendious work. Director Harlow Shapley of the Harvard Col-

lege Observatory described for me, in advance of publication, the results of his recent investigations on the structure of the Galactic System (page 418). Director James Robertson of the Nautical Almanac Office furnished advance proofs of the pages of the *American Ephemeris* which deal with the total solar eclipse of 1932. The Mount Wilson and Lowell Observatories again contributed photographs; the original of Plate 22, presenting results of recent observations of the trans-Neptunian planet, was sent to me by Professor C. O. Lampland of the latter observatory while the page proofs were being read. Valuable assistance in reading proof was given by Mrs. Duncan and by Miss Helen F. Story, Instructor in Astronomy in Wellesley College. My gratitude for these favors is here recorded.
J. C. D.

WHITIN OBSERVATORY
WELLESLEY, MASSACHUSETTS
May, 1930

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ASTRONOMY

ASTRONOMY

INTRODUCTION

Astronomy is the science of the heavenly bodies. Literally, the word ('άστρον, star, νέμειν, arrange) means mapping, classifying, or describing the stars.

From analogy with such words as Geology, Psychology, and Zoölogy, it might be expected that the science of the stars would be called Astrology ('άστρον, λέγειν, speaking about stars), and in fact this word was so used centuries ago; but astrologers wandered from the paths of reality into the realms of fancy and superstition, and the name astrology is now applied, not to any science, but to fortune-telling by the stars.

The **Heavenly Bodies** are:

1. **The Stars,** which appear to us as tiny glittering points in the night sky, and which are really vast globes of intensely heated gas shining by their own light. They are the most numerous of visible heavenly bodies. They are so far away that their relative motions do not become appreciable to the eye in centuries, and so they are sometimes referred to as "the fixed stars."

2. **The Sun,** one of the stars but much nearer than any other; the source of light, heat, and life upon the Earth.

3. **The Planets,** which look like stars to the unaided eye, but may be distinguished by the fact that their position among the stars may be seen to change in the course of a few nights. In reality, the planets are opaque spheres which shine by reflected sunlight and revolve in nearly circular orbits around the Sun. The Earth itself is a planet, and as such forms a part of the domain of Astronomy. The names of the principal planets, in the order of distance from the Sun, are: Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune, and Pluto.

4. **Satellites,** bodies resembling small planets, but revolving around planets instead of around the Sun.

5. **The Moon,** the satellite of the Earth and the nearest of the heavenly bodies.

6. **Comets,** flimsy bodies, some of great size, which revolve around the Sun in elliptic or parabolic orbits. The brightest comets are spectacular objects with long trains or tails that extend over a large part of the sky.

7. **Meteors,** tiny solid objects which fly through space around the Sun like the comets, but which are so small that they cannot be seen until they encounter the Earth and become entangled in its atmosphere, when the heat generated by the stoppage of their swift motion makes them luminous.

8. **Nebulæ,** vast aggregations of matter at distances comparable to those of the stars, but so large as to have perceptible size and shape instead of appearing, like the stars, as mere points. The nebulæ are all faint, only a few of the many thousand known being visible to the unaided eye.

The Sun, planets, satellites, comets, and meteors comprise the **solar system,** which, vast though it is, forms but a speck in the universe of stars and nebulæ.

The Constellations.—In contemplating the stars, which at first glance seem sprinkled at random over the sky, it is natural to group them into geometric figures—triangles, squares, and winding rows; and the imagination can easily people the sky with more elaborate forms. Star-groups to which definite names have been given are called **constellations.** Forty-eight were named by the ancients, mostly in prehistoric times, and mostly for objects or heroes of mythology. Since A.D. 1600 the spaces between these ancient groups, spaces which for the most part contain only inconspicuous stars, have also been named, bringing the whole number of constellations up to eighty-five.[1] Most of the groups bear little resemblance to the objects for which they are named: for example, Pegasus, the winged horse, is most easily recognized by a square of four stars; Andromeda, the chained maiden, is represented chiefly by a nearly straight row of three; and the most conspicuous part of Ursa Major, the bear into which the nymph Callisto was transformed, is called in America the Great Dipper and in Europe the Wain or the Plow. The names of all the constellations are given in Table 1 (page 5).

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[1] Or eighty-eight, if Argo is divided into four (page 5).

Designations of Stars.—The most common method of designating the brighter stars is by a letter of the Greek alphabet followed by the genitive form of the Latin name of the star's constellation. This method was introduced in A.D. 1601 by Bayer, who generally applied the letters, beginning with α, in the order of brightness (but there are notable exceptions). If the number of stars in a constellation exceeds the number of Greek letters, Roman letters also are used.

The Greek alphabet (small letters) is as follows:

α	alpha	ι	iota	ρ	rho
β	beta	κ	kappa	σ	sigma
γ	gamma	λ	lambda	τ	tau
δ	delta	μ	mu	υ	upsilon
ε	epsilon	ν	nu	φ	phi
ζ	zeta	ξ	xi	χ	chi
η	eta	ο	omicron	ψ	psi
θ	theta	π	pi	ω	omega

A few stars are well known by individual names, mostly Latin words such as Polaris (α Ursæ Minoris), Pollux (β Geminorum), Spica (α Virginis), and Bellatrix (γ Orionis); or words taken from the Arabic, as Algenib (γ Pegasi), Deneb Kaitos (β Ceti), and Mizar (ζ Ursæ Majoris).

For a few faint naked-eye stars which were not given letters in the Bayer system, numbers are used. These were assigned by Flamsteed in the seventeenth century in the order of the right ascension (page 20) of the star within its constellation.

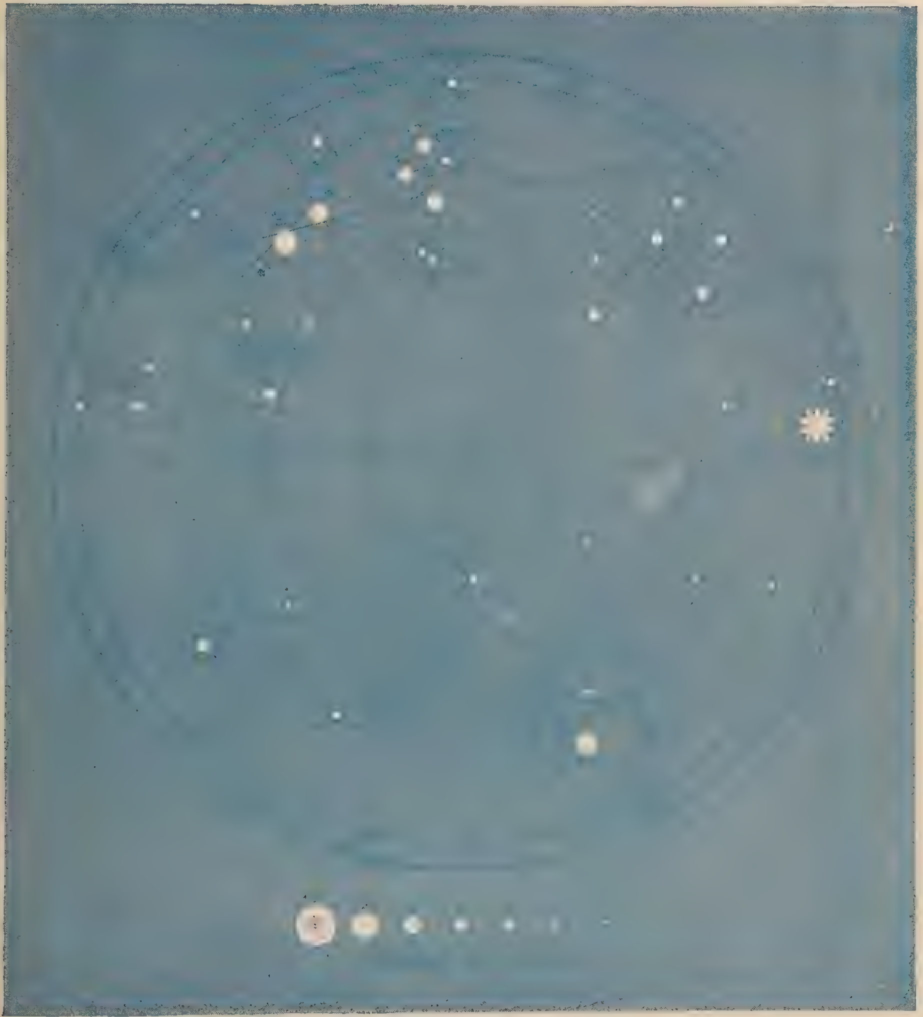
For the vast numbers of telescopic stars, to which no designation has been assigned in any of the above systems, it is customary to give the star's number in some well-known star catalogue (page 321), or, if it is not listed in a catalogue, to give its right ascension and declination at a designated epoch.

Star "Magnitudes."—The brightness of a star is denoted by a number known as its **magnitude.** This system has come down from Ptolemy, who, about A.D. 150, classified the brightest twenty stars as of the *first* magnitude, those just visible to the naked eye as of the *sixth*, and those of intermediate brightness by intermediate numbers which increase as the brightness grows less. The system has been greatly extended in modern times and is discussed in Chapter XIV.

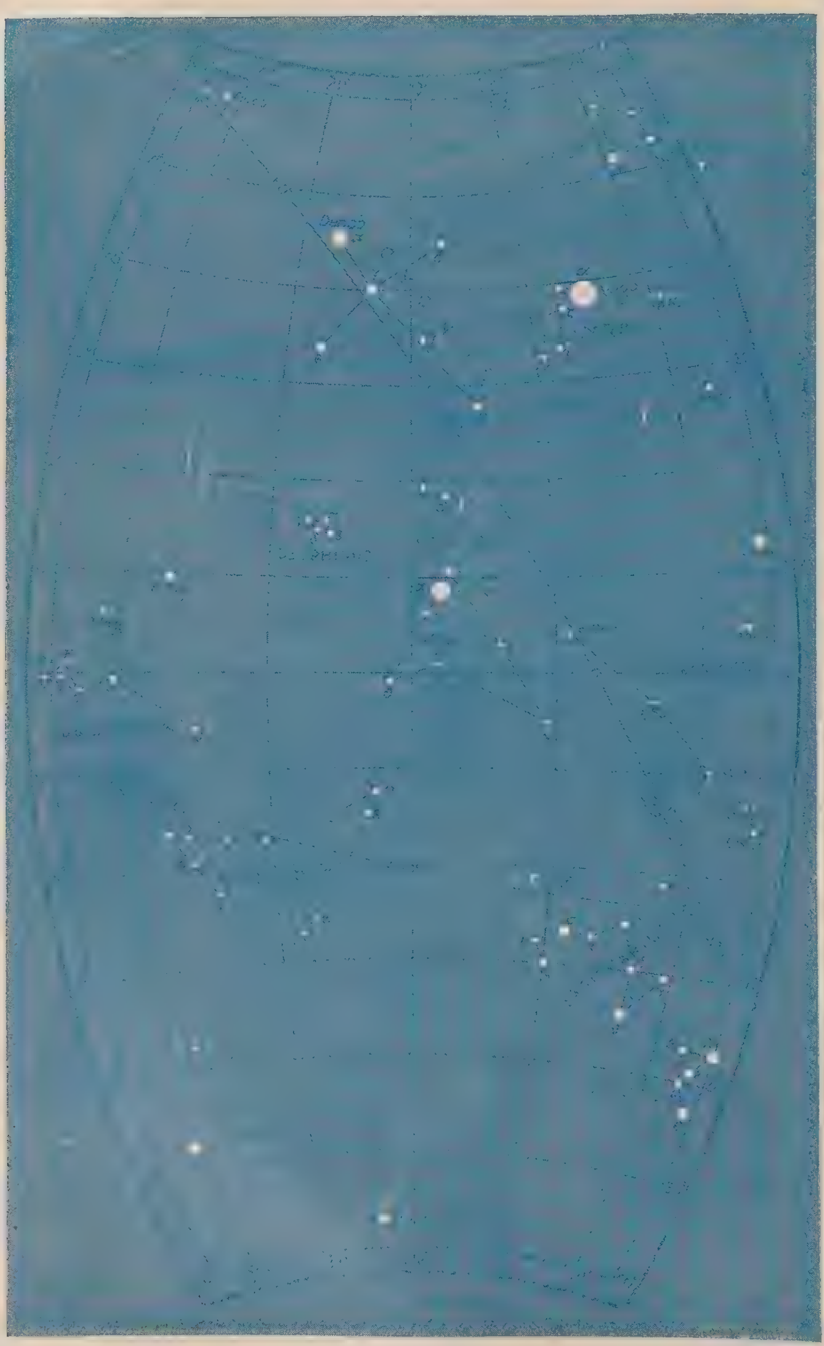
In beginning the study of Astronomy, the student should acquaint himself as soon as possible with the more important constellations then visible, and should add others to his acquaintance as they are brought to view in the progress of the year. For this purpose the star maps on the following eight pages should prove useful, especially if supplemented by the help of a friend who has already made a start. In these maps, only those stars are shown which are rendered important by their brightness or by their salient positions in the constellation figures, and many of the less important constellations are omitted. The more familiar constellation figures are traced by dark lines which connect the stars. The magnitude scale is such that the areas of the white dots are proportional to the amount of light which we receive from the respective stars.

For one who lives within the latitudes of the United States it is probably easiest to begin with the Great Dipper, since it is conspicuous, always above the horizon, and almost universally known. A line drawn through the two end stars of the Dipper bowl, which are called The Pointers, and prolonged five times the distance between them, passes very near Polaris, the North Star. On the opposite side of Polaris from the Dipper is Cassiopeia, which has the form of a W. After these groups are located in the sky, it is easy to find from the north polar map (Map 8) the remaining stars of Ursa Major, also Ursa Minor, Draco, and Cepheus; and these groups will then serve, with the help of the other maps, to identify constellations farther from the Pole.

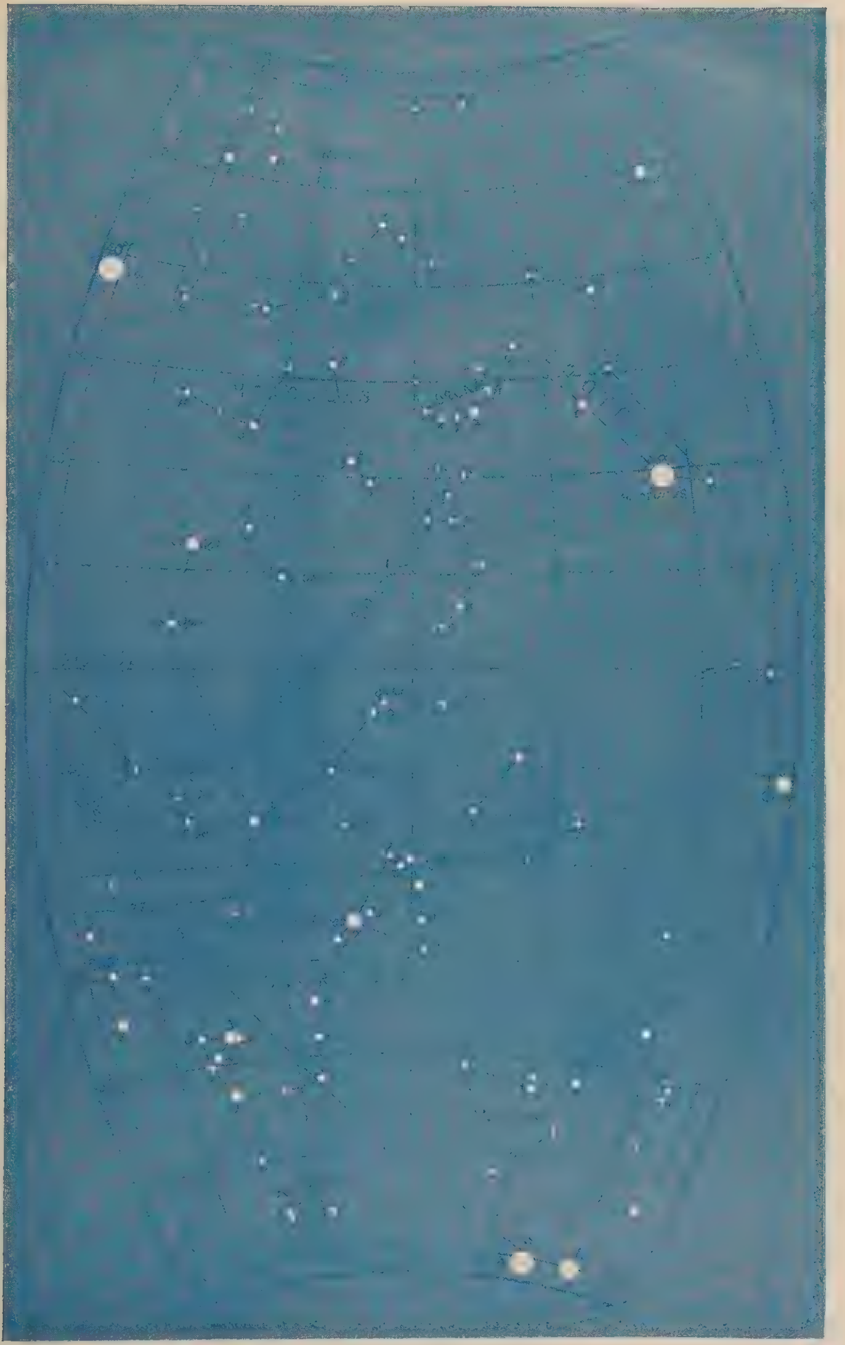
MAP I



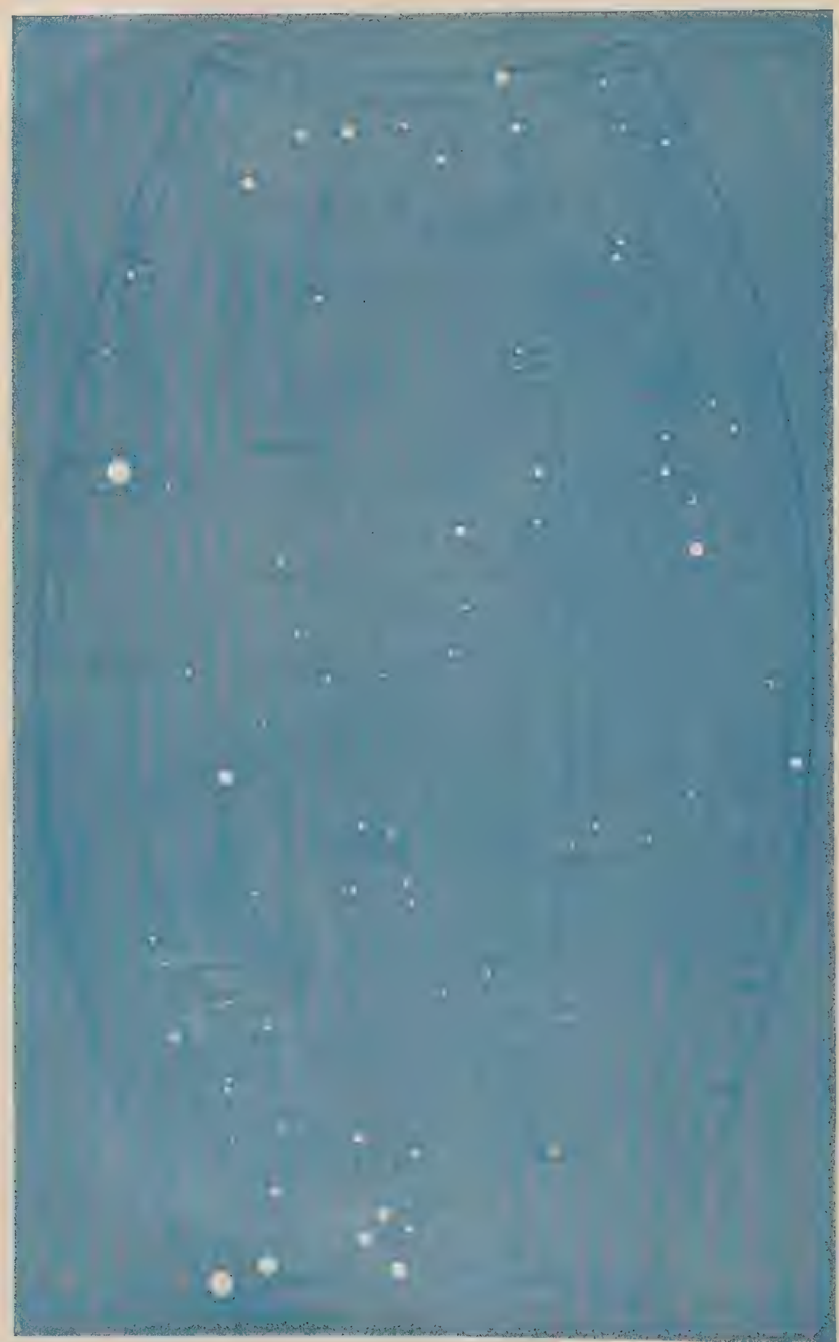
MAP 2



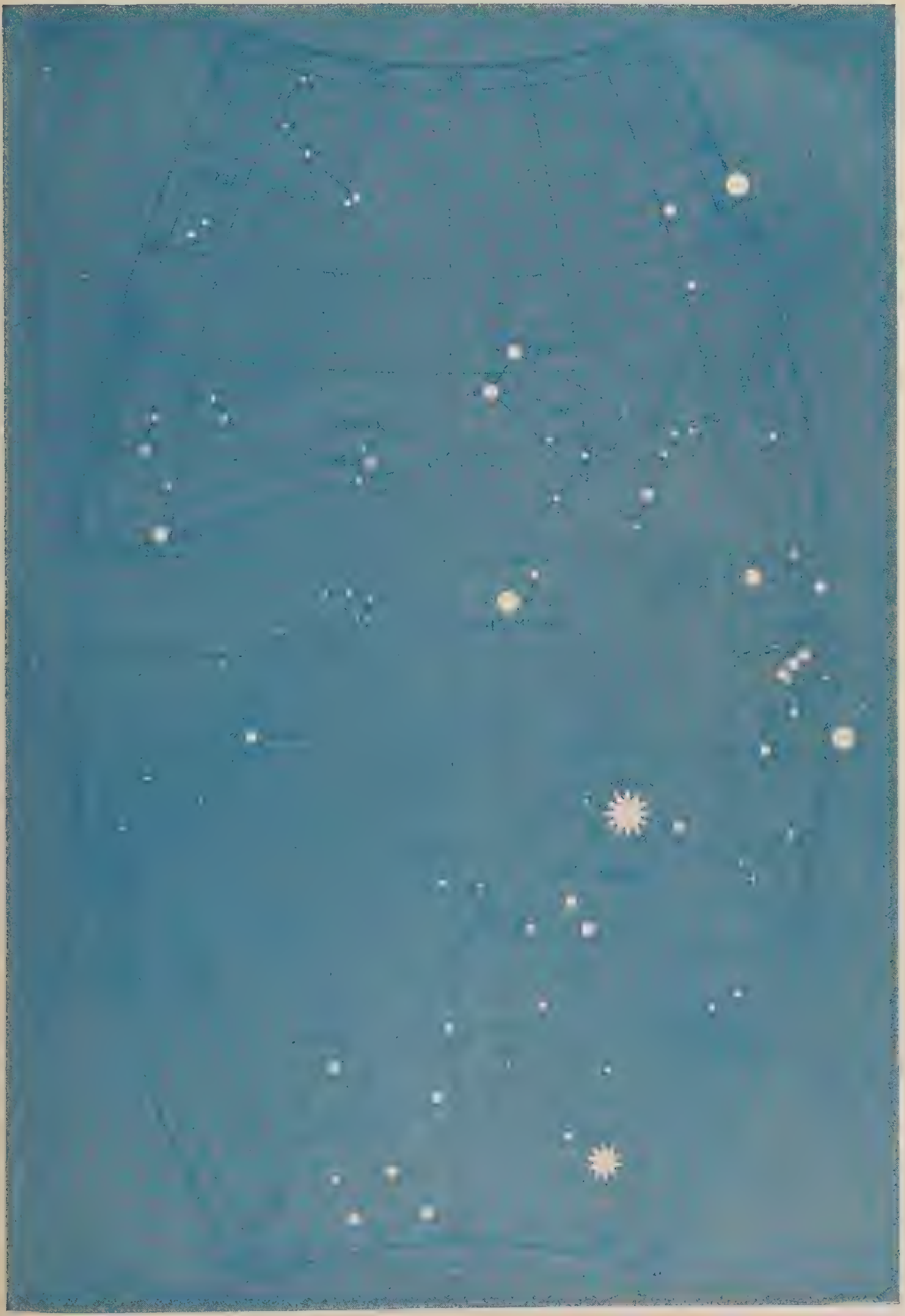
MAP 3



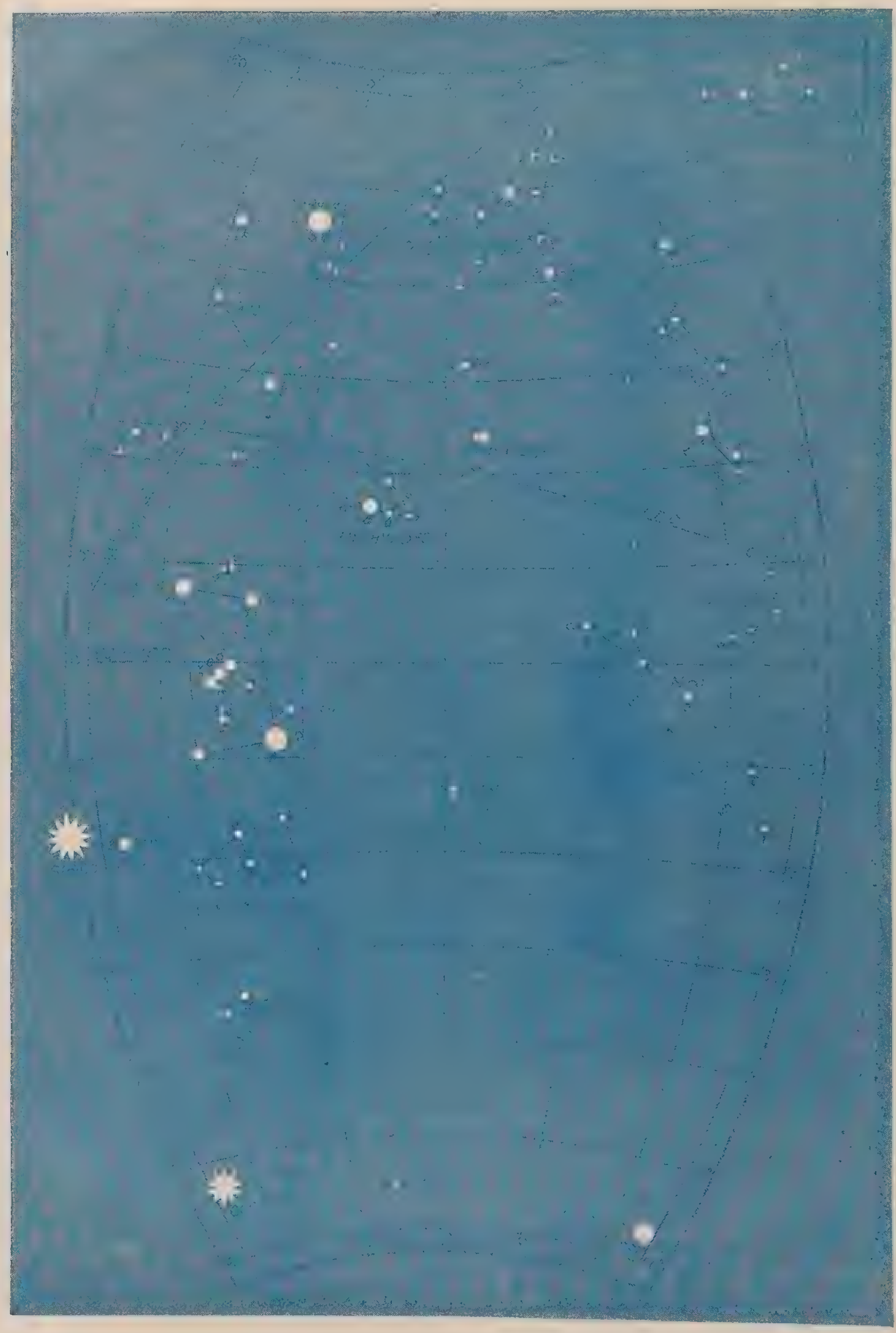
MAP 4



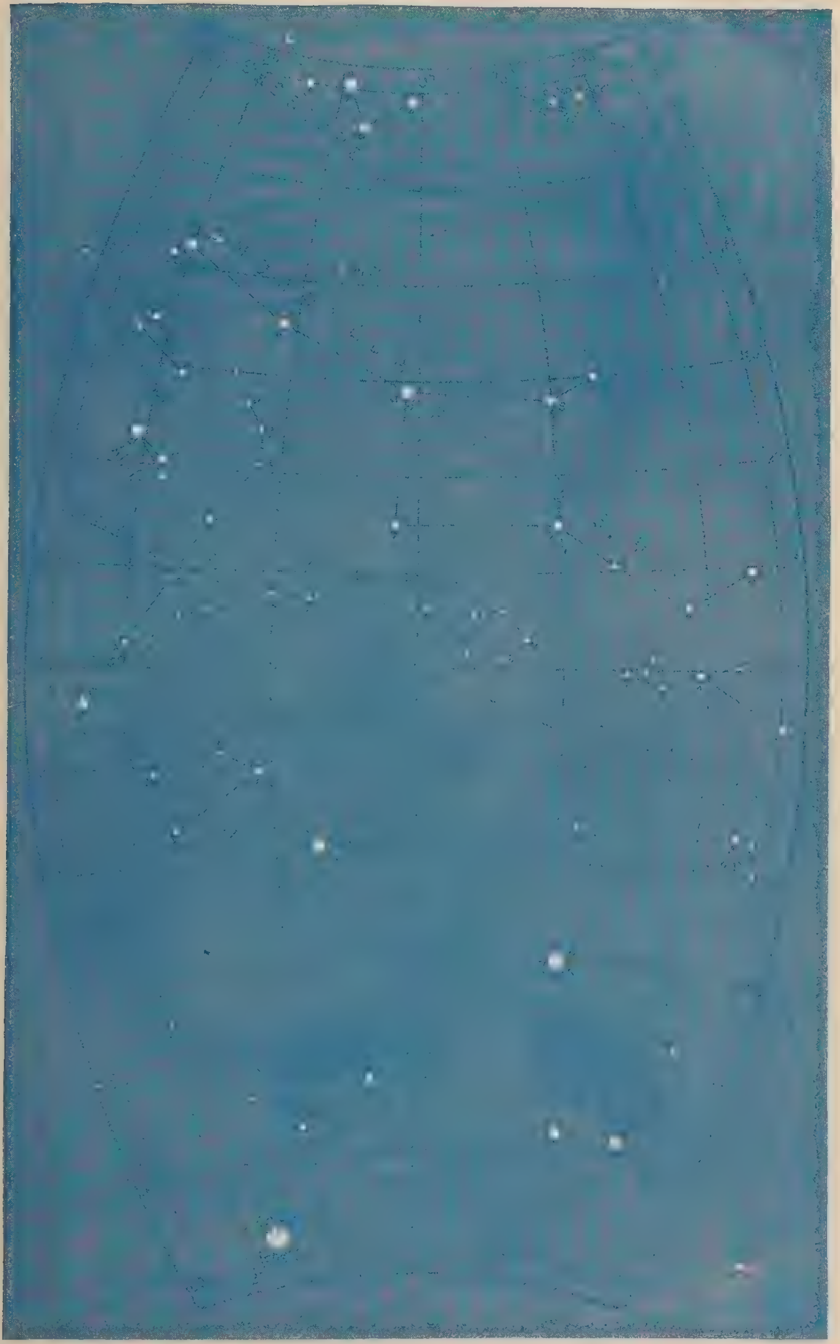
Map 5



MAP 6



Map 7



Map 8

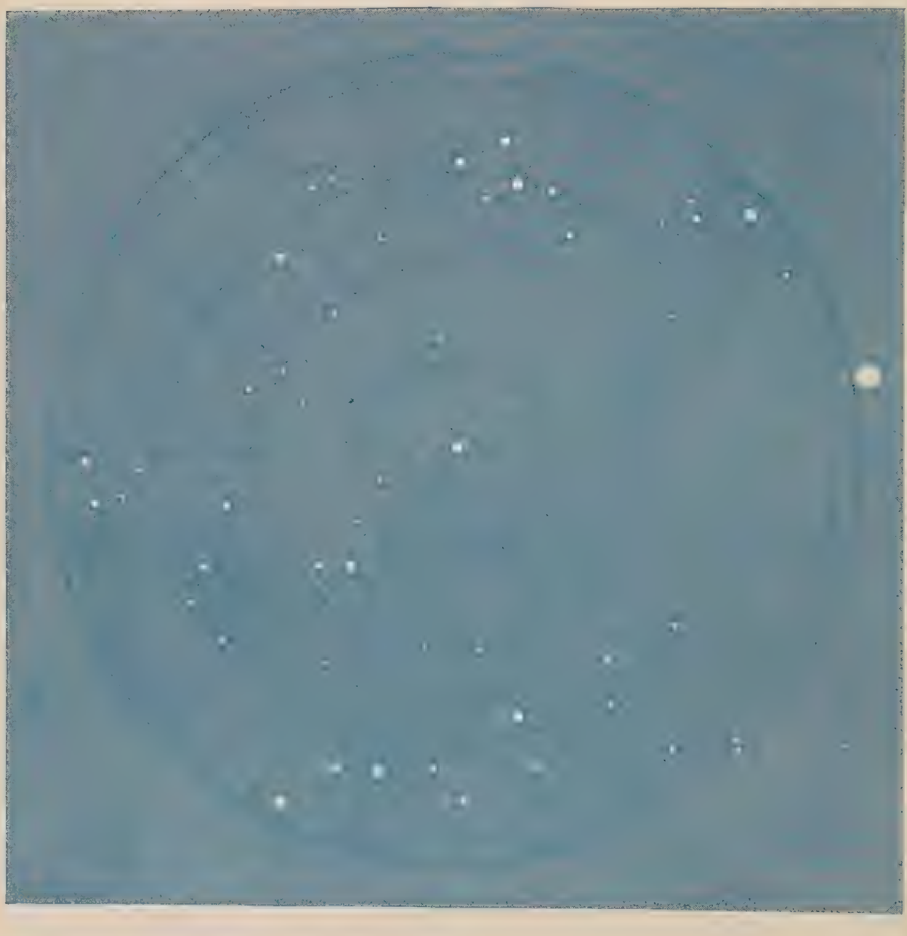


TABLE I

THE CONSTELLATIONS

Name	Genitive	Abbreviation[1]
Andromeda	Andromedæ	And
Antlia[2]	Antliæ	Ant
Apus[2]	Apodis	Aps
Aquarius	Aquarii	Aqr
Aquila	Aquilæ	Aql
Ara	Aræ	Ara
Argo (Navis)[3]	Argus	
Aries	Arietis	Ari
Auriga	Aurigæ	Aur
Bootes	Bootis	Boo
Cælum[2]	Cæli	Cæ
Camelopardalis[2]	Camelopardalis	Cam
Cancer	Cancri	Cnc
Canes Venatici[2]	Canum Venaticorum	CVn
Canis Major	Canis Majoris	CMa
Canis Minor	Canis Minoris	CMi
Capricornus	Capricorni	Cap
Cassiopeia	Cassiopeiæ	Cas
Centaurus	Centauri	Cen
Cepheus	Cephei	Cep
Cetus	Ceti	Cet
Chamæleon[2]	Chamæleontis	Cha
Circinus[2]	Circini	Cir
Columba[2]	Columbæ	Col
Coma Berenices[2]	Comæ Berenices	Com
Corona Australis	Coronæ Australis	CrA
Corona Borealis	Coronæ Borealis	CrB
Corvus	Corvi	Crv
Crater	Crateris	Crt
Crux[2]	Crucis	Cru
Cygnus	Cygni	Cyg
Delphinus	Delphini	Del
Dorado[2]	Doradus	Dor
Draco	Draconis	Dra
Equuleus	Equulei	Equ
Eridanus	Eridani	Eri
Fornax[2]	Fornacis	For
Gemini	Geminorum	Gem
Grus[2]	Gruis	Gru
Hercules	Herculis	Her
Horologium[2]	Horologii	Hor
Hydra	Hydræ	Hya
Hydrus[2]	Hydri	Hyi
Indus[2]	Indi	Ind
Lacerta[2]	Lacertæ	Lac
Leo	Leonis	Leo
Leo Minor[2]	Leonis Minoris	LMi
Lepus	Leporis	Lep
Libra	Libræ	Lib
Lupus	Lupi	Lup
Lynx[2]	Lyncis	Lyn
Lyra	Lyræ	Lyr
Mensa[2]	Mensæ	Men
Microscopium[2]	Microscopii	Mic
Monoceros[2]	Monocerotis	Mon
Musca[2]	Muscæ	Mus
Norma[2]	Normæ	Nor
Octans[2]	Octantis	Oct
Ophiuchus	Ophiuchi	Oph
Orion	Orionis	Ori
Pavo[2]	Pavonis	Pav
Pegasus	Pegasi	Peg
Perseus	Persei	Per
Phœnix[2]	Phœnicis	Phe
Pictor[2]	Pictoris	Pic
Pisces	Piscium	Psc
Piscis Australis	Piscis Australis	PsA
Reticulum[2]	Reticuli	Ret
Sagitta	Sagittæ	Sge
Sagittarius	Sagittarii	Sgr
Scorpius	Scorpii	Sco
Sculptor[2]	Sculptoris	Scl
Scutum (Sobiescianum)[2]	Scuti	Sct
Serpens	Serpentis	Ser
Sextans[2]	Sextantis	Sex
Taurus	Tauri	Tau
Telescopium[2]	Telescopii	Tel
Triangulum	Trianguli	Tri
Triangulum Australe[2]	Trianguli Australis	TrA
Tucana[2]	Tucanæ	Tuc
Ursa Major	Ursæ Majoris	UMa
Ursa Minor	Ursæ Minoris	UMi
Virgo	Virginis	Vir
(Piscis) Volans[2]	Volantis	Vol
Vulpecula[2]	Vulpeculæ	Vul

[1] Adopted in 1922 by the International Astronomical Union.

[2] Of modern origin.

[3] This constellation, which is very large, is often divided into four: Carina (Car), Puppis (Pup), Pyxis (Pyx), and Vela (Vel). The sequence of Greek letters applied to the stars, however, is not broken.

CHAPTER I

THE CELESTIAL SPHERE

The Celestial Sphere.—When we look upward from some point where the view is unobstructed by roofs or trees, we seem to see what we call the sky, which is like a great hollow sphere with the observer at the center. The space between the heavenly bodies is really empty and the light of the sky is due to the air within less than a hundred miles of the Earth, which reflects to our eyes a little of the light of the Sun and stars, the blue more copiously than the other colors; and yet this **celestial sphere** seems so real and of so firm a construction that it is sometimes called the *firmament.* If we attempt to reach the sky or to find out how far away it is, its distance turns out to be greater than that of any of the heavenly bodies, prodigious as these distances are. For convenience in describing the apparent positions of the stars, astronomers speak of the celestial sphere as if it really existed, and conceive of it as having mathematically infinite dimensions, in comparison with which the observer and the whole Earth shrink to a mere point located at the center of the sphere.

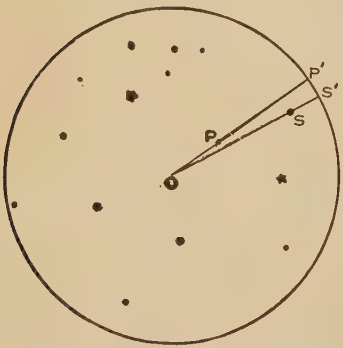


FIG. 1. APPARENT POSITION OF A HEAVENLY BODY

Apparent Position of a Heavenly Body.—The distances of the stars, the planets, and even the Moon are so great that they cannot be determined except by delicate and indirect measurements, and so, to all ordinary appearances, these distances are also infinite and the heavenly bodies appear as if set in the surface of the celestial sphere itself. Two bodies that are in reality separated by a vast space, as the star *S* and the planet P (Fig. 1), will, if nearly in the same direction from the observer O, appear close together, since S seems attached to the celestial sphere at S' and P at P'. The apparent position of a heavenly body is described by locating its projection upon the celestial sphere, and so only its *direction* from the observer is taken into account, its distance from him being disregarded.

Apparent, or Angular, Distance.—Distances laid out upon the surface of the infinite celestial sphere—for example, the apparent separation $P'S'$ of two heavenly bodies—can evidently not be expressed in such units as feet, miles, or kilometers, and such statements as that two stars "look to be about a foot apart" are without meaning unless the distance of the foot rule from the observer is specified. A foot rule would cover the space between the two Pointers of the Big Dipper if held about twelve feet from the eye; but if brought within four feet, it would appear larger and could cover a side of the Square of Pegasus. The apparent distance of P from S can in fact be definitely stated only as the difference of direction between the two objects as seen from O—that is, the angle POS or its equivalent arc $P'S'$.

The arc $P'S'$ is a portion of a circle whose center is O, the center of the sphere. Such a circle is called a **great circle,** since no larger circle can be drawn upon the surface of the sphere, although any number of smaller circles can be drawn parallel to it. A great circle divides the sphere into halves and lies in a plane that passes through the center. Apparent distances on the celestial sphere are always specified as arcs of great circles.

Units of Angular Measurement.—The most familiar unit of arc or of angular distance is the **degree,** the 360th part of a circumference or 1/90 of a right angle. It was chosen many centuries ago because it represents very nearly the apparent daily motion of the Sun among the stars. It is divided into sixty equal parts called **minutes,** and these into sixty **seconds** each, so that a degree contains 3,600 seconds. A side of the Square of Pegasus is about fifteen degrees long, the Pointers are about five degrees apart, and the diameters of the Sun and

Moon are each about thirty minutes. The smallest angular distance that can easily be perceived by the unaided eye is about $3\frac{1}{2}'$, which is the distance between the principal components of the multiple star ϵ Lyræ. Two stars that are only a second apart require a fairly good telescope to show them separately, and yet the results of modern astronomical measurements are reliable to the hundredth of a second—the angle subtended by the diameter of a ten-cent piece at a distance of about two hundred miles.

Certain angular distances in the sky are conveniently measured by the angle through which the Earth rotates in a given time. These are often expressed in **hours, minutes,** and **seconds,** the hour being one twenty-fourth of a circumference, or fifteen degrees. The subdivisions of the hour are called minutes and seconds of **time** and must be carefully distinguished from those of the degree, which are only one-fifteenth as long and are called minutes and seconds of **arc.** The units of the "time" system are abbreviated by ^{h}, ^{m}, ^{s}, while those of the "arc" system are written °, ′, ″. Thus, $22°\ 18'\ 44''.25$ designates the same angular distance as $1^h\ 29^m\ 14^s.95$.

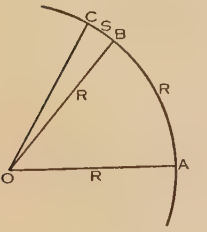
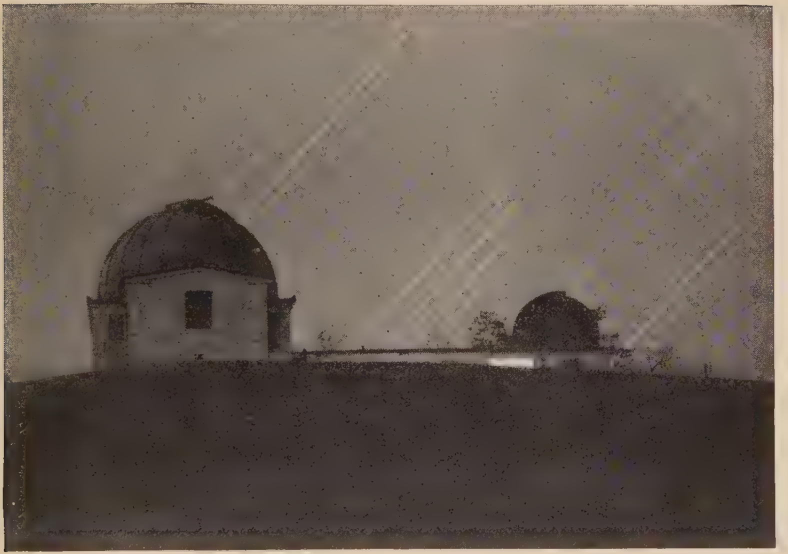


Fig. 2. The Radian

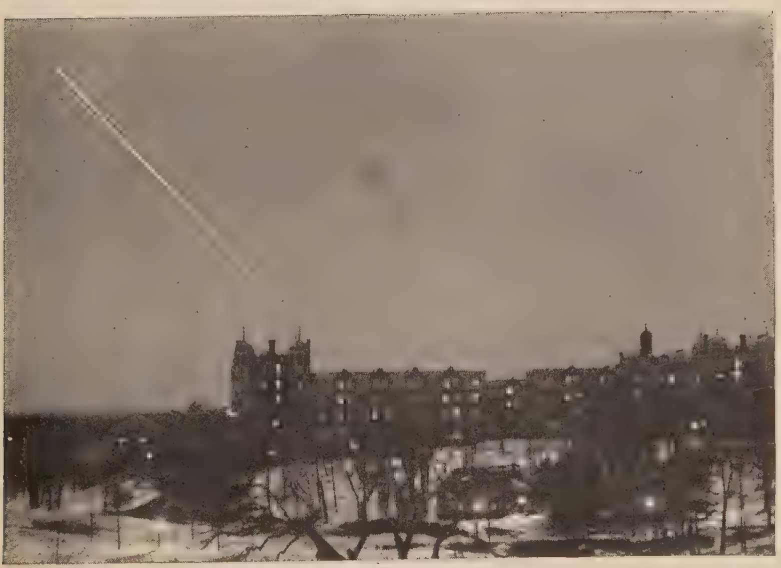
A third unit of angular measurement, very useful in computations, is called a **radian,** and is defined as the angle subtended at the center of any circle by an arc equal in length to the radius. In Fig. 2 the arc AB is laid off so that, measured along the curve, its length is equal to the radius, R, of the circle; hence, the angle AOB, with vertex at the center of the circle and sides extending to the extremities A and B of the arc, is a radian. Any other angle BOC is the same fraction of a radian that its arc BC is of AB, or of the radius R, because angles are measured by their subtending arcs; hence, in general, if we represent by R the radius of any circle and by S the length of an arc subtending an angle at its center, *the value of that angle in radians is* S/R.

There are 2π arcs of length R in the whole circumference,

PLATE I. PHOTOGRAPHIC STAR TRAILS (Whitin Observatory)



Orion rising



Venus setting

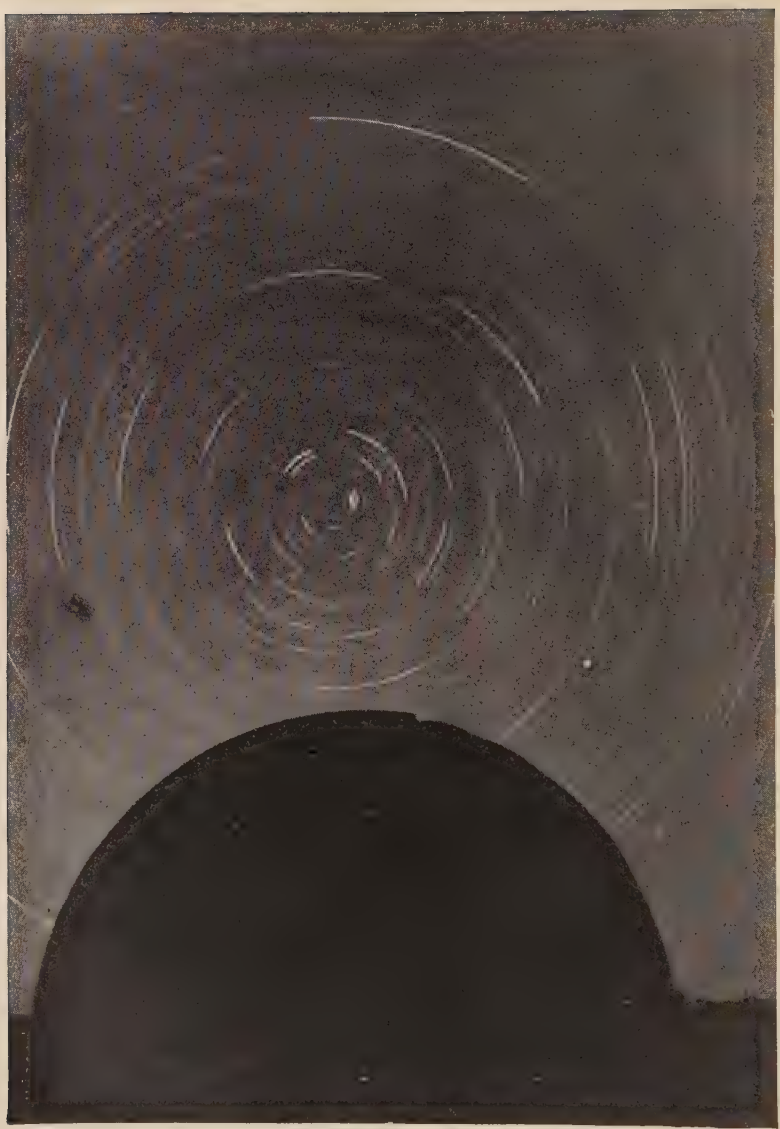


PLATE 2. POLAR STAR-TRAILS (Whitin Observatory)

and hence 360° = 2π radians. The value of a radian is therefore $360/2\pi$ degrees, and we may easily obtain

$$\begin{aligned} 1 \text{ radian} &= 57^\circ\!.3, \\ 1 \text{ radian} &= 3438', \\ 1 \text{ radian} &= 206{,}265''. \end{aligned}$$

These relations are so important that they should be memorized. An example of their usefulness is found in the solution of slender triangles without the use of trigonometric tables. In the triangle OMN, let the arc MP of a circle be drawn with center at O and with radius OM, and let the length of this arc be S. Then the value of the angle MON in radians is S/OM. If this angle be very small, the arc MP must be short and its curvature slight, so that its length is nearly equal to that of the side MN and also of PQ, and we may represent approximately any of these three lengths by S. Also, the sides OM and ON are nearly equal; call either of them D. Then, if x_r, x°, x', and x'' represent the value of the angle MON in radians, degrees, minutes, and seconds, respectively,

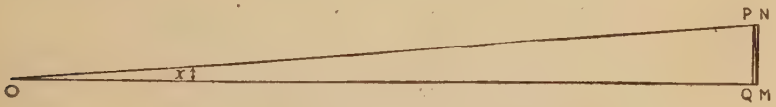


FIG. 3. SOLUTION OF A SLENDER TRIANGLE

$$\frac{S}{D} = x_r = \frac{x^\circ}{57.3} = \frac{x'}{3438} = \frac{x''}{206{,}265}.$$

If D represents the distance of a planet from the Earth and x its apparent diameter, S will be its real diameter, and any one of these three quantities may be obtained by the above relations if the other two are known.

For example, the apparent diameter of Mars is $24''$ when its distance from the Earth is 36,354,000 miles; its real diameter is therefore

$$S = \frac{24 \times 36{,}354{,}000}{206{,}265} = 4250 \text{ miles.}$$

The error involved is that of putting the value of an angle in radians $\left(\frac{PM}{D}\right)$ equal to that of its tangent $\left(\frac{MN}{D}\right)$ or its sine $\left(\frac{PQ}{D}\right)$. For angles less than 6′, it is no greater than that of using 6-place logarithmic tables, and even for angles as great as 50′ the error is no greater than that of using 4-place tables.

Practical Measurement of Angular Distance.—The actual measurement of apparent distances in the sky depends upon two principles—the physical principle that a ray of light in free space is straight and the geometric principle that the arcs of concentric circles included between two straight lines that meet at their common center have each the same value in degrees or other angular units as the angle between the lines.

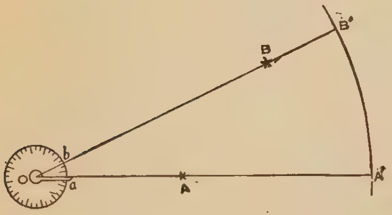


FIG. 4. PRINCIPLE OF THE ASTROLABE

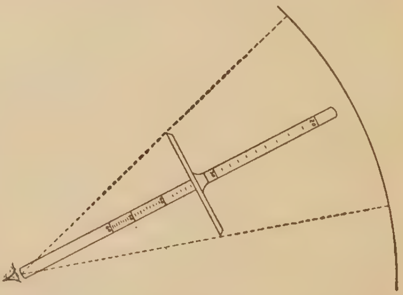


FIG. 5. PRINCIPLE OF THE CROSS-STAFF

One of the earliest of astronomic instruments, the **astrolabe,** which was used by the ancient Greek astronomers and for many centuries after them, consists in its simplest form of a circular plate, graduated to degrees around its circumference and having a movable pointer, called the **alidade,** pivoted at its center. To measure the angular distance between the stars *A* and *B* (Fig. 4) the circular plate is held in the plane of the observer and the two stars, and the position of the alidade is read upon the graduated circle, first when the alidade is pointed to *A* and again when it is pointed to *B*. The difference of the readings is the desired angular distance; for the center of the astrolabe, being on the Earth which is a mere point in comparison with the celestial sphere, is also the center of the sphere; and so the angle *AOB*, between the straight rays of light from the two stars, is measured by the arc *ab* between the two settings of the alidade, and this is equal to the arc *A′B′* of the great circle of the celestial sphere included between the apparent places of the two stars.

A second type of instrument, called the **cross-staff,** was also used by early astronomers and navigators. It consists of a straight staff of wood, about a meter long, with a cross-piece that slides upon it and is used as illustrated in Fig. 5. The end of the staff is placed near the eye and the cross-piece is adjusted so that it covers the arc in the sky which it is desired to measure. The number of degrees in this arc is the same as that in the angle formed at the eye by the rays of light that just pass the ends of the cross-piece; and this angle increases as the cross-piece is brought nearer the eye. Its value in degrees is read from a scale engraved on the staff.

Both the astrolabe and the cross-staff were used by Vasco da Gama and by Columbus. Measurements made with them can be relied upon to a limit of perhaps a tenth of a degree. The instrument commonly used for such measurements by modern navigators, which is accurate to less than a minute, is known as a sextant. It will be described in Chapter III (page 77).

Systems of Co-ordinates.—The position of a point in a plane may be completely described by two numbers which give the distances of the point from two lines or axes that intersect at right angles—a device familiar to every student of algebra. The two numbers are called **co-ordinates,** the horizontal distance being an **abscissa** and the vertical one an **ordinate.** A practical example is found in the method often used for designating an address in a city, the streets of which intersect at right angles. Thus, for a residence at 234 East 116th Street, 234 may be regarded as the abscissa and 116 as the ordinate.

For designating the positions of stars, astronomers conceive of the celestial sphere as covered with imaginary circles which intersect at right angles like the city streets. They are precisely analogous to the circles used for indicating the latitude and longitude of places on the Earth, which appear on all geographic maps. Four such **systems of co-ordinates** are in common use in Astronomy; they are known as the **horizon, equator, ecliptic,** and **galactic** systems.

Each system consists of a **fundamental circle,** which is a great circle of the sphere and for which the system is named; the **poles** of this great circle, which are the points of the sphere 90° from it; a set of **secondary great circles,** indefinite in number, which pass through the poles and cut the fundamental circle at right angles; and a set of **parallels,** small circles parallel to the fundamental circle. A point on the fundamental circle is chosen as **origin** and one co-ordinate of a star is counted from this point along the fundamental circle, while the other co-ordinate is measured along a secondary from the fundamental circle to the star. In the familiar geographic system the poles are the points where the Earth's axis intersects the surface of the Earth; the fundamental circle is the Earth's equator; the secondaries are the meridians,[1] which are cut at right angles by the parallels of latitude; the origin is the point where the meridian of Greenwich intersects the equator; and the co-ordinates of a place are its latitude and longitude.

The Horizon System.—The fundamental circle in the horizon system of co-ordinates is the **horizon** and its poles are the **zenith** and **nadir.** Their position is determined by the direction of gravity.

Suppose a weight is suspended by a string and allowed to come to rest. The string will take the direction of gravity, and if prolonged indefinitely upward will meet the celestial sphere at the zenith. The **zenith** is therefore defined as the point where the direction of gravity produced upward meets the celestial sphere.

The **nadir** is the point of the celestial sphere exactly opposite the zenith.

The **true horizon** is the great circle midway between the zenith and nadir, 90° from each. It may also be defined as the great circle along which a level plane meets the celestial sphere. The term horizon is loosely applied to the line where the sky seems to meet the surface of the Earth; this may properly be distinguished as the **visible** or **apparent** horizon. It is irregular in shape on land, and at sea it is a small circle lying below the true horizon.

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[1] The meridians, however, are not circles, but nearly ellipses.

Vertical Circles are the secondaries of the system and are defined as great circles passing through the zenith or at right angles to the horizon. The vertical circle that extends north and south is called **the meridian,** since it is the trace on the celestial sphere of the plane of the observer's geographic meridian; a more precise definition will be given later (page 19). The vertical circle that extends east and west is called the **prime vertical.**

Almucantars are small circles parallel to the horizon.

The co-ordinates of a star in the horizon system are its **azimuth** and **altitude.** As defined by the astronomer, **azimuth** is the arc of the horizon measured in the clockwise direction from the south point to the foot of the star's vertical circle. The south point of the horizon is thus chosen as the origin; its exact definition must be postponed a little (page 19), but the general idea is already familiar to everyone. Surveyors and navigators often prefer to measure azimuth, or "bearing," as they often call it, from the north point. Azimuth is expressed in degrees, and in astronomy is counted all the way around the horizon—*i.e.*, from 0° to 360°.

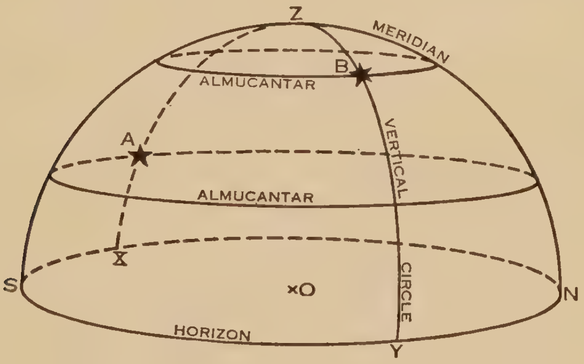


FIG. 6. THE HORIZON SYSTEM OF CO-ORDINATES

The **altitude** of a star is the arc of a vertical circle included between the star and the horizon, and is reckoned in degrees. The complement of the altitude is the **zenith distance.**

The base of Fig. 6 represents the horizon, O the observer, and Z the zenith. The azimuth of the star A is the arc SX—

about 35°—and its altitude is AX or about 25°. The azimuth of B is reckoned through the west and north around to Y and is about 250°, while its altitude BY is about 70°.

Practical Measurement of Altitude and Azimuth.—In the most common form of the ancient astrolabe, the instrument was supported by a ring held in the observer's hand so that the weight of the instrument caused it to hang vertically. The graduations of the circle being so arranged that the alidade read zero when pointing horizontally, its reading when pointed to a star gave the star's altitude—*i.e.*, the angle which its ray made with a horizontal line.

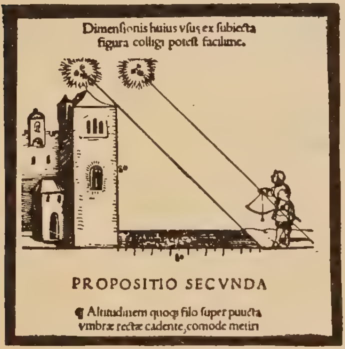

Dimenſionis huius vſus ex ſubiecta
figura colligi poteſt facilime.
PROPOSITIO SECVNDA
¶ Altitudinem quoq; filo ſuper puncta
vmbræ rectæ cadente, comode metiri

Fig. 7. Using a Small Quadrant

A more accurate measurement of altitude could be made with the **quadrant,** which was highly developed in the sixteenth century by the German Hevelius and the Dane, Tycho Brahe. This instrument had the form of a ninety-degree circular sector sometimes several feet in radius. It was arranged to turn in a vertical plane about a pivot at its center, from which point was suspended a fine plumb-line. Sights resembling modern rifle sights were attached to one edge and the arc was graduated to degrees and minutes, the zero being 90° from the edge that bore the sights. The altitude to which the sights were directed was read by using the plumb-line as an index on the graduated arc.

Fig. 7, from Regiomontanus's *De Trianguli,* 1532, shows the use of a small quadrant in measuring the altitude of the Sun for the purpose of calculating the height of a tower. The latter can be obtained from the Sun's altitude together with the length of the tower's shadow, since the Sun is so far away that its rays are sensibly parallel, and its altitude is thus equal to the angle at the base of the right triangle formed by the tower and its shadow. The height of the tower equals the length of the shadow multiplied by the tangent of this angle, and the value of the tangent may be found in trigonometric tables when the value of the angle is known.

At sea, the altitude of a heavenly body is found by measuring with a sextant the shortest distance of the body from the visible horizon and correcting for the "dip" (page 55), or depression of the visible horizon below the true, which depends on the observer's height above the water.

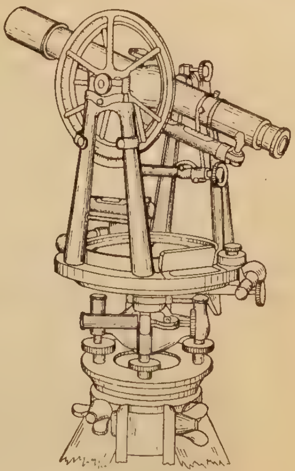


FIG. 8. A SURVEYOR'S TRANSIT

Altitude and azimuth may be determined from a single pointing by means of the **altazimuth instrument,** of which the modern **surveyor's transit** or **theodolite** (Fig. 8) is an example. It consists of a small telescope mounted on an axis which is in turn supported upon a second axis at right angles to the first. Each axis bears a graduated circle. The base of the instrument is provided with spirit levels by means of which the second axis may be made vertical; the circle attached to the first axis then reads the altitude of the point to which the telescope is directed. If the base is set with the zero of the horizontal circle due south, this circle reads the azimuth.

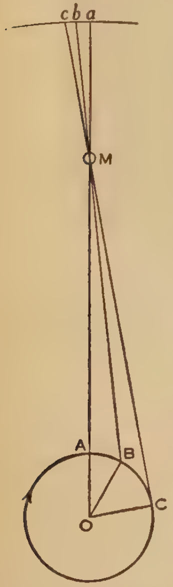


FIG. 9. GEOCENTRIC PARALLAX

Geocentric Parallax.—A few of the heavenly bodies are so near us that, in accurate work, they cannot be regarded as infinitely distant; for, when seen from different places on the Earth, such a body is projected to perceptibly different points of the celestial sphere. The angular distance between these points, which is the difference of direction of the body from the two observing stations, is called the body's **parallax.** In Fig. 9, let *O* and *M* be the centers of the Earth and Moon, respectively, and let *A* and *B* be two observers. The position of *A* is here chosen upon the line joining *O* and *M*; hence, *A* sees the Moon at the zenith and in the same direction as if he were at the center of the Earth. The point *a* on the celestial sphere, where the

Moon's center appears to A, is therefore called the **geocentric** position of the Moon, and the angle AMB, or aMb, is the Moon's **geocentric parallax** as seen from B. This parallax grows greater if the observer recedes from A until he reaches C, where the Moon appears upon the horizon. This maximum parallax, AMC, is therefore called the Moon's **horizontal** geocentric parallax. It may be defined as the apparent semi-diameter of the Earth as seen from the Moon.

The geocentric parallax of a body varies with the body's altitude from a maximum at the horizon to zero at the zenith, being proportional to the cosine of the altitude; also, it is greater for a body near the Earth than for a distant body. The horizontal parallax of the Moon is about one degree, but that of every other body of the Solar System is less than a minute of arc, while the geocentric parallaxes of the stars and nebulæ are far too small to detect.

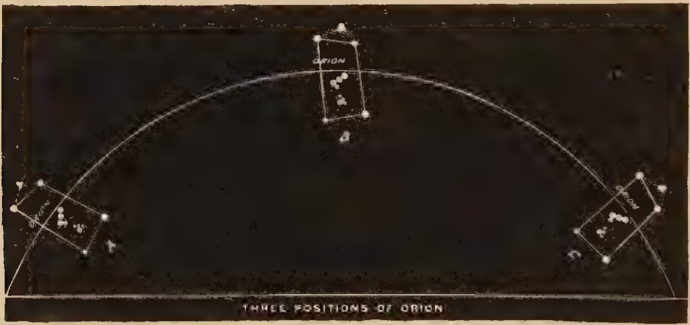


Fig. 10. Three Positions of Orion (From McKready: *A Beginner's Star Book*)

The positions of the bodies of the Solar System that are given in almanacs are geocentric positions, and so, to be strictly comparable with them, the observed positions of such bodies must be corrected for parallax. When the distance and apparent altitude of the body are known, the computation of this correction is a simple matter.

The Diurnal Motion.—If the Earth were flat and stationary, no other system of co-ordinates than the horizon system would be needed for describing the positions of the stars; but since the plane of the horizon is tangent to the Earth's surface, the motions of the Earth and also any change in the observer's geographic position must move his horizon and all its associated circles and so change the altitudes and azimuths of the heavenly bodies. The rotation of the Earth on its axis, which is completed each day, results in an apparent revolution of all the heavenly bodies around the Earth in the same time. This apparent revolution[1] is called the **diurnal motion.**

One result of the diurnal motion is the most obvious of all astronomical phenomena—the rising and setting of the Sun; but the manner of this rising and setting is not so obvious. At no place except the Equator does the Sun ever rise or set vertically, and for no observer outside the Torrid Zone does it ever reach the zenith. As seen within the latitudes of the United States, the Sun follows in the forenoon a path that slants upward toward the right, crosses the meridian south

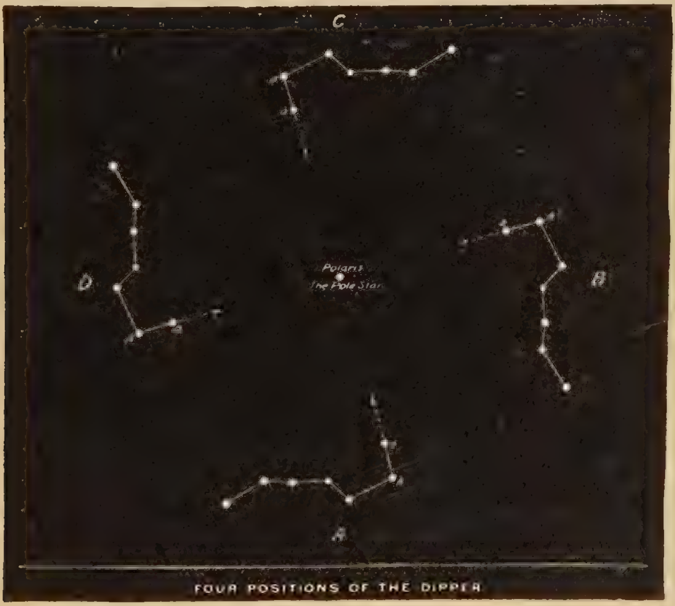

C
D
Polaris
The Pole Star
B
A
FOUR POSITIONS OF THE DIPPER

FIG. 11. FOUR POSITIONS OF THE DIPPER (FROM MCKREADY: *A Beginner's Star Book*)

[1] Throughout this book the word **rotation** will be used to designate a turning or spinning of a body around an axis that passes through the body itself, while **revolution** will mean motion around an external point.

of the zenith at midday, and descends toward the right in the afternoon—moving like the hands of a watch that lies on a sloping roof facing the north. Such stars as rise approximately in the east follow similar paths, and the constellation Orion, for example, may be seen on a December night successively in the positions shown in Fig. 10.

If one faces the north, he sees stars that do not descend below the horizon, but follow diurnal circles centered upon a point which, in the northern part of the United States, is about halfway between the horizon and the zenith. Four successive positions occupied by the Great Dipper at intervals of six hours are shown in Fig. 11. The stars far in the south, on the other hand, are visible but a short time, following diurnal arcs that extend but little above the horizon.

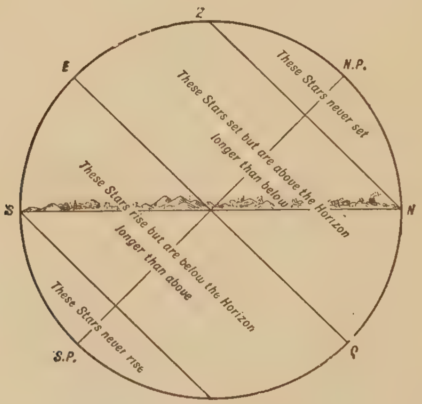

Z
E
N.P.
These Stars never set
These Stars set but are above the Horizon longer than below
S
N
These Stars rise but are below the Horizon longer than above
These Stars never rise
S.P.
C

Fig. 12. The Diurnal Motion (After Todd)

The nature of the diurnal motion is readily made evident by photographing the stars, using a fixed camera and exposing the plate an hour or more, so that the star images trail upon the plate. The three photographs reproduced in Plates 1 and 2 were obtained in this manner, the camera being pointed east, west, and north, respectively, in latitude 42°.3.

The diurnal motion is shown diagrammatically in Fig. 12. It is as if the whole celestial sphere, with the heavenly bodies attached to its inner surface, rotated each day around an axis passing north and south and inclined to the plane of the horizon

—just what the Earth is really doing, but in the opposite direction. Evidently, the celestial sphere must contain stars that are south of us and that never rise; and this inference is confirmed by the observations of people living farther south, to whom these stars become visible. The boundary of the portion of the celestial sphere containing stars that never rise is called the **circle of perpetual occultation**; that of the region of stars that never set, the **circle of perpetual apparition.**

The Equator System of Co-ordinates.—The diurnal motion both limits the usefulness of the horizon system of co-ordinates and affords a basis of another system, the **equator system.** The fundamental circle of this system is the **celestial equator** and its poles are known as the **north and south poles of rotation,** or often simply the poles.

The **poles of rotation** are those points in the sky about which the diurnal motion takes place, or, more accurately, those points which have no diurnal motion. They may also be defined as the points where the Earth's axis, produced, intersects the celestial sphere. The **north** pole is the one in the constellation of Ursa Minor around which the diurnal motion appears counter-clockwise.

The **celestial equator** is the great circle of the celestial sphere that lies midway between the poles—90° from each.

Hour circles are great circles that pass through the poles. Each hour circle in turn, as the diurnal motion proceeds, passes through the zenith, when it coincides with the **meridian,** which therefore is common to the two systems and may be defined as the hour circle that passes through the zenith or as the vertical circle that passes through the poles.

The **cardinal points** are the north, south, east, and west points of the horizon. The north point is that one of the intersections of the meridian with the horizon which is the nearer to the north pole. The cardinal points are 90° apart, the south being opposite the north, and the east being at the observer's right when he faces the north.

Parallels of declination are circles parallel to the celestial equator.

The **declination** of a star is the arc of an hour circle included between the star and the celestial equator. It is reckoned in degrees and is considered + if the star is north of the equator and – if south. Declination is often abbreviated by the Greek letter δ. Since the diurnal motion takes place parallel to the celestial equator, it does not change the declination of the star as it does the altitude and azimuth.

The **hour angle** of a star is the arc of the celestial equator included between the meridian and the star's hour circle. It is usually measured westward so that it continually increases with the time. The rate of increase of hour angle is uniform, and in fact can be measured by a clock, and so the hour angle is usually expressed in hours.

It is very desirable to use co-ordinates which, unlike the altitude, azimuth, and hour angle, are not changed by the diurnal motion. We have seen that the declination is not changed, and we may obtain an equally permanent co-ordinate to accompany it if, instead of measuring the arc of the equator from the meridian, we measure it from a point that is carried along by the diurnal motion. The point chosen by astronomers for this purpose is called the **vernal equinox,** and is located on the celestial equator in the comparatively starless region just south of the Square of Pegasus, as shown in Fig. 13. Its exact definition will be given on page 25.

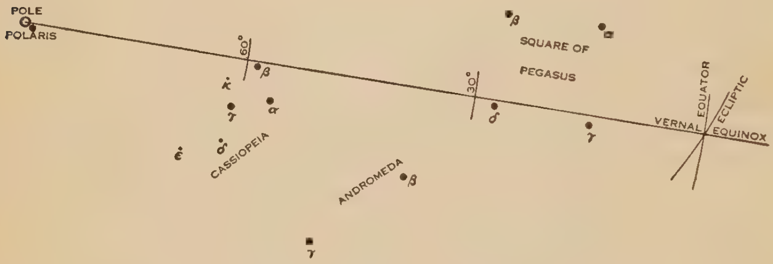


FIG. 13. HOW TO LOCATE THE VERNAL EQUINOX

The **right ascension** of a star is the arc of the celestial equator included between the vernal equinox and the star's hour circle. It is reckoned eastward from the vernal equinox and is usually expressed in hours. Its common abbreviation is the Greek letter α.

The Equatorial Telescope.—Most large telescopes are supported on what is called an **equatorial** mounting, the principle of which may be seen in Plate 3, which is a photograph of the great refractor of the Lick Observatory. A massive pier supports an axis of steel which is placed parallel to the Earth's axis and therefore points to the pole of rotation and is called the **polar axis** of the instrument. Attached to this is a steel sleeve that holds the **declination axis** in a direction at right angles to that of the polar axis; and the tube of the telescope is fixed at right angles to this. Motion of the telescope around the declination axis causes it to sweep along an hour circle, and the angle turned through from the equator is the declination of the star to which the telescope is directed, which may be read upon a graduated circle, called the **declination circle,** attached to the declination axis. Motion around the polar axis is parallel to the equator, and the hour angle may be read upon the graduated circle, called the **hour circle** (although not corresponding to the hour circles in the sky) attached to the polar axis. To follow a star in its diurnal motion, the instrument need only be turned about the polar axis, the declination setting being kept fixed; and, since the motion in hour angle is uniform, the telescope is ordinarily provided with a clock which keeps a driving wheel upon the polar axis turning westward at a uniform rate of one revolution in twenty-four hours. When the telescope is once set on an object the polar axis is clamped to the driving wheel, which thus keeps the object in the field of view.

To set the equatorial telescope on a star, whether the star is visible to the unaided eye or not, it is only necessary to know the star's declination and its hour angle at the moment, and to set the circles accordingly. On some modern equatorials, a second circle is mounted on the polar axis in such a way that its zero may be made to correspond with the vernal equinox instead of the meridian. So long as the clock is kept running correctly, this circle reads the right ascension.

Sidereal Time.—The period of one rotation of the Earth relative to the stars, which is the time interval occupied by one complete apparent rotation of the celestial sphere, is called the **sidereal day.** It is about four minutes shorter than the ordinary solar day, and is divided into twenty-four sidereal hours, each a little shorter than an ordinary solar hour. **Sidereal noon** occurs at the moment when the vernal equinox is on the meridian, which is at different times of the day or night at different times of year. The **sidereal time** at any moment is the hour angle of the vernal equinox, or, what is exactly the same thing, the right ascension of the meridian. The sidereal time is therefore the interval in sidereal hours, minutes, and seconds since sidereal noon.

The hour angle and right ascension of a star are connected very simply with the sidereal time; for, the hour angle being measured westward from the meridian and the right ascension eastward from the vernal equinox, each to the star's hour circle, their sum is the angular distance between the meridian and equinox (Fig. 14)—the hour angle of the vernal equinox, or the right ascension of the meridian, which we have already defined as the sidereal time. This is a relation that is often used. In setting an equatorial telescope on a planet, for example, the right ascension and declination for the date are found in an almanac and the sidereal time is read from an astronomical clock. Subtracting the right ascension from the sidereal time gives the hour angle, which with the declination gives the necessary settings.

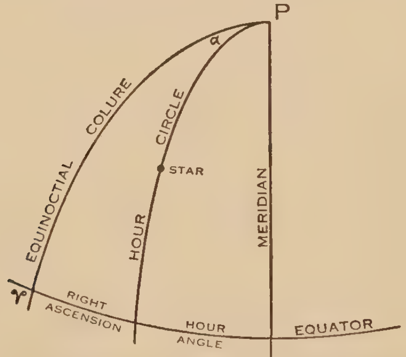


FIG. 14. RIGHT ASCENSION, HOUR ANGLE, AND SIDEREAL TIME

Solar Time.—Sidereal time would be inconvenient for regulating the affairs of life, since its noon occurs at all times of the day and night. **Apparent solar time** is the hour angle of the

Sun, which differs from sidereal time by the Sun's right ascension, and is the time indicated by a properly erected **sun dial.** Many forms of this instrument are known, now valued chiefly for ornamental purposes. In each, a gnomon, or straight edge, placed parallel to the Earth's axis, casts its shadow upon a dial which is graduated to hours and subdivisions and which is placed in such a way as to show the hour angle of the Sun. The interval between two successive meridian passages of the Sun is called the apparent solar day. Its length is not quite the same throughout the year, and so our clocks are regulated to **mean solar time,** which is the hour angle that the Sun would have if the solar days were all of their average length. The subject of Time is discussed more completely on pages 71 *et seq.*

The Annual Apparent Motion of the Sun.—In the temperate zones, a striking feature of the change of the seasons is the difference of the length of day and night in summer and winter. In midsummer in the northern hemisphere the Sun rises far to the north of the east point, crosses the meridian high in the sky, and sets far north of the west point, the part of its diurnal path that is above the horizon being much greater than that which is below, so that the day is longer than the night. In midwinter, it rises and sets far south of the prime vertical, reaching but a moderate altitude even at noon, so that the day is shorter than the night (Fig. 15). Evidently, the Sun crosses the celestial equator twice each year. A study of the constellations throughout the year shows that the apparent motion of the Sun is not directly northward or southward. In late September, for example, the constellation Scorpius is low in the southwest just after sunset, but by November this group cannot be seen and its place in the evening sky is taken by Sagittarius, which adjoins it on the east. By January, Sagittarius has disappeared and an early observer may see Scorpius rising in the southeast just before sunrise. During the four months, in fact, the Sun seems to have passed eastward through these constellations, and it may be traced in similar fashion until in the following September it will be found again at the starting point west of Scorpius. Measurements of the Sun's right ascension and declination show accurately the nature of its path on the celestial sphere, and it is found to be a great circle that is inclined to the equator at an angle of 23½°, in which the Sun moves always toward the east, completing the circuit in one year. Of course it is now known that this apparent motion is due to the revolution of the Earth, which, like all the other planets, moves in an orbit around the Sun.

We might represent the celestial sphere by a hollow globe with dots on it for the stars. The diurnal motion would be a spinning of the globe upon its axis, and the Sun might be represented by a trained firefly if it would accommodatingly creep around the globe in a direction making an angle of 23½° with the equator and opposite the diurnal motion, and at such a rate as to complete the circuit in 365¼ turns of the globe. The firefly would then be carried around by the globe, but more slowly than the dots of the star map, which it would pass on its way.

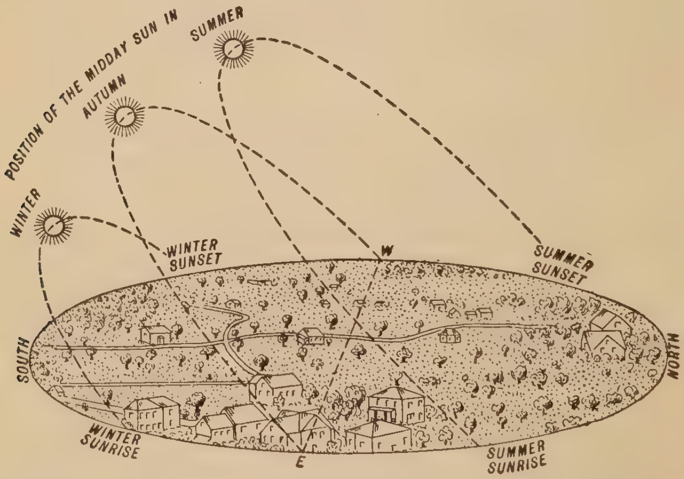


FIG. 15. DIURNAL PATHS OF THE SUN IN DIFFERENT SEASONS (AFTER TODD)

The Ecliptic System of Co-ordinates.—The apparent path of the Sun among the stars is called the **ecliptic,** the angle at which it intersects the equator (23½°) the **obliquity,** and the points of intersection the **equinoxes.** The equinox where the Sun

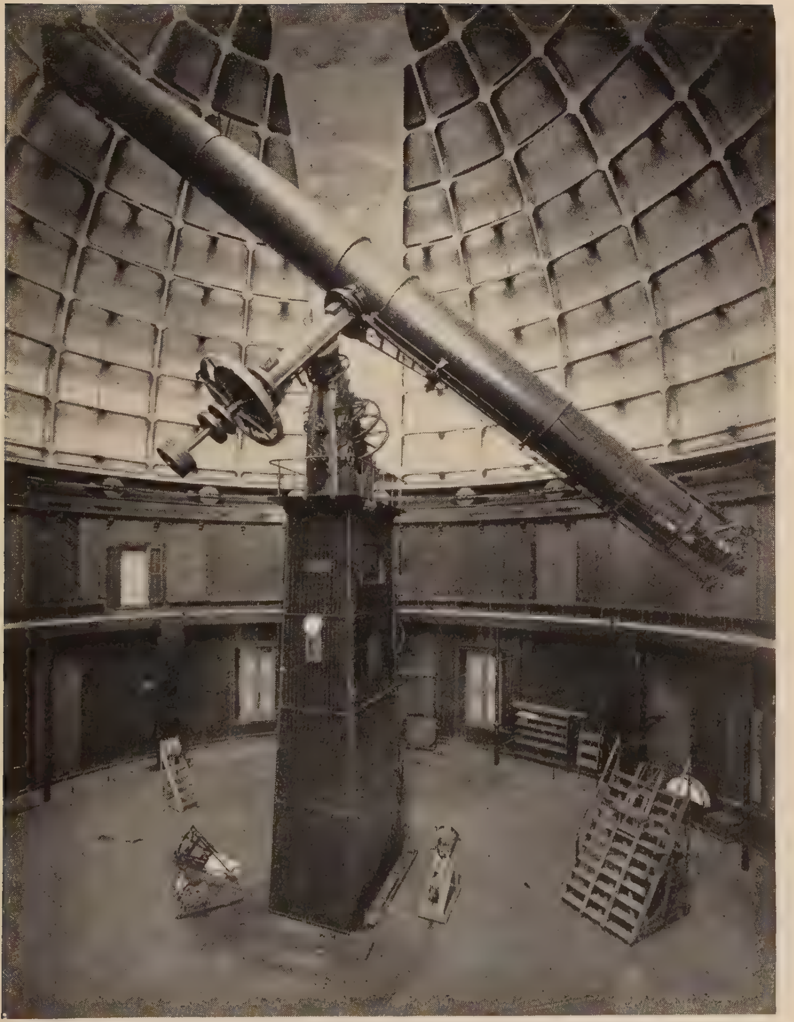


PLATE 3. THE 36-INCH REFRACTOR OF THE LICK OBSERVATORY

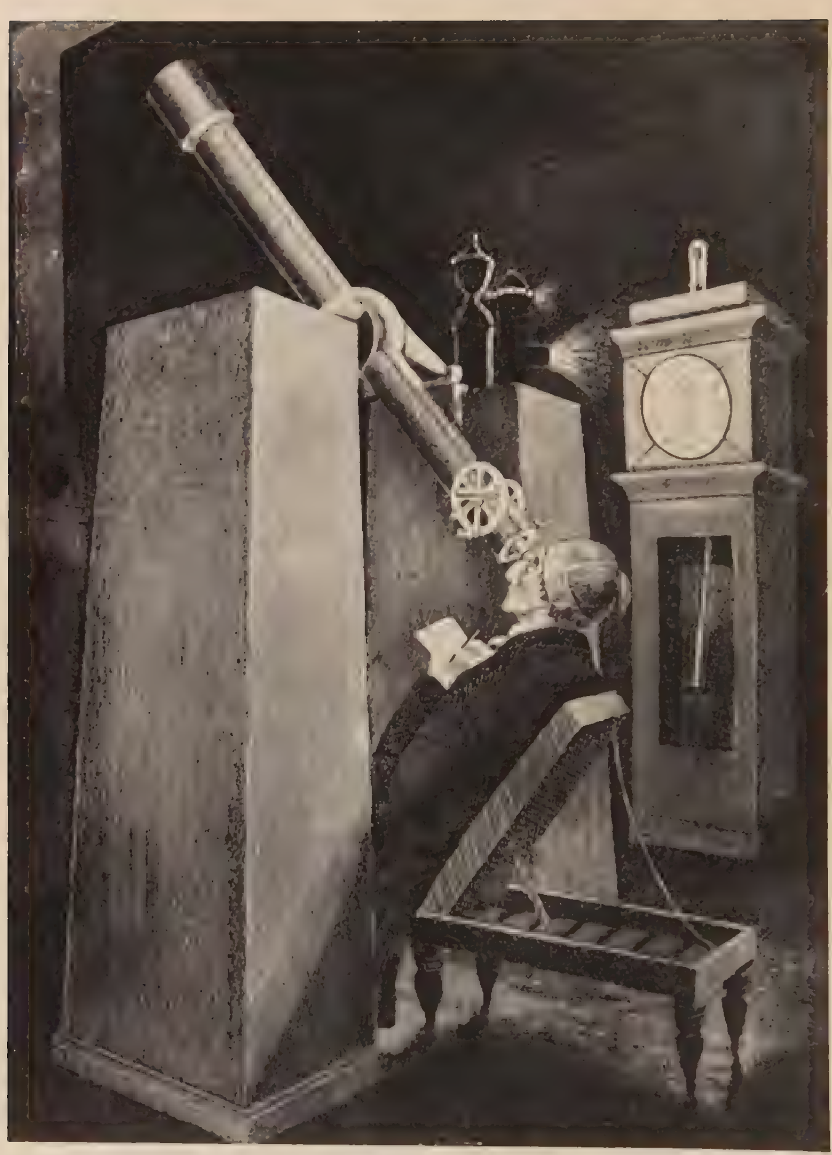


PLATE 4. A TRANSIT INSTRUMENT OF THE EARLY NINETEENTH CENTURY
From *Speculum Hartwellianum*, by W. H. Smyth

crosses from south to north of the equator, since it is passed by the Sun in the spring, is called the **vernal** equinox, and the other is the **autumnal** equinox. Ninety degrees from the equinoxes, as measured on the ecliptic, are the summer and winter **solstices.** The hour circles that pass through the equinoxes and solstices are known as the **equinoctial** and **solstitial colures,** respectively.

The belt of sky 18° wide which has the ecliptic as its central line is called the **zodiac** (circle of animals) because, from the earliest recorded times, its twelve constellations with the exception of Libra, the Balance (which was probably formed at a later epoch from the claws of the Scorpion), were given the names of living creatures. Within the zodiac are always to be found not only the Sun, but also the Moon and all the principal planets. It was divided into twelve parts of 30° each, which were called the **signs** of the zodiac and were named for the twelve zodiacal constellations; and the position of a planet was indicated by the sign in which it could be seen —a practice still followed by many makers of almanacs.

Ninety degrees from the ecliptic are the north and south **poles of the ecliptic,** and passing through them are the great circles called **secondaries to the ecliptic;** while small circles parallel to the ecliptic are called **parallels of latitude.**

The **celestial longitude** of a star is the arc of the ecliptic, measured eastward from the vernal equinox to the secondary that passes through the star. The star's **celestial latitude** is the arc of its secondary between the star and the ecliptic, + if the star lies north of the ecliptic and – if south.

The Galactic System of Co-ordinates.—Every watcher of the sky is familiar with the beautiful band of misty light that is known as the Milky Way or **Galaxy.** It is composed of a vast number of faint stars and a few nebulæ and is of great importance in studies of the distribution of the stars. Its central line is approximately a great circle of the celestial sphere which makes an angle of 62°.8 with the celestial equator. This circle is called the **galactic circle** and is the fundamental circle of the galactic system of co-ordinates. The north pole of the galaxy is in the constellation Coma Berenices, in right ascension $12^h\ 42^m$ and declination + 27°.2, and the opposite pole is in the constellation Sculptor, about 10° south of the bright star β Ceti. **Galactic latitude** and **longitude** are related to the galacict circle exactly as celestial latitude and longitude are related to the ecliptic. The origin from which galactic longitude is reckoned is the intersection of the galactic circle with the celestial equator, in $\alpha = 18^h\ 42^m$.

General Remarks on Astronomical Co-ordinates.—The co-ordinates of the horizon system, altitude and azimuth, may be said to depend on the personal outlook of the observer; they are based on the direction of gravity, which is very nearly the line joining the observer with the center of the Earth. Wherever the observer goes, he takes his zenith and his entire horizon system with him. Right ascension and declination depend on the direction of the Earth's rotational axis, and are the same for every observer on the Earth, while an observer on another planet would doubtless have a different set of equatorial co-ordinates depending on the plane of that planet's rotation. Celestial latitude and longitude are also on a terrestrial basis, but in this case it is the plane of the Earth's revolution that is the determining factor instead of the plane of its rotation. The position of the circles of the galactic system of co-ordinates depends on the structure of the visible universe of stars and would be the same for an observer on any planet of the Solar System or among its neighboring stars.

Probably the altitude and azimuth of stars are more easily estimated by the eye than are any of the other co-ordinates, since they are referred to the horizon, the position of which can be easily noted; but the co-ordinates of the equator system can be measured with greater precision than any of the others, and it is usual to obtain the others from them by a trigonometric transformation.

A better understanding of the different systems of co-ordinates may perhaps be obtained by a study of the comparison shown in Table 2 (page 27).

Star Maps and Celestial Globes.—A star map is a chart that shows the relative apparent positions of stars in the sky as an ordinary map shows the relative positions of places on the Earth. Like the terrestrial map, it represents on a flat surface bodies that lie upon a sphere; and as a result, if a large area is shown on a single map, there is considerable distortion. For this reason, in the best star atlases only a limited portion of the sky is shown in each map.

As in the terrestrial map, it is customary to represent north at the top; but since the star map represents objects seen overhead, the right-hand side of the equatorial maps is west instead of east. It must be understood

TABLE 2

COMPARISON OF SYSTEMS OF CO-ORDINATES

System	Geographic	Horizon	Equator	Ecliptic	Galactic
Basis	Rotation of Earth	Direction of gravity	Diurnal motion of celestial sphere	Earth's orbital motion	Structure of visible universe
Fundamental Circle	Terrestrial equator	True horizon	Celestial equator	Ecliptic	Central line of Milky Way
Poles	Terrestrial poles	Zenith and nadir	Poles of rotation	Poles of ecliptic	Poles of galaxy
Secondary Great Circles	Meridians	Vertical circles	Hour circles	Secondaries to ecliptic	Secondaries to galactic circle
Parallels	Parallels of latitude	Almucantars	Parallels of declination	Parallels of latitude	Parallels of gal. latitude
Co-ordinates	Latitude, φ Longitude, λ	Altitude Azimuth	Declination, δ Right ascension, α Hour angle	Latitude, β Longitude, λ	Galactic lat. Galactic longitude
Origin for second co-ordinate	Meridian of Greenwich	South point of horizon	Vernal equinox, meridian	Vernal equinox	Intersection of galactic circle and equator
Circles fixed with respect to	Surface of Earth	The observer	The stars	The stars	The stars

that "west" means the direction of the diurnal motion of the heavens, or contrary to that of the Earth's rotation; and hence it is really a circular direction, and on the north polar map "west" follows the parallels of declination counter-clockwise around the pole. Astronomers often use the words

preceding and **following** instead of west and east in describing directions in the sky.

The star maps in this book (Maps 1 to 8, between pages 4 and 5) are drawn on what is known as the **globular projection**, each map representing a portion of the sky approximately as if the stars had been plotted on the inside of a hemispherical shell and photographed from a point on the opposite side of its center, at a distance from the center of 1.7 times the radius of the hemisphere. The use of this projection and the fact that each map shows only a small section of the sky render the distortion of the constellation figures almost negligible. Hour circles, parallels of declination, the ecliptic, and the galactic circle are all drawn as dark lines. Right ascension, declination, celestial longitude, and galactic longitude are indicated by dark figures. To follow the stars in the order of their diurnal or seasonal appearance, the pages containing the equatorial maps (Maps 2 to 7) should be used in order from right to left.

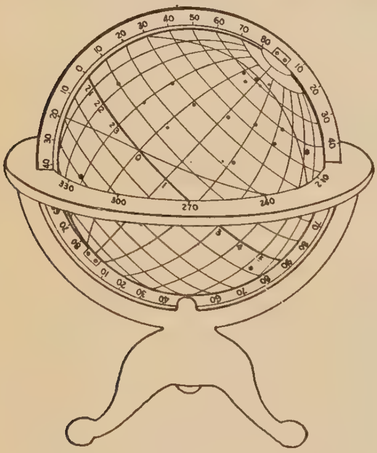


Fig. 16. A Celestial Globe

A celestial globe (Fig. 16) is a small model of the celestial sphere, on which the positions of stars are represented without distortion. It is only at great expense that a globe can be made large enough for the student to go inside of it, and the usual type is only a few inches in diameter. The map on the surface of the globe must therefore show the stars as they would appear if we could go outside the celestial sphere. If the globe be viewed from the north, "west" is the direction clockwise around the globe—the reverse of the case of maps in star atlases.

The globe is mounted on an axis passing through the poles of rotation. On its surface, in addition to the map of the stars, are printed the equator and ecliptic, and sometimes the galactic circle, together with either the secondaries to the ecliptic and parallels of latitude or the hour circles and parallels of declination. In either case, the two points where each of the twelve principal hour circles intersects the equator are indicated by numerals along the equator, reading from the vernal equinox toward the east from 0 to 24 and designating the hours of right ascension. The ecliptic is divided into degrees, and on most globes there are printed along the ecliptic dates showing the position of the Sun throughout the year. The globe is supported at the ends of its axis by a vertical ring, usually made of brass and called the brass meridian. The center of the globe lies in the plane of one side of this ring, and this side is graduated to degrees. The graduations on the upper half read from 0 at the equator to 90 at the poles to designate declination, while those on the lower half read from 0 at the poles to 90 at the equator and are used in setting the globe to correspond to the observer's latitude. The brass meridian is supported in a frame, the upper part of which is a ring that represents the horizon, which is also graduated to degrees to designate azimuth. A thin brass or paper quadrant, when attached to the zenith, or highest point of the brass meridian, represents a vertical circle. It is graduated to designate altitude. When set so that the altitude of the pole equals the observer's latitude (page 66) and the right ascension of the meridian equals the sidereal time, the globe exhibits correctly the aspect of the sky and is said to be "rectified."

Practical Determination of Sidereal Time, Right Ascension, and Declination.—The declination and hour angle of a star may be found with a considerable degree of accuracy from the circle readings of an equatorially mounted telescope; and if the sidereal time be known, the right ascension is obtained by subtracting from it the hour angle (page 22). Similarly, the sidereal time is obtained by setting the equatorial on a star of known right ascension, reading the hour angle, and adding hour angle and right ascension together. The tube of the equatorial, however, when turned into different positions, bends by a small amount that is difficult to determine, and the observed positions are affected by this and by the even more troublesome effects of atmospheric refraction (page 60).

The most accurate determinations of star positions and of sidereal time are made by observing the stars as they cross the meridian. This method was used extensively by Tycho Brahe before the invention of the telescope, and is practiced with the modern **meridian circle** and **transit instrument,** which are often referred to as the fundamental instruments of practical astronomy. The meridian circle is a small or moderate-sized telescope attached firmly at right angles to a rigid axis which is supported in a level, east-west direction on massive piers, and on which is mounted concentrically a finely graduated circle. The telescope and circle are thus free to swing in the plane of the meridian, but in no other direction. The transit instrument is like the meridian circle except that its circle is not so finely graduated and is used only for setting. Placed in the tube of the telescope in such a way as to appear to the observer along with the star (page 38), is a **reticle** which consists of threads of the finest spider-web or of lines ruled on a thin plate of optical glass with a fine diamond, and which is illuminated at night by a small lamp. An odd number of lines—five, seven, sometimes as many as fifteen—are placed at equal intervals in the north-south direction, and when the instrument is in perfect adjustment the middle line of the group lies exactly on the meridian. At right angles to these parallel lines are one or more similar lines to mark the center of the field (Fig. 17).

Since no instrument can be made perfect, it is never assumed that the middle wire follows the meridian with absolute precision. Its departure from the meridian can be expressed in terms of the "errors of the instrument," of which there are three: (1) the collimation error, the angle which the line joining the middle wire of the reticle with the center of the object glass makes with a plane perpendicular to the axis; (2) the level error, the angle made by the axis with the plane of the horizon; and (3) the azimute error, the angle made by the axis with the plane of the prime vertical. These errors can all be determined and allowed for, as explained in works on practical astronomy, but their discussion is outside the scope of this book.

For determining sidereal time or the right ascension of a star, the meridian circle or transit instrument is used by noting the reading of a clock at the instant the star is on the meridian. Instead of noting the time when the star crosses the middle line of the reticle only, a more accurate determination of the clock time of meridian passage is obtained by taking the average of the times of crossing the different lines. Since the star's hour angle at the moment of meridian passage is zero, its right ascension equals the sidereal time at that moment; and so, if the clock gives the correct sidereal time, the right ascension of the star is determined, or, *vice versa*, if the right ascension is known, the error of the sidereal clock is found by subtracting from its reading the star's right ascension. No effort is ordinarily made to set a sidereal clock "right"; it is better to leave it undisturbed and to keep a record of its error.

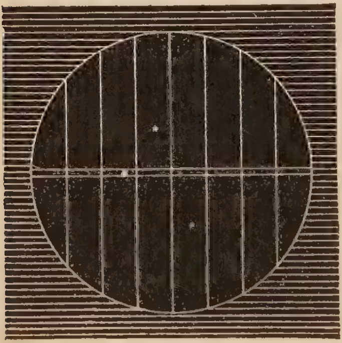


FIG. 17. THE FIELD OF VIEW OF A TRANSIT INSTRUMENT OR MERIDIAN CIRCLE

Declination cannot be determined with the transit instrument, but is measured with the meridian circle. For this, the reading of the circle when the telescope is directed to the pole must first be found by observing a circumpolar star as it crosses the meridian above the pole (upper transit) and again at lower transit, twelve hours later. When corrected for atmospheric refraction (page 60), the mean of these two readings gives the polar reading, which, so long as the meridian circle is undisturbed, remains unchanged. The angular distance of any star from the pole may then be determined by observing it at meridian passage and subtracting the reading of the circle (corrected for refraction) from the polar reading. This **polar distance** is the complement of the star's declination.

The best meridian circles of the present day are graduated with exquisite accuracy and are read by means of microscopes to the tenth of a second of arc. With modern astronomical clocks and transit instruments, sidereal time and the right ascension of stars may be determined to the hundredth of a second of time.

The positions of objects that cannot be conveniently observed with the meridian circle, such as faint comets that cross the meridian in daylight, are determined by measuring, with a micrometer (page 47) or upon a photograph, the *difference* of right ascension and of declination between the object and a star whose position is already known.

Locating the Equinox.—The meridian circle method of determining sidereal time requires a knowledge of the right ascension of the stars observed, while the determination of right ascension requires a knowledge of sidereal time. The reader may naturally inquire how the astronomer makes a start. The fact is that the right ascensions of hundreds of standard stars are now known with great accuracy and that the methods we have described are the ones actually used in practice; but since right ascension is measured from the vernal equinox, the system must rest ultimately upon a knowledge of the position of that point among the stars. The vernal equinox being defined as the point where the Sun crosses from the south to the north side of the equator, the determination of its position must be made by observations of the Sun. Observations of the Sun's declination throughout the year (but best near the times of the equinoxes) give an accurate value of the obliquity of the ecliptic, which, with the observed declination, gives by a trigonometric calculation the Sun's right ascension independently of the sidereal time or observations of the stars; hence, observations of the clock time of the meridian passage of the Sun made with the meridian circle simultaneously with those of the declination give the error of the sidereal clock, and the possession of this error provides for the determination of the right ascensions of the standard stars.

EXERCISES

1. At a distance of 39,080,000 miles, the planet Venus subtends an angle of 40″. What is the approximate diameter of Venus?

Ans. 7580 miles

2. At what distance must a toy balloon one foot in diameter be placed in order to appear the same size as the Moon (apparent diameter ½ degree)?

Ans. 115 feet

3. If two stars are seen at the same altitude below the pole by an observer in the United States, which is "east" of the other—the right or the left? Which, if the two stars are between the pole and the zenith?

4. What are the altitude, azimuth, and hour angle of the west point of the horizon? Of the north point?

5. What is the sidereal time when β Cassiopeiæ is directly above Polaris? Below Polaris?

6. What is the approximate solar time when β Cassiopeiæ is directly above Polaris on December 21?

7. What are the right ascension and declination of the north pole of the ecliptic?

CHAPTER II

THE OPTICS OF THE TELESCOPE

Invention of the Telescope; Its Importance to Astronomy.—The first telescope of which we have any authoritative account was made in 1608 by Jan Lippershey, a Dutch spectacle-maker. He made no astronomical application of his invention, but in 1609 news of this wonderful instrument which could extend the power of the human eye reached Galileo Galilei, the great Florentine physicist, who at once discovered the principle for himself, made a number of telescopes, and within a year had used them to make discoveries that revolutionized the astronomy of his time. With improvements in the making of telescopes their usefulness has increased, and at the present time almost all observations that contribute to the advancement of astronomy are made with the help of this instrument.

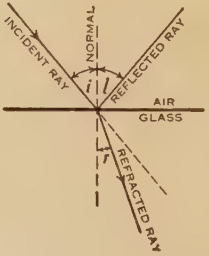


FIG. 18. ANGLES OF INCIDENCE AND REFLECTION

The telescope is essential for three chief purposes: first, for revealing objects or their details that are too faint or too distant for study with the unaided eye; second, to serve as the most accurate form of pointer for determining the position of an object on the celestial sphere; and third, to gather and intensify the light of faint bodies for study in other instruments such as spectroscopes and photometers.

Reflection and Refraction of Light.—When light travels through empty space, as between the Sun and the Earth, or through a uniform transparent substance, it travels in straight lines. If a ray of light, traveling in such a medium, encounter the surface of a second medium, say a piece of glass, the ray will be divided (Fig. 18); one part will be turned back into the first medium and the remainder will pass into the second, If this second medium be homogeneous like the first, the portion of the original ray that enters it will also travel in a straight line, but not, in general, the same straight line as that of the original ray. In other words, the light is bent abruptly at the common surface of the two media. This bending is called **refraction,** and the turning back of the light into the first medium is called **reflection.**

The angle i, made with the normal to the common surface of the two media by the original (incident) ray is called the **angle of incidence;** that made by the reflected ray, the **angle of reflection** (l); and that made by the refracted ray, the **angle of refraction** (r). The **law of reflection** states that the angles of incidence and reflection are equal. The **law of refraction,** known as Snell's law, states that the ratio of the sines of the angles of incidence and refraction is a constant which depends on the color of the light and the nature of the two media; that is,

$$\frac{\sin i}{\sin r} = \mu.$$

This constant, μ, is called the relative **index of refraction** of the two media.

Path of a Light Ray Through Glasses of Different Forms.—When light enters glass from air or empty space, it is refracted toward the normal, and when it passes from glass to air it is refracted away from the normal. In passing through a plate of glass with plane, parallel sides (Fig. 19), the two refractions are equal and the emergent ray is parallel to the incident ray. The total effect is thus to shift the ray sidewise without changing its direction.

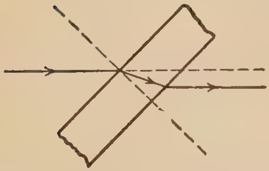


FIG. 19. PATH OF LIGHT THROUGH PLATE WITH PARALLEL SIDES

The case is different if the two glass faces through which the light passes are not parallel. This is true of the optical **prism,** a block of glass with three plane faces that meet in parallel edges (Fig. 20). Here the normal to the second face is not parallel to that of the first, and the ray suffers a deviation, which is always in a direction away from the edge in which the two refracting faces meet. Light of different colors is deviated by different amounts; for the index of refraction of glass is greatest for violet light and least for red, the indices for the different colors being in the order violet, blue, green, yellow, orange, red. White light is in reality a mixture of all the different colors, and so when white light is passed through a prism, it is spread out in a colored **spectrum,** red at the end toward the refracting edge and violet at the other. This property of transparent substances is called **dispersion,** and is exceedingly useful to astronomers, although somewhat troublesome to makers of telescopes.

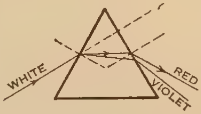


FIG. 20. PATH OF LIGHT THROUGH AN OPTICAL PRISM

Suppose that two prisms are placed with their bases together (Fig. 21) and illuminated by a beam of parallel rays—say the light of a star, which is always so far away that its rays that fall on the Earth are sensibly parallel. If the rays through the extreme parts of the combination could be made to deviate more than those through the middle part, all the rays might be made to meet. This is in fact accomplished by the **convex lens** (Fig. 22). The point where rays that were originally parallel are made to meet is called the principal **focus,** and its distance from the center of the lens, the latter's **focal length.**

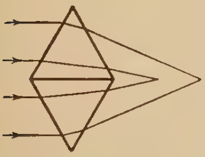


FIG. 21. PATH OF LIGHT THROUGH A DOUBLE PRISM

If the rays are not parallel, but diverge from a point within a finite distance from the lens, they are converged by the lens to a point more distant than the principal focus. Thus light diverging from the point *A*, Fig. 23, is focused to the point *B*. The points *A* and *B* are called **conjugate foci.** In the use of a projecting lantern, as in a moving-picture theater, one of a pair of conjugate foci of the projecting lens is occupied by the slide or film carrying the photograph, and the other by the screen on which the audience sees the picture.

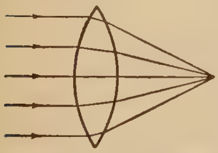


FIG. 22. PATH OF LIGHT THROUGH A CONVEX LENS

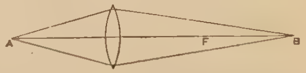


FIG. 23. CONJUGATE FOCI

Fig. 24 shows a **concave lens,** the effect of which upon parallel rays is to make them diverge from each other.

The Photographic Camera.—A photographic camera is, in its simplest form, a light-tight box with a convex lens in an

1-32

opening in one wall and means for holding a sensitive plate or film in the opposite wall, at a distance from the lens equal to its focal length. If the camera be directed at night to the sky, the parallel rays from each star within its field are brought to a focus upon the plate, which registers a dot in a position corresponding to that of the star. Each point of an extended object may be similarly recorded, the aggregate of the point-images making up a picture of the object. For "taking close-ups" of terrestrial objects, where each point of the object sends a divergent pencil of light to the lens, it is necessary to "focus" the camera by changing the distance of the plate from the lens, so that the plate and the object are at conjugate foci; and all except the simplest box cameras are provided with means for doing this.

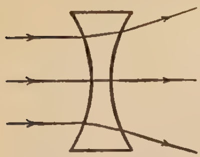


FIG. 24. PATH OF LIGHT THROUGH A CONCAVE LENS

The Human Eye.—Physiologically considered, the eyeball is a camera, in which light is focused by the **crystalline lens** and **cornea** upon the **retina,** a network of sensitive nerves which, instead of photographing the light, transmit to the brain the sensation produced by its presence. In the normal eye, when it is at rest, the retina lies in the principal focus of the lens-system. The near-sighted, or myopic, eye is too long, and in it the retina is behind the principal focus; in the far-sighted, or hypermetropic, eye it is in front.

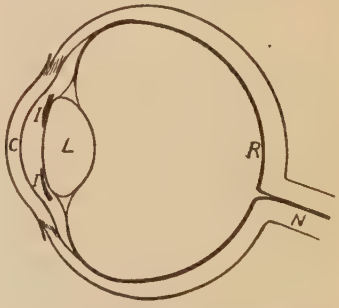


FIG. 25. DIAGRAM OF THE EYE *C*, CORNEA; *I*, IRIS; *L*, CRYSTALLINE LENS; *R*, RETINA; *N*, OPTIC NERVE

Light coming from a point less than a hundred feet from the observer, instead of being parallel, is perceptibly divergent; the normal, relaxed eye converges it to a point farther back than the retina, and for distinct vision it is necessary that the focal length of the lens be changed. This is accomplished by a muscular effort which causes the crystalline lens, which is elastic, to change its curvature, a process that is called by physiologists **accommodation.** Young children can accommodate their eyes to distinct vision at a distance of less than three inches, but the power is gradually lost, the "near point" being on the average four inches from the eye at the age of twenty, and nine inches at forty, while in old age it recedes to infinity and the power of accommodation is lost. This wonderful ability of the lens of the eye to change its focal length is not possessed by any artificial optical instrument.

The opening through which light enters the eye, which appears as a black circular hole at the center, is called the **pupil**; it is surrounded by the **iris,** the colored portion of the eye, which has the power of contracting so as to change the size of the pupil. In very bright light, the pupil is reduced to a mere pin-point, while in darkness it is enlarged to its maximum diameter, which averages about a third of an inch. In the observation of faint stars, it is probable that the pupil has about this diameter. This property of the eye is imitated in most camera lenses by the **iris diaphragm.**

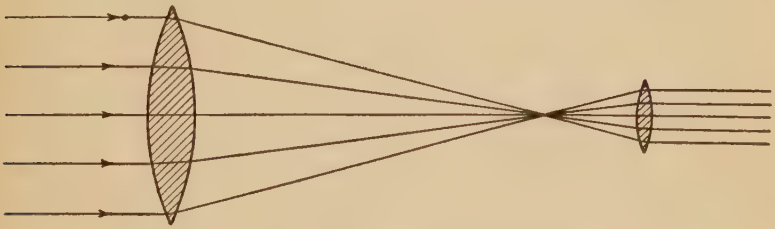


FIG. 26. SIMPLE REFRACTING TELESCOPE

Simple Refracting Telescope.—The action of an astronomical refracting telescope, in its simplest form, is shown in Fig. 26. The large lens, which receives the rays from the star, is called the **objective,** and the small lens the **eyepiece.** The objective brings the originally parallel rays together at its principal focus, forming an image of the star which can be seen as a bright point on a white screen or registered as a fine dot on a photographic plate held in the focal plane. In the photographic telescope, which is essentially a camera of great focal length, the objective is thus the only optical part needed. If not stopped by a screen or plate, the light proceeds through the focus in straight lines, which diverge beyond the focus. An eye placed in this cone of diverging rays receives light from the star, but in directions as if the star were a bright point situated at its image, so that the observer seems to see the star in the tube of the telescope, just before his eye—a rather interesting observation. In order, however, to receive in the pupil (never more than one-third inch in diameter) all the light that has come through the objective, the eye must be placed very close to the focus, closer than its power of accommodation could allow for. To obviate this difficulty the eyepiece is added.

The astronomical eyepiece is essentially a short-focus convex lens which, for normal vision, is placed at a distance from the focal plane of the objective equal to its own focal length, so that the distance between the two lenses is equal to the sum of their focal lengths. The property of the lens by which it converges parallel rays to its principal focus is thus reversed, and the rays that diverge from the image of the star emerge from the eyepiece in a parallel beam. If the focal length of the eyepiece is small enough, so that the lens is placed near the apex of the cone of rays, the emergent beam is narrower than the pupil of the eye, which thus may receive all the light transmitted through the large objective.

The eyepiece is ordinarily supported in a draw-tube that slides within the tube supporting the objective so that the instrument may be "focused" for the eyes of different observers by changing the distance between the objective and the eyepiece.

The Telescope as a Pointer.—One difficulty in using sights or pointers for determining the direction of a star or other object, as was done in the ancient astrolabes, arose from the fact that the eye cannot be accommodated for clear vision of the near-by sight and the distant object at the same time. This difficulty is eliminated in the telescope by placing in the common focus of the objective and eyepiece a pair of crossed spider-threads or other form of reticle and illuminating it with a lamp. The light reflected from the threads, since it diverges from the focus of the eyepiece, is made parallel by the latter, and so the threads are seen, not in their real position just before the eye, but as exquisitely fine, bright lines drawn on the surface of the sky, among the stars. By using the intersection of the threads as a fiducial point the direction of the line of sight is thus made definite.

Rays from Two Stars; Magnifying Power.—Suppose the telescope directed to a close pair of stars, one above the other and let the parallel rays *AAA* come from the upper of the two stars and the rays *BBB* from the lower (Fig. 27). The rays that pass through the center of the objective, like the ray passing through the plane plate in Fig. 19, are undeviated, and this central ray from the upper star meets all the others from that star in the focal plane at *a*, while the rays from the lower star are focused at *b*. After diverging from the image the rays from each star are bent toward the thick, central part of the eyepiece and emerge parallel to the line that joins the image with the center of the lens. An eye placed behind the eyepiece thus sees star *A* in the direction *xa* and star *B* in the direction *xb*. One effect of the telescope is therefore to invert the image, the upper star appearing below and the lower one above. Though objectionable in viewing terrestrial objects, this inversion causes no inconvenience in studying heavenly bodies, whose position with respect to a vertical line changes in any case with the hour angle.

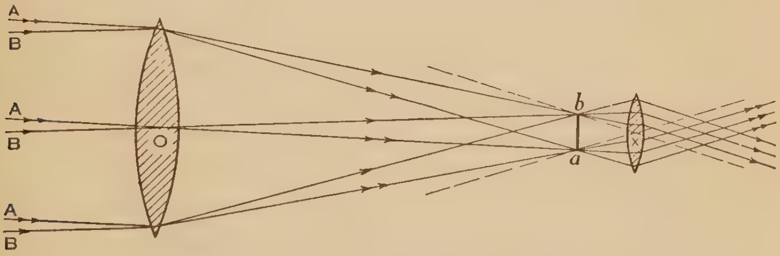


FIG. 27. MAGNIFYING POWER OF A TELESCOPE

The angular separation of the two stars, as seen by the naked eye, is the angle *AOB* which is equal to *aOb*; as seen in the telescope, their separation is *axb*, which is larger. In the figure, the angles are much exaggerated—two stars to be visible at the same time in a telescope must be more nearly in the same direction than here represented; hence, in practice, the angle *AOB* (or *aOb*) is small, and its value in radians may be taken as the distance ab divided by Oa or Ob. Similarly, the angle axb may be taken as ab divided by xa or xb. But Oa is equal to the focal length, F, of the objective and xa equals the focal length, f, of the eyepiece; hence, the **magnifying power,** which is the ratio of the apparent distance between the two stars as seen in the telescope to their apparent distance as seen by the unaided eye, is

$$M = ab/f \div ab/F = F/f.$$

That is, the magnifying power of a telescope is equal to the ratio of the focal length of the objective to that of the eyepiece. Since the eyepiece is small and inexpensive compared to the objective, most telescopes are provided with a number of eyepieces of different focal lengths and therefore of different magnifying powers.

It may be seen in Fig. 26 that M is equal also to the ratio of the diameter of the beam of star-light that enters the objective to that of the beam that emerges from the eyepiece; for these diameters are the bases of similar triangles having a common vertex at the principal focus and the focal lengths F and f as their respective altitudes. The lower the magnifying power, the larger is the emergent beam, and if too low a power is used the beam will be too large to enter the pupil of the eye and the full aperture of the objective will not be utilized. Assuming the maximum diameter of the pupil as a third of an inch, the minimum useful magnifying power is seen to be three times the diameter of the objective in inches.

Resolving Power.—So far, we have assumed that the telescopic image of a star is a geometric point and therefore without size, but this is not quite true. Owing to the fact that light consists of waves of sensible though exceedingly small length, the rays that meet at the focus of a lens "interfere" and produce a star image which is a small, bright disk surrounded by a series of concentric dark and bright rings, theoretically infinite in number, but growing fainter so rapidly with distance from the center that usually only one or two are visible and these only with a high magnifying power. The reason for this "diffraction pattern" is set forth in works on Physical Optics, but its discussion is beyond the scope of this book.

If two stars are so close together that their images overlap, they appear as a single object, although perhaps somewhat elongated, and an increase of magnifying power serves only to make the combined image larger without separating the stars. If the images could be made smaller, the two stars might be seen separated, or "resolved," by the telescope. Now, it is a fact that the larger the objective of the telescope the smaller is its image of a star, the diameter of the image being inversely proportional to that of the objective; hence, the **resolving power** of a telescope, or its ability to separate close stars, is directly proportional to the diameter of the objective. The angular distance of the closest pair of stars that can be resolved in a one-inch telescope is, according to Dawes, 4.5 seconds of arc; hence, in a telescope a inches in diameter, the minimum resolvable distance is

$$4''.5/a.$$

One might infer that the naked eye could resolve a pair of stars that were only $4''.5 \div \frac{1}{3} = 13''.5$ apart; actually, the minimum distance for resolution by the eye is much greater than this, the two stars of ϵ Lyræ, $3\frac{1}{2}'$ apart, being a pretty good test for acute vision. The reason for this lies not in the lenses of the eye, but in the structure of the retina, the sensitive elements of which are the microscopic "rods and cones." If a complete cone lies between the images of two stars, untouched by either, the stars are seen separately; if the images are too close for this to occur, the observer is conscious of but a single point of light. The action of a photographic plate, which is granular in structure, is in this respect analogous to that of the retina.

The resolving power of a telescope sets a practical limit to the magnifying power that can advantageously be applied to it; for there is no advantage in enlarging the image of a star beyond the point where it is easily perceived as a disk. Assuming that the eye can perceive a disk 180″ (a little less than the separation of ϵ Lyræ) in diameter, the maximum useful magnification in a telescope would be found by

$$4''.5/a \cdot M = 180'', \text{ whence } M = 40a;$$

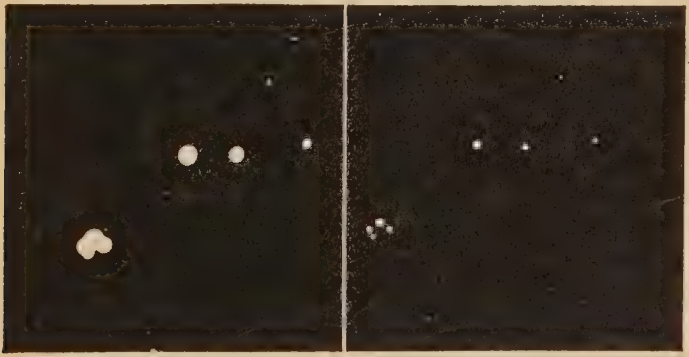
or, the greatest practicable magnifying power is forty times the diameter of the objective in inches. Greater magnification than this is in fact seldom used. It has already been shown, on the other hand (page 40), that the *minimum* useful magnifying power is $3a$.

"Bad Seeing."—One of the most obvious characteristics of a star, and one which appeals to poets far more than to astronomers, is its twinkling. This is a rapid fluctuation in the star's brightness, accompanied by minute and equally rapid changes of its apparent position and sometimes by variations of color. It is caused by motions in the air above us and is analogous to the effect that would be produced if we should view the star from the bottom of a clear lake, the surface of which was agitated by waves. The telescope magnifies the twinkling, so that it is usually great enough to interfere with the observation of fine detail or of stars that are close together. This condition is called **"bad seeing."** Its effects are worse in a large telescope than in a small one because the larger objective receives light from a wider air-path in which the total disturbance is greater. It often happens that a great telescope is rendered practically useless by bad seeing on a perfectly cloudless night. Fig. 28 shows two photographs of the "trapezium" of stars in the Orion nebula, made with the 100-inch Hooker telescope. The one on the right, made in fairly good seeing, shows two faint stars whose existence would hardly be suspected from the evidence of the other picture, which was made in poor seeing.

To the unaided eye, a planet usually does not seem to twinkle. This is because its angular diameter is appreciable, and while one point of its surface increases in brightness, that of another diminishes, keeping the total effect about constant. Bad seeing, however, is sufficiently evident in telescopic observation of planets, causing the fine detail to run together and the whole image to become blurred.

Since the aërial disturbances that cause bad seeing are worse in some localities than in others, this factor is very important in choosing a site for an observatory. Other desiderata are a large number of clear nights in the year and freedom of the air from dust and haze. A high altitude, to place the observer above a part of the air, a dry climate to assure cloudless skies, and distance from the smoke and artificial lights of cities are all desirable.

Brightness of the Image.—The **light-gathering power** of a telescope may be defined as the ratio of the amount of light from a given star entering the objective to the amount entering the pupil of the unaided eye. The amount of light that can pass through a given aperture is proportional to the area of the aperture, and therefore to the square of its diameter; and hence, calling the diameter of the objective a and that of the pupil b, we have for the light-gathering power,



Poor seeing   Good seeing

FIG. 28. TWO PHOTOGRAPHS OF THE "TRAPEZIUM" IN ORION

$$L = a^2/b^2.$$

If we let $b = \frac{1}{3}$ inch, the maximum diameter of the pupil, we have for the light-gathering power of a telescope of a inches aperture,

$$L = 9a^2.$$

The stars are all at such prodigious distances that, although their real diameters are very great, their angular diameters are less than that of the diffraction-image produced by the objective of any existing telescope; and, except when excessive magnification is used, the image is too small to be seen as a disk, and so the effective size of all star images on the retina is the same. The apparent brightness of a star as seen in a telescope as compared with its brightness as seen by the unaided eye depends, therefore, only on the light-gathering power. A ten-inch telescope makes a star appear 900 times, and a hundred-inch telescope 90,000 times, as bright as it appears to the unaided eye. Large telescopes thus disclose millions of faint stars that without their aid would be quite invisible.

The brightness of the image of a planet, nebula, or other object that presents a perceptible surface depends upon the focal length of the objective as well as upon its diameter; for the brightness is diminished by enlarging the image, and the diameter of the latter is proportional to the focal length, as may be seen in Fig. 27. The area of the image varies directly, and its brightness inversely, as the square of the focal length. Calling the focal length of the objective F and its linear aperture a, the brightness of the image of a planet or nebula in the focal plane is therefore proportional to a^2/F^2. The "speed" of an objective for photographing nebulæ and planets, or for ordinary landscape and portrait photography, is proportional to this ratio, while for photographing stars it is proportional merely to a^2. If the telescope is used visually, the area of the image on the retina is proportional to the magnifying power, and its brightness to a^2/M^2.

It can be shown that the brightness of a widely extended surface, which more than fills the field of view, cannot be increased by any optical device. Such an object is the sunlit sky. The reason we cannot ordinarily see the stars in the daytime is that the background of sky is too bright to afford sufficient contrast to the light of the star. The telescope, by increasing the apparent brightness of the stars without changing that of the sky, enables us to see the brighter stars in daylight without difficulty.

Chromatic Aberration.—It has already been pointed out (page 34) that, when light is deviated by refraction, it is dispersed into its component colors, the red being deviated the least and violet the most of the colors to which the eye is sensitive. A prism thus produces a spectrum. The dispersion that takes place in a simple convex lens produces a series of colored images of a star as shown in Fig. 29, the violet one being nearest the lens and the red one farthest removed. With an eyepiece focused on any one of these images, an observer sees a small bright point of the corresponding color surrounded by a halo that is composed of the out-of-focus light of the other colors. By thus enlarging the image of a star, the colored halo greatly impairs the resolving power of the lens, and it also makes it impossible to judge correctly the color of any object viewed. This defect of the simple lens, by which light of different colors fails to arrive at the same focus, is called **chromatic aberration.**

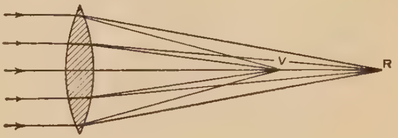


FIG. 29. CHROMATIC ABERRATION

Soon after Galileo's application of the telescope to the study of the sky, it was found that chromatic aberration could be mitigated by making the focal length of the objective very great in comparison with its diameter, and some telescopes of the seventeenth century attained fantastic lengths, up to two hundred feet. These long telescopes must have been exceedingly unwieldy, and their use led to no results commensurate with their size.

Achromatic Lenses.—In 1758 John Dollond, an English optician, patented a method of reducing the effects of chromatic aberration by combining two lenses made of different kinds of glass. The glasses ordinarily used for this purpose are called **crown** and **flint** glass, names derived from the manner in which the glasses were formerly made. Their principal constituents are silica (sand) and sodium sulphate or carbonate or potassium carbonate, to which is added lime in making crown glass and lead in making flint glass. Modern varieties of optical glass intended for special purposes contain also a great variety of other substances.

The refractive index of flint glass is greater than that of crown, and its dispersive power—*i.e.*, the difference of its refractive index for light of different colors—is also greater. By properly combining a convex crown lens with a concave flint one, a compound objective is formed which will bring to the same focus rays of light of any *two* colors that may be chosen. Such a lens is shown in Fig. 30. The colors to which the eye is most sensitive are those near the red end of the spectrum, and so objectives intended for visual use are made to combine the red and green rays; but the ordinary photographic plate is more sensitive to the blue end of the spectrum, and so photographic objectives are made to combine the blue and violet. Such compound lenses are called **achromatic** (color-free), but the images they produce are not wholly free from color, since the light of all colors except two is focused at different points. However, the colored halo around the image of a star produced by such a lens is not nearly so large or troublesome as in the case of a single lens. With some of the special modern glasses it is possible to produce a compound lens that is very nearly achromatic over the whole range of the visible spectrum, but these glasses have not yet been produced in blocks large enough to make a lens of very great size.

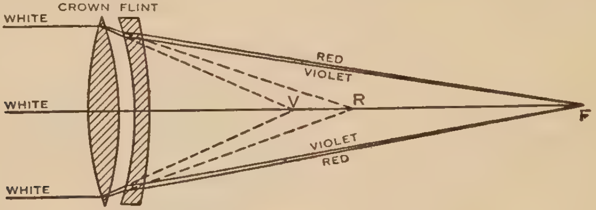


FIG. 30. ACHROMATIC OBJECTIVE

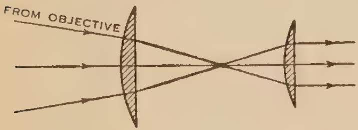


FIG. 31. NEGATIVE EYEPIECE

Eyepieces.—A simple convex lens used as an eyepiece introduces color and distorts the image unless the object is at the center of the field. The eyepieces of modern telescopes are usually composed of two lenses, and are usually of either of two forms, the **negative,** or **Huyghenian,** and the **positive,** or **Ramsden,** illustrated in Figs. 31 and 32, respectively. In the Huyghenian eyepiece, the rays from the objective are intercepted by the first of the two lenses before reaching

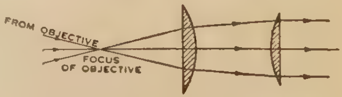


FIG. 32. POSITIVE EYEPIECE

a focus, and cross between the lenses; in the Ramsden form they reach their focus before encountering either lens. The Huyghenian eyepiece gives better definition at the center of the field, but of course cannot be used with a reticle; the Ramsden can be used with a reticle, and has a wider field.

Galileo employed in the first telescopes a concave eyepiece, placed between the objective and its focus. Such an eyepiece gives an erect image, but its field of view is very small if it is made of any considerable magnifying power. It is used in the modern opera glass, in which a high power is undesirable and inversion of the image would be objectionable.

The Filar Micrometer.—An important accessory to a visual telescope is the **filar micrometer,** an instrument used for measuring small angular distances. A rectangular frame, *A* (Fig. 33), is made to slide over a larger frame, *B*, by turning the fine-threaded screw *S*, which has a graduated head for reading small fractions of a revolution. The springs *P*, *P* take up the back-lash of the screw. Across each frame is stretched a fine thread of spider web, *T*, *T*, and these lines are illuminated by the little electric lamp *L*. The two frames are arranged to rotate about the center of the circular plate *C*, the edge of which is graduated to degrees read by the index *I* for determining the "position angle" (page 335) of the lines. A positive eyepiece is attached to the frame *A* and focused on the spider lines, and the plate *C* is attached to the tube of the telescope so that the lines lie in the focus of the objective. The parallel spider lines then appear as if drawn in light upon the sky, and the distance between the images of two close stars may be measured by setting one line on each star and reading the graduated head of the screw.

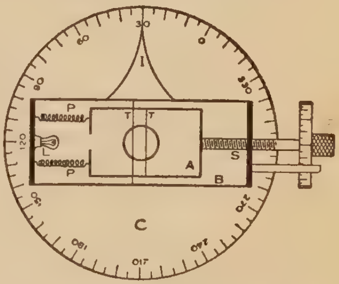


Fig. 33. Filar Micrometer

To reduce this distance to seconds of arc, it must be multiplied by the "micrometer constant," which is the number of seconds that corresponds to one complete turn of the screw. This constant may be determined by setting the lines parallel to an hour circle, separating them by a known number of turns, and finding the time required for a star on the equator to pass from one line to the other. The number of seconds of *time* in this interval, multiplied by fifteen, is the number of seconds of *arc* between the spider lines.

Accessories for Celestial Photography.—For photographing any celestial objects except the brightest ones, long exposures, often of many hours' duration, are necessary. To follow the diurnal motion during this time, the photographic telescope or camera must of course be equatorially mounted and clock-driven, but even then the position of the image upon the plate will be changed by the effects of atmospheric refraction, bad seeing, and the imperfections of the clock unless these errors are corrected by the observer. Short-focus cameras are often attached rigidly to a **guiding telescope** provided with crossed spider lines on which the image of the object being photographed is kept by the observer, using the slow-motion devices of the telescope. With long-focus instruments, a **double-slide plate carrier** is used. The plate is clamped to a rigid frame which may be moved parallel or perpendicular to the equator by fine screws in the hands of the observer. A positive eyepiece fitted with a pair of crossed spider lines is clamped firmly beside the plate, so that the intersection of the lines may be held upon the image of a "guide-star" located just outside the limit of the field that is being photographed. The difference in the use of the guiding telescope and the double-slide plate carrier is this: in the former the errors of following are corrected by moving the whole instrument, while in the latter only the plate and eyepiece, with their supporting frame, are moved.

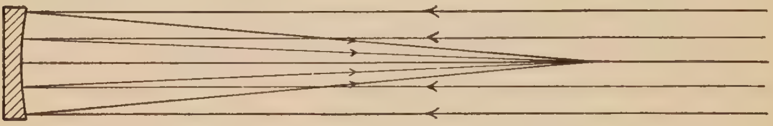


FIG. 34. PARALLEL RAYS REFLECTED BY A PARABOLIC MIRROR

Reflecting Telescopes.—Instead of a convex lens, it is possible to use as the objective of a telescope a concave mirror. In order to reflect parallel rays to a common focus, the cross-section of the surface of the mirror must have the form of a parabola (page 228). For a star situated on the axis of the parabola, the path of the light is as shown in Fig. 34. To receive the light from such an objective, the eyepiece or photographic plate would have to be placed inconveniently in the middle of the tube unless an additional reflection is provided for. Different ways of doing this give different types of telescopes. In the Newtonian type, a plane mirror set at an angle of 45° to the axis of the parabola intercepts the light before it reaches a focus and sends it to the side of the tube near the top. The observer must then stand on a high platform, if the instrument is large, and look in through the side of the tube. In the Herschelian form, the objective is inclined sufficiently to focus the light at the side of the tube, and a second reflection is avoided; but the image then lies at a distance from the axis of the parabola and is subject to considerable distortion.

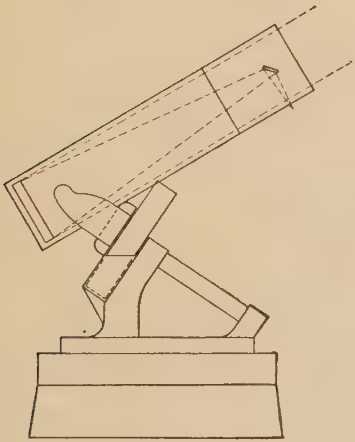


Fig. 35. 60-inch Reflector, Newtonian Form

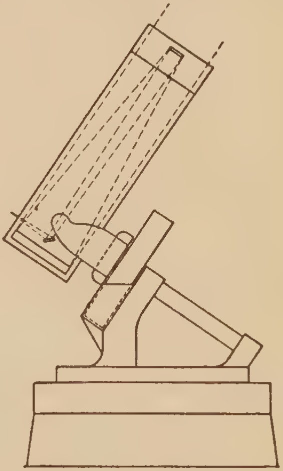


Fig. 36. 60-inch Reflector, Cassegrain Form

This form was used by William Herschel late in the eighteenth century, but is no longer employed. In the Cassegrain form a convex, hyperboloidal mirror at the upper end of the tube sends the rays back toward the objective, where they either pass through a hole in the latter, as in the earlier Cassegrains and in the great reflector of the Dominion Astrophysical Observatory, or are reflected to the side by a plane mirror placed just above the objective. Large modern reflectors are provided with a number of secondary mirrors mounted in removable sections of the tube, so that the type and equivalent focal length of the telescope may be changed. The sixty-inch reflector of the Mount Wilson Observatory is shown in the Newtonian form in Fig. 35 and in the Cassegrain form in Fig. 36.

Reflecting telescopes have been in use since the time of Newton, about 1670. During the first two hundred years the mirrors were usually made of speculum metal, a hard, brittle alloy of tin and copper; but modern mirrors for astronomical purposes are made of glass, covered on the front surface with a thin coating of metallic silver. If they were silvered on the back like an ordinary looking-glass, the image would be spoiled by the faint reflection from the unsilvered front.

Relative Advantages of Refractors and Reflectors.—As compared with the refracting telescope, the reflector has the great advantage of perfect achromatism. For an instrument of given size, the cost of a mirror is less than that of an achromatic lens, since in the former there is only one surface to bring to an accurate figure, while in the latter there are four, and the glass of the mirror need not be of so high a quality since the light does not enter it. On the other hand, the mirror is more sensitive than the lens to changes of temperature and its surface tarnishes after a few months, requiring resilvering, while a lens, if well treated, is permanent. Also, unless mounted with skill and at considerable expense, a large reflector is more unwieldy than a large refractor.

The Largest Telescopes.—The largest telescope yet constructed (1930) is the Hooker reflector of the Mount Wilson Observatory near Pasadena, California. It has a parabolic mirror of 100 inches diameter and 500 inches focal length, which is 13 inches thick and weighs 5 tons. It is sometimes used in the Newtonian form with its natural focal length, and sometimes in the Cassegrain form with either of two hyperbolic secondary mirrors giving equivalent focal lengths of 135 and 250 feet. The moving parts of the telescope weigh 100 tons, and this weight is supported principally by two steel drums attached to the polar axis, which float in troughs of mercury. The instrument is of course too massive to be moved by hand, and fifty electric motors, controlled by switchboards near the eyepieces and at the assistant's desk, are provided for operating the telescope, observing platforms, and dome. A photograph of this magnificent instrument is shown in Plate 6.

A six-foot reflector, with a speculum-metal mirror, was built and used by the Earl of Rosse in Ireland in the middle of the nineteenth century, but it is no longer in use. The second largest telescope now employed is the 72-inch reflector of the Dominion Astrophysical Observatory at Victoria, British Columbia. The Mount Wilson Observatory, in addition to the 100-inch, possesses a fine 60-inch reflector.

Of refractors, the largest is that of the Yerkes Observatory of the University of Chicago, situated at Williams Bay, Wisconsin. The objective is 40 inches in diameter and of 64 feet focal length. The second largest is the 36-inch refractor (Plate 3) of the Lick Observatory of the University of California, situated on the top of Mount Hamilton. Both these telescopes were designed for visual observations, but are used also for photographic purposes by employing special plates. There are many refractors, both visual and photographic, of 24 inches aperture and larger.

Funds have been allotted by the International Education Board to the California Institute of Technology for the erection of a new reflector, of *two* hundred inches aperture, at a mountain site in Southern California or Arizona. In the construction and use of this instrument, the California Institute and the Mount Wilson Observatory will co-operate. It is probable that the mirrors will be made of fused quartz, a material resembling glass but having so low a coefficient of expansion that the form of the mirrors will be very little affected by changes of temperature. This new enterprise is due largely to the energy and vision of Dr. George E. Hale, who has to his credit also the organization of the Yerkes Observatory (1895) and the Mount Wilson Observatory (1904).

EXERCISES

1. The objective of a certain telescope has an aperture of 12 inches and a focal length of fifteen feet. An eyepiece frequently used with it has a focal length of one inch. What is the magnifying power?

Ans. 180

2. What is the least apparent distance of the components of a double star which can be seen clearly separated in the above telescope?

Ans. 0.″375

3. How does the brightness of a star as seen in the above telescope compare with its brightness as seen by the naked eye?

Ans. The telescope increases the brightness 1,296 times

4. One telescope has an aperture of 20 inches, and the focal lengths of its objective and eyepiece are respectively 25 feet and one inch; the corresponding numbers for a second telescope are 60 inches, 25 feet, and ½ inch. How do the two telescopes compare in (1) magnifying power, (2) resolving power, (3) the brightness of the image of a star, and (4) the brightness of the image of a planet (observed visually)?

Ans. The properties of the second as compared to the first are:
(1) 2; (2) 3; (3) 9; (4) 9/4

5. If a fly alights on the objective of a telescope which is directed to the Moon, will an observer at the eyepiece seem to see the fly upon the Moon?

CHAPTER III

THE EARTH

General Description of the Earth.—The Earth is one of the smallest of the eight principal planets that revolve around the Sun. In form it is almost a perfect sphere, 7,920 miles in mean diameter (exclusive of the atmosphere); but the equatorial diameter is about 27 miles longer than the polar, giving the planet more nearly the form of an oblate spheroid. The mass of the Earth is $6 \cdot 10^{21}$ (6 followed by twenty-one ciphers) metric tons. Its mean density, as compared with water, is 5.53. This great ball rotates on its axis in twenty-four sidereal hours, and moves in its vast orbit around the Sun at an average speed of eighteen miles, or thirty kilometers, a second.

The Earth acts as a huge, irregular, spherical magnet, having in each hemisphere a magnetic pole situated about 20° distant from the geographic pole.

The Earth possesses a **lithosphere,** which is the main body of the planet; a **hydrosphere,** consisting of the water on its surface; and an **atmosphere,** the gaseous envelope that surrounds them both.

The outer portion of the lithosphere is a solid crust, on the surface of which we live. It consists mainly of rocks, the principal constituents of which are oxygen and silicon (always in chemical combination), with smaller quantities of most of the other chemical elements. The average density of the crust is considerably less than that of the Earth as a whole, from which fact we may infer that the interior of the lithosphere is compressed by the great weight of the crust. This compression, however, is regarded by geophysicists as insufficient to explain all the difference in density, and by many it is believed that the central core of the Earth is composed of solid nickel-iron having at the center a density of about 10. This view is supported by several lines of evidence, including the magnetic properties of the Earth and the analogy of iron meteorites which are believed by some to be fragments of a cosmic body. Such phenomena as volcanoes and hot springs show that, at no great distance below the surface, the temperature is very high, and it is believed that the main body of the lithosphere is at a temperature above the melting point of rocks at ordinary pressures. It might be expected that the interior of the Earth would in these circumstances be liquid, and until late in the nineteenth century this was thought to be true; but a study of the speed with which earthquake shocks are transmitted through the body of the Earth, and other evidence as well, show that the lithosphere is both elastic and rigid like a solid, the rigidity being between that of glass and that of steel.

The hydrosphere consists of all the oceans, lakes, and rivers, and covers about four-fifths of the surface of the lithosphere. As the molecules of a liquid are free to yield to whatever forces are applied to them, the surface of the ocean is (except for the slight effects produced by winds and tides) a *level* surface, everywhere normal to the direction of gravity. It is the form of this surface that we mean when we speak of the shape of the Earth.

Proofs of Approximate Sphericity; Dip of the Horizon.—To the casual observer the surface of the Earth, especially that of the sea, appears flat, and until its circumnavigation by the fleet of Magellan in 1592 it was generally believed that it really was flat; but well-informed thinkers of all times, beginning with the ancient Greeks, have seen good reasons for believing the Earth to be spherical. Some of the ways open to every careful observer for confirming this belief are the following:

1. The shadow of the Earth, seen on the Moon during a lunar eclipse, is always sensibly circular, whatever may be the face of the Earth that is turned toward the Sun.

2. When a ship moves away from a stationary observer, the first part to disappear is the hull and the last is the highest point of the mast or funnel—showing that the surface of the water is convex (Fig. 37).

3. The angular depression of the visible sea horizon below the true horizon increases with the height of the observer and,

for a given height, is sensibly the same in all azimuths, thus showing that the curvature of the water is the same in all directions—a property possessed only by a spherical surface. This angular depression is called the **dip of the horizon** and is a correction that must be subtracted from all measurements of the altitude of a heavenly body above the visible horizon to reduce them to the true horizon. It is the angle between a level plane and a tangent to the Earth's surface, both passing through the observer's eye.

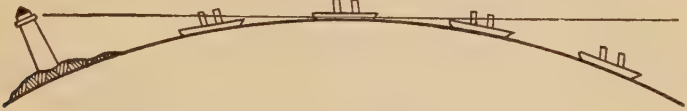


FIG. 37. PROOF OF THE EARTH'S CONVEXITY

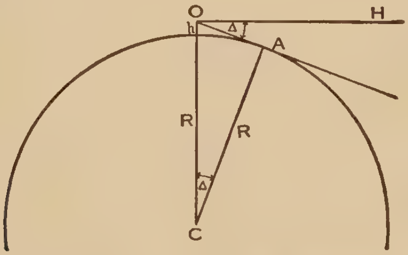


FIG. 38. DIP OF THE HORIZON

Knowing from other considerations, as we now do, the radius of the Earth, we may easily compute both the dip and the distance of the visible horizon if the height of the observer above the water is known. In Fig. 38, C represents the center of the Earth (considered spherical), O the observer, and OH the direction of the true horizon, at right angles to OC. The visible horizon lies at A, the point of contact of a tangent to the sphere drawn from O. If R be the radius of the Earth in miles and h the observer's height in feet, we have in the right triangle OAC

$$\overline{OA}^2 = \left(R + \frac{h}{5280}\right)^2 - R^2$$

$$= \frac{2Rh}{5280} + \left(\frac{h}{5280}\right)^2.$$

R is 3960 (page 86), so that the first term has the value $\frac{2 \times 3960}{5280} h$, while the second, for all values of h which occur on shipboard, is so small as to be negligible. Hence the distance of the sea horizon is given in miles by

$$\overline{OA} = \sqrt{\frac{7920}{5280} h} = \sqrt{\frac{3}{2} h}.$$

The dip, Δ or HOA, equals the angle OCA since the sides of the two angles are respectively perpendicular. As these angles are always small, their value in radians may be taken as $\frac{\overline{OA}}{R}$, or

$$\Delta \text{ in radians} = \frac{\sqrt{\frac{3}{2}h}}{3960}.$$

Hence (page 9),

$$\Delta \text{ in minutes of arc} = \frac{3438}{3960}\sqrt{\frac{3}{2}h}$$
$$= \sqrt{h}, \text{ approximately.}$$

We thus have the sailors' rough but convenient rules:

1. *The distance of the sea horizon in miles is equal to the square root of three-halves of the observer's height in feet;* and

2. *The dip of the horizon in minutes of arc is equal to the square root of the observer's height in feet.*

For example, from the deck of a ship 25 feet above the water, the circle of visible water is about 6 miles in radius, and the value of the dip is about 5′.

The Earth's Atmosphere.—The atmosphere is a mixture (not a chemical combination) of a number of different gases held to the surface of the Earth by gravitational attraction. The lower portion of the air is about three-fourths nitrogen and one-fourth oxygen, with small quantities of water vapor, carbon dioxide, and argon. Suspended in the lower air is a considerable amount of solid material in the form of dust and smoke, and usually also liquid water in the form of the droplets that make up the haze and lower clouds; while the higher clouds are often composed of ice crystals—particles of solid water. The upper air is believed to be made up of the light gases—mostly hydrogen and a small amount of helium.

The distinction between solids, liquids, and gases has to do with the arrangement of the molecules. In a solid, the molecules resist any force that tends to change their relative positions or distances, so that the body possesses **incompressibility** and **rigidity.** In a liquid, the molecules pass freely over one another, but resist any tendency to change the distance between adjacent molecules. A liquid thus possesses incompressibility, but not rigidity. The molecules of a gas constantly move about among one another, each one darting in a straight line until it encounters another molecule or the wall of the containing vessel. The speed of a molecule of gas is the greater the greater the temperature, and the length of its free path is the greater the less the density. Increasing the pressure on a given mass of gas both decreases its volume and increases its density by diminishing the free paths of the molecules. If the pressure is relieved, the molecules at once spring apart and the volume increases, decreasing the density. Most substances can be made to pass successively through the three states by varying the temperature or the pressure. Thus, in air at ordinary atmospheric pressure, at a temperature of 0° Centigrade, solid water (ice) is transformed into liquid, and at 100° Centigrade liquid water boils—*i.e.*, is rapidly transformed into an invisible gas called water vapor.

The weight of each layer of air produces a pressure upon the portion of the atmosphere that lies below it, and the total pressure and density are therefore greatest at the level of the sea and diminish with increasing height. The total weight of the atmosphere is some 10^{15} tons and the pressure at sea level amounts to about fifteen pounds to every square inch of surface; but we are not ordinarily conscious of this pressure because it is the same in all directions. When a part of the air in a vessel is artificially removed, the atmospheric pressure forces other air to take its place. Philosophers before the time of Galileo were unaware of the pressure produced by the weight of the air, and attributed its effects to the false but famous principle, "Nature abhors a vacuum."

Illumination and Color of the Sky; Twilight.—When light passes through a medium which contains particles that are small compared with the wave-length of the light, a part of the light is scattered by the particles in all directions, and the ratio of the intensities of the scattered and incident lights varies inversely as the fourth power of the wave-length. The molecules of the air, and also the particles of dust suspended in it, have this effect upon sunlight and starlight; and so the light that reaches the eye directly through the air has been deprived of some of its shorter waves (composing the violet end of the spectrum), and the heavenly bodies appear of a redder or yellower hue than they would if the Earth had no atmosphere. Just before sunset, the sunlight passes through a greater depth of air than near the middle of the day, and its redness is increased both by this cause and by the greater dustiness of the air at that time of the day. The brilliant colors of sunset clouds are due to their illumination by light that has passed through different depths of air. On the other hand, when we turn to directions nearly at right angles to that of the Sun, we receive light that has been scattered by particles in the air, and which is therefore blue or, in the pure air of lofty mountains, a deep violet. The illumination and color of the sky are thus due to the scattering of light by the small particles of the air. If the Earth had no atmosphere, the sky would be black and the stars could be seen at all times, day or night.

Twilight and **dawn** are the names applied to the illumination given to the sky by the Sun when it is below the horizon. Just after sunset, the Sun is still shining on the air above our heads, and the entire sky is still bright; but this brightness rapidly diminishes, especially near the eastern horizon, where, if the sky be very clear, the dark shadow of the Earth (known as the **twilight arc**) may be seen mounting the sky as the invisible Sun descends. Twilight ends when the depression of the Sun below the horizon, as measured on a vertical circle, is 18° For an observer at the Earth's equator, where the diurnal motion takes place vertically, the duration of twilight is only about $1^h\ 12^m$; but it varies with the observer's latitude and the Sun's declination, both of which modify the angle made by the Sun's diurnal path with the horizon, and in summer, in latitudes greater than 48°.5, twilight lasts all night.

The Depth of the Atmosphere.—Since the pressure produced by the weight of the air diminishes with increasing height, the density also diminishes in the same way, and it is difficult to locate the boundary of the atmosphere if indeed any definite boundary exists. It is possible, however, to determine approximately the height to which air of a certain density extends. The tops of the highest mountains are about five miles above sea level, and manned balloons and airplanes have risen to heights somewhat greater than this. At these heights the air is too rare to support life for any length of time, and is very cold, because of the rapid loss of heat into space through the still thinner air above. Sounding balloons, carrying recording meteorological instruments, have risen twenty-one miles and their records have shown that, at heights greater than about seven miles, the temperature is constant at about − 55° C. This region of uniform temperature is called by meteorologists the **strato-**

sphere. Ordinary clouds and dust are not detected at heights much greater than seven miles. The fact that twilight persists until the Sun is 18° below the horizon shows that the air is dense enough to scatter light perceptibly at heights above forty miles.

This result is obtained as follows: In Fig. 39 let the two concentric arcs represent the surface of the Earth and that of the highest reflecting layer of atmosphere; C the center of the Earth, O the observer, OH the plane of the observer's horizon, and AS the direction of the Sun at the end of twilight. Twilight ends when the Sun ceases to shine at A, and observation shows that this occurs when the angle HAS is 18°. This angle is equal to OCB since their sides are respectively perpendicular, and OCB is bisected by the line CA, whose length exceeds the radius R of the Earth by the height h of the atmosphere. In the right triangle OCA we have

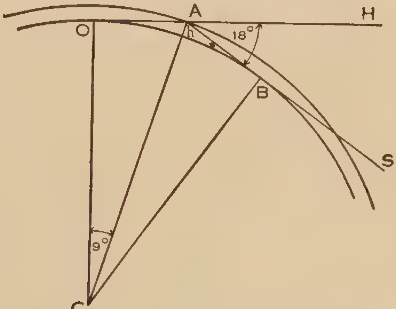


FIG. 39. DEPTH OF THE ATMOSPHERE

$$(R + h)/R = \sec 9°$$

or

$$h = R(\sec 9° - 1).$$

The secant of nine degrees is 1.0125 and the radius of the Earth is 3,960 miles, and so h by the above formula is 3,960 × 0.0125 = 49 miles. The rays AO and AS, however, are slightly curved downward by refraction (page 60), and the actual depth of the reflecting atmosphere is less than that given by the formula.

The light of "shooting stars" or meteors is due to the heat generated by their swift passage through the air (page 276), and these bodies are often seen at heights exceeding one hundred miles, showing that the air extends to that height. Finally, the arcs and streamers of the polar aurora (page 184), which is believed to be due to the impact of free electrons from outer space upon the particles of the air, have been observed by Störmer of Oslo at heights as great as six hundred miles.

Atmospheric Refraction and Dispersion.—When a ray of light from any heavenly body encounters the atmosphere it obeys the law of refraction and is bent toward the normal to the surface. The index of refraction of air increases with the density, and so the bending increases as the light passes downward, and the ray follows a curved path through the air as shown in Fig. 40, where a ray from the star S reaches the observer at O. We "see" an object in the direction from which its light enters the eye, and so the star appears to be at S', in the direction of a tangent drawn to the curve at O. The effect of atmospheric refraction is thus to make the heavenly bodies appear slightly higher in the sky than they really are, and a "refraction correction" must accordingly be applied to all measured altitudes.

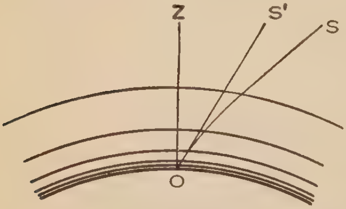


FIG. 40. ATMOSPHERIC REFRACTION

If the body is observed at the zenith, the incident ray coincides with the normal and the refraction is zero. It is greatest at the horizon, and varies nearly as the tangent of the zenith distance, an approximate formula for the refraction correction being

$$r = 60''\!.7 \tan \zeta$$

where ζ is the zenith distance. This formula obviously breaks down at the horizon, where it would give an infinite displacement of the star, and in fact is of little value at altitudes below 25°. The exact calculation of atmospheric refraction must take into account the barometric pressure and temperature of the air, and is very complicated.

At the horizon, under average conditions, the refraction is about 35′, which is greater than the apparent diameter of the Sun or Moon. The result is that we see these bodies while they are still entirely below the true horizon, and refraction has the effect of lengthening the day and shortening the night. Near the horizon, the refraction changes very rapidly, and at an altitude of half a degree it is about 6′ less than at the horizon. For this reason, the upper limb of the rising or setting Sun is elevated 6′ less than the lower, and the disk appears noticeably flattened on the under side.

Violet light is refracted more than red by air, just as by glass, but the dispersion is much less. It is noticeable, however, in the case of a star near the horizon, especially if viewed through a telescope, when the image appears as a short spectrum. The effect is not conspicuous in ordinary telescopes except at altitudes of 5° or less, but in the modern giant reflectors it affects the quality of star images when the star is as high as 30°.

Experimental Evidence of the Earth's Rotation.—That the Earth rotates on an axis was taught as long ago as 395 B.C. by Herakleides of Pontus, but the doctrine was not accepted by many of the early philosophers, and until after the work of Copernicus (1543) it was generally believed that the Earth was stationary and that the diurnal motion of the celestial sphere was a real motion in which all the heavenly bodies took part. With the expansion of ideas regarding the vast distances of these bodies, especially the stars, the simplicity of the view of Herakleides as opposed to the complexity of its alternative became evident, and throughout the last three centuries the rotation of the Earth has not, by intelligent persons, been seriously questioned.

In 1851 Foucault performed a famous experiment which for the first time demonstrated the Earth's rotation without the use of any point of reference outside the Earth itself. He suspended in the dome of the Pantheon at Paris a pendulum consisting of a heavy iron ball attached to a slender wire more than two hundred feet long, the upper end of which was pivoted on a small round point under the top of the dome. The pendulum was thus free to swing in any azimuth, and a rotation of the support could not readily be transmitted to it. Such a pendulum, if left undisturbed, continues to swing for several hours very nearly in the same plane. At the bottom of the ball was fixed a pin which, as the pendulum swung, just touched the surface of a circular ridge of sand heaped upon a table beneath, thus showing by a mark in the sand the direction of the plane of vibration. If the Earth did not rotate the pin would continue to cut the sand in the same place; but Foucault found, and showed to a great crowd of spectators, that the sand was cut in a fresh place at each swing, the floor of the building visibly turning under the pendulum. The experiment has been repeated many times and in many places, including some in the southern hemisphere, where the apparent rotation is, as would be expected, in the direction opposite that observed in the northern.

If the experiment were performed at the Earth's pole, the rate of apparent rotation of the plane of vibration would be the same as the real rate of the Earth—one complete turn in twenty-four hours. At the equator, there would be no apparent rotation at all, because the rotation of the plane of the pendulum must take place around a vertical axis, and a building located at the equator does not rotate in this way, but is carried bodily in a circle around the center of the Earth. The motion of a building located between the pole and the equator may be regarded as a combination of rotation and revolution, the former predominating the more as the pole is approached; and it may be shown by the principle of composition of angular velocities in Dynamics that the time of an apparent rotation at a place whose latitude is φ is $24^h \div \sin \varphi$. At the latitude of Paris the time of a complete apparent rotation is about 32 hours.

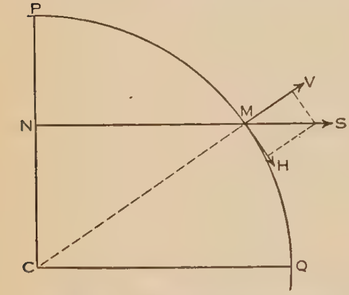


FIG. 41. EFFECT OF EARTH'S ROTATION ON ITS FORM

There are a number of other ways in which the rotation of the Earth has been detected independently of observations of the stars.[1] Perhaps the most important, certainly so from a practical point of view, is the action of the gyroscopic compass which is used on many large ships and which depends upon the directive effect of the Earth's spin upon the axis of a rapidly rotating, massive wheel. The explanation of this instrument is too difficult to be entered upon here.

Effect of the Earth's Rotation upon Its Form and upon Gravity.—Owing to the rotation of the Earth, every particle of its substance (except those exactly on its axis) moves in a circle in a plane parallel to the equator and centered on a point on the axis. Thus, a particle at M, Fig. 41, revolves in a circle of radius MN with center at N. Now, motion in a circle results in a tendency of the moving body to fly away, known in Mechanics as "centrifugal acceleration," the formula for which is

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[1] For an elementary discussion of several of these experiments the reader is referred to Young's *General Astronomy* or Barlow and Bryan's *Mathematical Astronomy.*

$$C = V^2/R = 4\pi^2 R/T^2,$$

where V is the velocity of the moving body, R the radius of the circle, and T the time of a complete revolution. The particle at M would have a tendency to fly out in the direction MS, a tendency proportional, according to the second part of the above formula, to the radius MN, and therefore a maximum for a body at the equator Q and zero at the pole P.

Although, because of the large value of T in the case of the Earth, the centrifugal force acting on a body of ordinary mass is small (force is equal to mass times acceleration), the Earth is so enormous that the aggregate of the forces acting on all its parts is greater than any known material could withstand, and it would fly apart like an overstrained flywheel were it not held together by the mutual gravitation of its parts.

Suppose the Earth were spherical and homogeneous. The resultant gravitational attraction of all its parts upon M would be directed to the center C. The centrifugal acceleration MS is, according to the principle of the parallelogram of forces (page 216), the equivalent of two accelerations, represented by the sides MV and MH of a rectangle of which MS is the diagonal. The first of these is directed away from C and is overpowered by the gravitational attraction, its only effect being to reduce the weight of the body M; but the component MH acts at right angles to the gravitational attraction, tending to make the body slide along the surface of the sphere toward the equator, and has only the rigidity of the planet to resist it. The result is that the Earth, instead of being an exact sphere, is bulged at the equator and has approximately the form of an **oblate spheroid.**

An oblate spheriod is the solid figure generated when an ellipse (page 100) is rotated about its minor axis. Rotation of an ellipse about its major axis generates a **prolate spheroid.** The former is shaped somewhat like an orange or pumpkin, the latter like a football or an airship.

The effect of the Earth's rotation upon its form is illustrated by the simple piece of apparatus shown in Fig. 42.

By **gravitation** is meant the universal attraction of all particles of matter for each other, which will be discussed in Chapter 10. **Gravity** is the resultant effect of the Earth's gravitation and its centrifugal acceleration. Gravity is measured by the acceleration produced in a freely falling body, which is about thirty-two feet (981 cm.) per second per second—that is, a body falling under the influence of gravity alone increases its velocity by thirty-two feet per second during each second of its fall. This acceleration is commonly denoted by g. The **weight** of a body is its mass multiplied by g. The value of g is greater at the poles than at the equator by about 1/190 of itself; hence, an object that weighs 190 pounds at the pole would weigh only 189 at the equator (if weighed on a spring balance). One pound in 289 of this difference is due to centrifugal force; the remainder, about one pound in 555, is due to the difference of the attractive power of the Earth, which depends on the fact that the equator is farther from the center of the planet than is the pole.

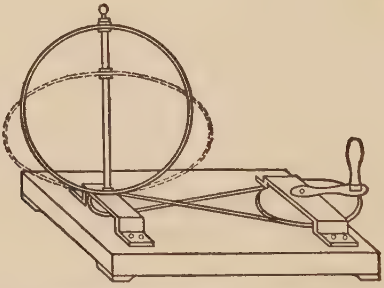

FIG. 42. A ROTATING SPHERE BECOMES OBLATE (From Young's *Manual of Astronomy*)

Determination of the Earth's Form by Pendulum Observations.—In the year 1672 Jean Richer, a French astronomer, was sent by Louis XIV on an expedition to the island of Cayenne and took with him a clock whose pendulum was regulated to beat seconds in Paris. He found from time-determinations by astronomical methods that in Cayenne this clock lost about two and a half minutes a day, although the length of the pendulum had not been altered. It is well known that the time of swing of a pendulum depends only on its length and the value of g, being for a simple pendulum,

$$T = \pi \sqrt{l/g},$$

where l is the length of the pendulum. Since l had not changed, the change in T must have been due to a variation in g; and it was in this way that the difference in gravity at different parts of the Earth's surface was discovered.

The part of this difference that is due to centrifugal force can be calculated by the formula on page 63, so that from pendulum observations the effect upon gravity of changing distance from the center of the Earth can be ascertained, and from this the form of the Earth deduced. The result is expressed in terms of the **oblateness** of the Earth, which means the fraction obtained by dividing the difference between the equatorial and polar diameters by the equatorial. The most reliable observations indicate for the Earth an oblateness of

$$\Omega = 1/297.$$

Geographic Co-ordinates.—The position of any place on the surface of the Earth is completely described by stating its co-ordinates in the geographic system—that is, its longitude and latitude. For example, if a ship in distress sends out a radio SOS call accompanied by a statement of its longitude and latitude, its distant rescuers know at once what course to pursue in order to reach the spot. Since the Earth is not a sphere, it is best not to attempt to define these co-ordinates as arcs of circles as we defined the co-ordinates in the various celestial systems in Chapter I, but rather as angles.

The points where the Earth's axis pierces the surface of the spheroid are called the north and south terrestrial **poles.** The surface is cut along **meridians** by planes that intersect one another along the axis, and on **parallels of latitude** by planes perpendicular to the axis. The parallel of latitude that lies midway between the poles is the terrestrial **equator.**

The **longitude** of a point on the Earth's surface is the angle between the plane of its meridian and the plane of the meridian of some place chosen as a standard—called the **prime meridian.**

Civilized countries have now agreed to use as a standard the meridian of the Royal Observatory at Greenwich, England. Longitude is counted either in degrees or in hours, and either east or west of Greenwich up to 180°.

It should be noted that longitude is defined as a *dihedral*, or wedge angle, and that it is measured by the arc of either the terrestrial or the celestial equator included between the two planes.

The **astronomic latitude** of a place is the angle between the plane of the equator and the direction of gravity at the place. The **geocentric latitude** is the angle between the plane of the equator and a straight line passing from the place to the center of the Earth. The difference between them is caused by the oblateness of the Earth, and amounts at its greatest to about 11′. The two kinds of latitude are illustrated in Fig. 43, in which the departure of the Earth's form from a sphere is of course greatly exaggerated, and in which the astronomic latitude is indicated by ϕ and the geocentric by ϕ'.

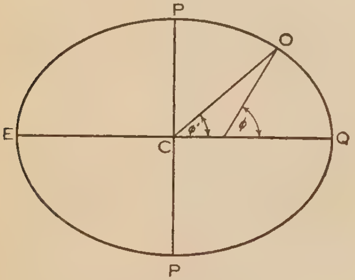

FIG. 43. ASTRONOMIC AND GEOCENTRIC LATITUDE

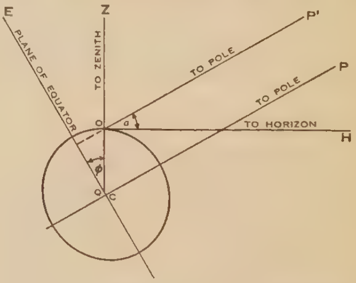

FIG. 44. EQUALITY OF LATITUDE AND POLE HEIGHT

Reference is sometimes made to a **geographic latitude,** defined as the angle between the plane of the equator and a normal to the standard spheroid. It differs from astronomic latitude only by the effects of local deviations of the direction of gravity caused by the attraction of mountains, etc.—never by more than 30″ or 40″.

The Astronomic Latitude of the Observer Equals the Altitude of the Celestial Pole.—In Fig. 44, let *O* be the place of an observer on the Earth's surface, *C* the center of the Earth, and *CP* the Earth's axis. The plane of the paper will then be the plane of the observer's meridian, and this meridian may itself be represented by a nearly circular ellipse passing through

O and with center at C. Draw CE perpendicular to CP; by definition, CE is then the intersection of the plane of the equator with the plane of the meridian. Let ZO be the direction of gravity at O and let it intersect CE at Q. By definition, the angle ZQE, or ϕ, is the astronomic latitude of O, and OZ is directed to the observer's zenith. Draw OH perpendicular to OZ; by definition it is directed to the observer's horizon. Draw OP' parallel to CP; it meets the infinitely distant surface of the celestial sphere at the same point as does CP, and is therefore directed to the celestial pole.[1] The angle $P'OH$, or a, is subtended on the celestial sphere by the arc of a vertical circle included between the pole and the horizon, and is therefore the altitude of the pole. But the sides of the angles a and ϕ are respectively perpendicular, and therefore the angles are equal; hence, we have the important relation: *The observer's astronomic latitude is equal to the altitude of the celestial pole.* Q. E. D.

Right, Parallel, and Oblique Spheres.—At the Earth's equator, the latitude is zero and both celestial poles lie upon the horizon. The celestial equator coincides with the prime vertical, all diurnal circles intersect the horizon at right angles, and each star is above the horizon just twelve sidereal hours (neglecting refraction) and below for the same length of time. This aspect of the heavens is called the **right sphere.** The term *right ascension of a star* was originally defined as the degree of the celestial equator, counted from the vernal equinox, that rises with the star in a right sphere. A little reflection will show that this is equivalent to the definitions given in Chapter I.

At the Earth's pole the latitude is 90° and the celestial pole appears at the zenith. The celestial equator coincides with the true horizon and each star sails around the sky on an almucantar, parallel to the horizon. This appearance of the sky is called the **parallel sphere.**

In intermediate latitudes, the diurnal motion is oblique to the horizon as it was described in Chapter I, the celestial pole being the more elevated the greater the latitude, and this state of affairs is called the **oblique sphere.**

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[1] Because the pole is the "vanishing point" of the two lines. Suppose a line drawn from O to any point X of CP within a finite distance of O; it would make a certain angle θ with OP'. Now let X recede along CP. As it does so, θ grows smaller, approaching zero as a limit. Hence, when X reaches the surface of the celestial sphere, $\theta = 0$ and the line is parallel to CP, coinciding with OP'.

As the observer travels toward the pole, the circles of perpetual occultation and apparition (page 19) enlarge, approach each other, and become more nearly level until at the pole itself they coincide with the equator and horizon. At the pole, the stars of half the sky never set; and those of the other half never rise.

As the observer approaches the equator, the circles of perpetual occultation and apparition contract and move apart and finally become mere points coinciding with the poles. If the observer crosses the equator, they exchange places and the direction of the diurnal motion becomes reversed with respect to the horizon, being clockwise in the northern hemisphere and counter-clockwise in the southern.

The *ZPS* Triangle.—As a plane triangle is made up of straight lines, a **spherical triangle** is made up of arcs of great circles on the surface of a sphere. The sides as well as the angles of a spherical triangle are measured in degrees, and by the formulæ developed in spherical trigonometry it is possible to compute any of the six parts (three sides and three angles) when any three are given.

Denoting the three angles of any spherical triangle by A, B, and C and the opposite sides by a, b, and c, the triangle can be completely solved by the three fundamental formulæ

$$\begin{aligned}\cos a &= \cos b \cos c + \sin b \sin c \cos A,\\ \sin a \cos B &= \cos b \sin c - \sin b \cos c \cos A,\\ \sin a \sin B &= \sin b \sin A.\end{aligned}$$

In many cases, the computation is shortened by judicious transformation of these formulæ. Computation by logarithms is of course always employed.

Many important problems in spherical astronomy require a transformation of co-ordinates in the horizon system to co-ordinates in the equator system, or *vice versa*, and involve the solution of the triangle whose vertices are at the zenith, the pole, and some star—commonly called the *ZPS* triangle. Suppose the observer is in the northern hemisphere and that the star is north of the equator and west of the meridian, as in Fig. 45. Representing by ϕ the astronomic latitude of the observer and by δ, t, h, and A, respectively, the declination, hour angle, altitude, and azimuth of the star, we have

$$\begin{aligned} \text{Side } PZ &= 90^\circ - \phi, & \text{Angle } P &= t \\ ZS &= 90^\circ - h, & Z &= 180^\circ - A \\ PS &= 90^\circ - \delta, \end{aligned}$$

The angle at S, between the vertical circle and hour circle of the star, is known as the **parallactic angle.**

Determination of the Latitude of an Observatory.—The *geocentric* latitude of a place cannot be determined by direct observation, but is computed from the astronomic latitude by means of a knowledge of the figure of the Earth. There are many ways, of varying degrees of accuracy and convenience, by which the astronomic latitude may be determined, each depending upon a measurement of the altitude of the pole or of its equivalent, the declination of the zenith. Of these, we shall here describe only one, the **method of circumpolar stars.**

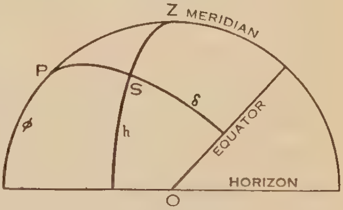

FIG. 45. THE ZPS TRIANGLE

This method consists of determining the altitude of the pole by observations with the meridian circle. In Chapter I we described this instrument and also the method by which the "polar reading" is found from observations of a circumpolar star at its transits above and below the pole (page 31). In addition to this, for finding the latitude the exact reading of the circle must be obtained when the telescope is pointed to the horizon. The altitude of the pole and hence the latitude is then found at once by subtracting the two readings. It is not possible to locate the horizon accurately by a direct observation, and so recourse is had to the **nadir reading,** which is exactly 90° from the horizon reading, and which is obtained as follows: A dish of mercury is placed below the axis of the meridian circle and the telescope is directed downward toward it. The light reflected from the lines of the reticle (which must be illuminated from the side next the eye) and passing downward is made parallel by the objective, reflected by the bright surface of the mercury, and focused again by the objective in the plane of the reticle. The observer adjusts the instrument so that the reticle coincides with its own reflected image, in which case the line of sight is exactly vertical—*i.e.*, the telescope is directed exactly to the nadir; for, by the law of reflection, the incident and reflected rays then both coincide with the normal to the mercury surface, and this surface is precisely level because the mercury is a free liquid.

The polar and nadir readings of every meridian circle in regular use are determined many times, so that the latitude of the instrument is well established. Latitudes determined from a good series of observations of this kind are reliable to about $0''.01$—that is, to about one foot as measured on the Earth's surface.

As compared with other accurate methods, the method of circumpolars has the advantage that it is independent of previous observations, as it is not even necessary to identify the star used for determining the polar reading; but it cannot be used at observatories near the equator and has the further disadvantage of requiring two observations twelve hours apart. For rapid and accurate determinations of latitude such as are needed, for example, in boundary surveys, the method introduced in 1845 by the American engineer Talcott is extensively used. It requires the use of a special instrument known as a **zenith telescope,** and although simple in principle, will not be discussed here.

Variation of Latitude.—It was shown by the Swiss mathematician Euler in the latter part of the eighteenth century that if a rigid oblate spheroid be set in rotation about its shortest axis, it will continue to rotate uniformly about that axis as long as it is not acted on by any external force; but that if the rotation be started about any other axis, its axis of rotation will itself rotate about its axis of symmetry in a period that depends on the form, mass, and speed of rotation of the spheroid. In the case of the Earth (assuming perfect rigidity) this period should be 305 days. This does not mean that the Earth's axis of rotation should change its position with respect to the stars (though it does this, too—see Precession and Nutation, pages 108 *et seq.*), but that the Earth should so move that its poles must wander slightly upon its surface. Since the latitude of any place is reckoned from the plane of the equator and the equator is fixed with respect to the poles, any such wandering of the poles must result in a variation of latitude.

A variation of latitude was actually detected about a century after Euler's work (1888) by Küstner and by Chandler. Chandler found from a laborious investigation of a great quantity of observations that the motion of the pole could be regarded as the resultant of two motions, one of these being in an ellipse with a period of a year and the other in a circle with a period of 428 days. The actual motion is very complicated but very slight, the greatest departure of the pole from its mean position being less than forty feet, and resulting in a total variation of latitude of only about $0''.6$.

The annual component of this motion of the Earth is probably due to meteorological causes such as the deposition of snow in one hemisphere in one half of the year and in the other during the other half. The other component may be due to the way in which the Earth originally started rotating, or possibly to some event in its past history that modified its rotation; if so, the difference between the 428-day period and Euler's theoretical 305 days is to be explained by the Earth's not being perfectly rigid.

Time and Longitude.—As the subject of longitude is intimately connected with that of time, it is appropriate that we review and extend what was said in Chapter I about the latter before discussing methods of determining the former. By the word *time* is here meant the number of hours and subdivisions that indicate the time of day, and no discussion of the recondite subject of Time in the abstract or of its relations to Space is intended. The time of day is always equivalent to the hour angle of some reference point in the sky. Astronomers are accustomed to count the hours of the day from 0 to 24 instead of dividing the day into twelve A.M. hours and twelve P.M. hours.

Sidereal time is the hour angle of the vernal equinox, or the right ascension of the meridian. The method by which it is determined by observations of stars with the transit instrument has already been described in Chapter I (page 30). This is the most accurate method known for determining time.

True or apparent solar time is the hour angle of the Sun.[1] It is the time shown by a correctly adjusted sun dial and differs from sidereal time by the right ascension of the Sun.

The apparent motion of the Sun in the ecliptic, caused by the Earth's orbital revolution, is not uniform, and even if it were, the Sun's right ascension would not increase uniformly because the right ascension is counted along the equator, which makes an angle with the ecliptic of 23½°. The **mean sun** is a fictitious sun that moves in the celestial equator with the mean speed with which the true Sun moves in the ecliptic. **Mean solar time** is the hour angle of the mean sun.

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[1] These definitions refer to **astronomical time,** which is reckoned from noon. The **civil day** begins at midnight, and to obtain **civil time** it is necessary to add 12 hours to astronomical time. In 1925 the principal national almanacs, which up to that year had used the astronomical day, agreed to discontinue its use and to adopt the civil day; but as it is inconvenient for astronomers to change the date in the middle of their night's work, the records of many great observatories are still kept in astronomical standard time—*i.e.*, time counted from standard noon.

The **equation of time** is the difference between mean and apparent solar time. It amounts, at its maximum, to about sixteen minutes. The right ascension of the Sun and the equation of time depend upon the position of the Earth in its orbit and can be computed for any given instant from a knowledge of the position, form, and size of that orbit. They are given for every day of the year in the almanacs and ephemerides published by the principal governments, such as the *American Ephemeris and Nautical Almanac.*

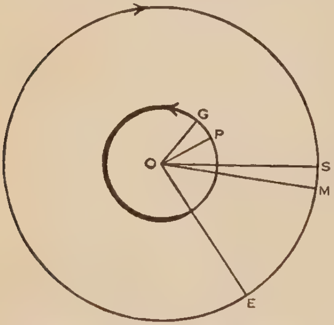


FIG. 46. TIME AND LONGITUDE

Since the celestial meridian, from which hour angles are counted, is the trace on the celestial sphere of the plane of the observer's terrestrial meridian, observers in different longitudes have different times at the same instant. Moreover, the difference of their times is the same number of hours as the difference of their longitudes, and the longitude of any place is the difference between the time at that place and the time at Greenwich. This may be seen in Fig. 46, in which *O* represents the center of the Earth, the inner circle the terrestrial equator, and the outer circle the celestial equator. From the northern hemisphere, the rotation of the Earth appears counter-clockwise and that of the heavens clockwise, as indicated by the arrows. Let *E* represent the vernal equinox, *M* the mean sun, *S* the intersection of the real Sun's hour circle with the celestial equator, *G* the intersection of the meridian of Greenwich with the terrestrial equator, and *P* the point corresponding to *G* for any place other than Greenwich. By the definitions already given, we have

Angle GOP = the west longitude of P
GOS = Greenwich apparent solar time
POS = local apparent solar time at P
GOM = Greenwich mean solar time
POM = local mean solar time at P
GOE = Greenwich sidereal time
POE = local sidereal time at P

The angle GOP may be obtained by any one of the subtractions $GOS - POS$, $GOM - POM$, or $GOE - POE$; hence, the longitude of P from Greenwich is equal to Greenwich time minus the time at the place, whether the time used be sidereal, apparent solar, or mean solar.

Standard Time.—Prior to the year 1883, the people of the United States used the local mean solar time of the cities in or near which they lived, as was natural before the advent of rapid means of transportation. With such a system it was necessary for a traveler, in order always to have the "right" time, to change his watch at the rate of about one minute for every fourteen miles traveled eastward or westward. Each railroad had its own arbitrary time system in which trains were scheduled throughout the length of a line or division, and in most of the towns along the way two kinds of time were recognized—"Sun time" and "railroad time."

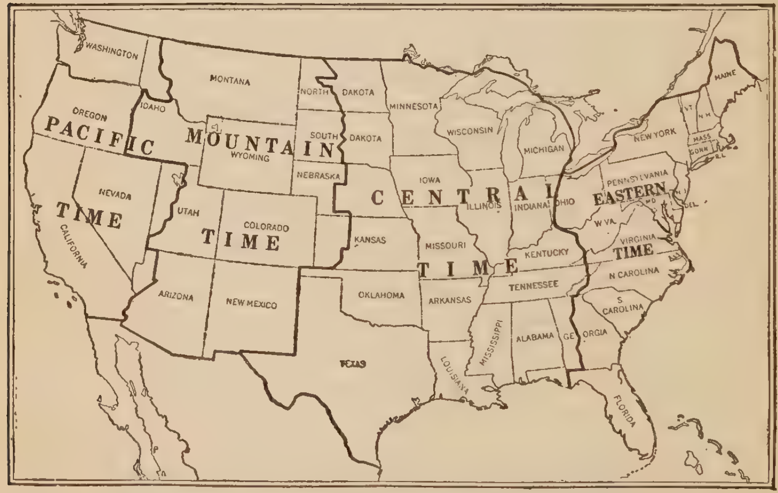
In November, 1883, the railroads of the United States agreed to adopt a system of standard time by which four **standard meridians** were established in longitudes exactly five, six, seven, and eight hours west of Greenwich, respectively. The country was then divided into four belts having these meridians approximately central, and the railroads of each belt agreed to use the time of the corresponding meridian. This system was legalized by Congress the following year, and standard time based similarly on Greenwich mean time has since been adopted in all the important countries of the globe.

The four standard times of the United States are:

Eastern time = Greenwich civil time − 5 hours
Central time = Greenwich civil time − 6 hours
Mountain time = Greenwich civil time − 7 hours
Pacific time = Greenwich civil time − 8 hours

The actual boundaries of the time belts are somewhat irregular, the change from one standard to another being ordinarily made at an important city or at the end of a railway division. They are shown in Fig. 47, which is corrected for readjustments made in 1926.

The Date Line.—Traveling westward from Greenwich, one passes through regions where the time is increasingly *less* than Greenwich mean time, and traveling eastward he finds the time increasingly *greater* than Greenwich. Just east of the 180th meridian the standard time is twelve hours less than Greenwich, and just west of that line it is twelve hours greater.


WASHINGTON
MONTANA
NORTH DAKOTA
MINNESOTA
OREGON
IDAHO
PACIFIC TIME
MOUNTAIN TIME
SOUTH DAKOTA
WYOMING
WISCONSIN
MICHIGAN
NEW YORK
MAINE
VT
N H
MASS
CONN
R I
PENNSYLVANIA
NEBRASKA
IOWA
CENTRAL TIME
NEVADA
UTAH
COLORADO
ILLINOIS
INDIANA
OHIO
EASTERN TIME
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VIRGINIA
CALIFORNIA
KANSAS
MISSOURI
KENTUCKY
N CAROLINA
TENNESSEE
ARIZONA
NEW MEXICO
OKLAHOMA
ARKANSAS
S CAROLINA
MISSISSIPPI
ALABAMA
GEORGIA
TEXAS
LOUISIANA
FLORIDA

FIG. 47. STANDARD TIME IN THE UNITED STATES

These two regions thus have times differing by a whole day, and in passing from one to the other it is necessary to change the date. Suppose a ship sailing eastward arrives at this **date line** on the evening of December 25; the date in the log is changed to December 24 and the next day the crew may claim a second Christmas dinner. A ship that reaches the date line on a westward voyage just before midnight on December 24 misses Christmas, for the date is changed to December 25 and shortly after, at midnight, it becomes December 26. Were it not for this change of date, a person traveling around the globe would find upon his return that his reckoning differed by a day from that of his friends at home. The date line does not follow exactly the course of the 180th meridian, but, like the boundaries of the time belts in the United States, is somewhat irregular. Its exact location depends upon the reckoning adopted by the inhabitants of the adjacent lands, which are mostly small islands.

Distribution of Time by Telegraph and Radio.—Time is distributed telegraphically from a number of large observatories, of which the most important in the United States is the Naval Observatory at Washington. Here the error of a very accurate sidereal clock is determined regularly by observations of the stars with the six-inch meridian circle and a record kept of both the error and the rate of its change. Shortly before noon every day, the Eastern standard time corresponding to a certain reading of this clock is computed from its known error and from the data given in the *American Ephemeris*, and a mean-time "distributing clock" is adjusted to within a few hundredths of a second of the correct Eastern standard time. The distributing clock is provided with a device for breaking an electric circuit at the beginning of each second. Three minutes before Eastern standard noon, this circuit is connected through a relay with the lines of the Western Union telegraph system, on which all other business is suspended, and the sounders in all the stations of this company are thus made to tick in unison with the clock. The circuit-breaker omits from each minute the ticks corresponding to the twenty-ninth second and to those from the fifty-fifth to the fifty-ninth, inclusive; and from the last minute before noon it omits the ticks of the fiftieth to fifty-ninth, inclusive, and gives a final tick at the instant of twelve o'clock. These signals may be received

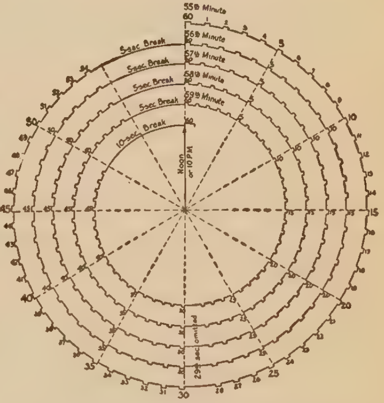

FIG. 48. DIAGRAM OF WASHINGTON TIME SIGNALS (*from Circular No. 51* OF THE U. S. BUREAU OF STANDARDS)

on the west coast at 9 A.M. Pacific time, apparently three hours before they were sent out in Washington. By the intervals of silence where ticks are omitted, any second in the three minutes can be easily identified, and many opportunities are thus afforded for comparing a watch or other timepiece with the Washington clock. Similar signals are broadcast by radio from a number of stations in the United States during the five minutes preceding noon and 10 P.M., and time signals of a different arrangement are broadcast from several foreign stations.

Determination of Longitude.—To determine the longitude of a place where the standard time signals can be received—and, with a good radio receiving set, this includes practically every place on the Earth—it is only necessary to determine from observation of the heavenly bodies the local mean solar time at the instant the signals are received. This usually involves transit observations of stars and the use of a clock whose rate can be relied upon for the interval between the observations and the reception of the signals. The difference between the local mean time of reception and the time indicated by the signals is the difference between the longitude of the standard meridian and that of the place of observation. The possible sources of error are the error of the distributing clock, which is seldom more than a tenth of a second; the time of transmission of the signals which is exceedingly small since radio waves are propagated with the speed of light; and the error of the local time determination.

When it is desired to obtain the longitude of a point with the utmost accuracy, observers are stationed both at that point and at a point whose longitude is already known, and the time of reception of signals from some station is determined by local clocks whose errors are found from transit observations of stars. The times of the stars' crossing of the lines of the reticles are recorded electrically on a *chronograph* and the "personal errors" of the observers are eliminated, either by repeating the work after interchanging observers or by using some form of *transit micrometer*. Space does not permit of a description of these instruments here.

Before the advent of radio telegraphy, telegraph wire lines or cables were used, and before that, observations of celestial phenomena such as the eclipses of Jupiter's satellites or, for short distances, artificial signals such as flashes of gunpowder. The difficulty in the use of eclipses as signals is that they are not sufficiently sudden in their occurrence.

Instruments of Navigation.—Probably the most frequent "practical" application of a knowledge of astronomy is found in the art of navigation. From a position out of sight of land, the view from a ship is the same in every direction except for the heavenly bodies, and so it is upon these that the navigator relies for knowing his position on the Earth and the direction he must take to reach his destination.

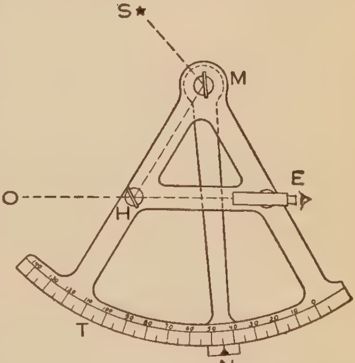

FIG. 49. THE SEXTANT

The principal problems that confront the navigator are the determination of the position of his ship—that is, her latitude and longitude—and that of the direction and speed of the ship's motion. The position is determined by observations of the stars, or more often the Sun. Up to the middle of the eighteenth century, the instrument used for this purpose was some form of the astrolabe or the cross-staff; since then the **sextant** has been almost universally used. The direction of the ship's course is found by the **compass,** and the speed by some form of **"log"** or, in the case of modern steamships, by counting the revolutions of the screw.

The **sextant** is a small, light instrument that can easily be held in the hand. It is represented in Fig. 49. The circular arc, *T*, about 60° long, is graduated to half-degrees which are numbered as whole degrees; the sextant can measure an angle as large as 120°. At the center of the circle is pivoted the **index-arm** *MN*, which carries the **index-mirror** *M* and the index or vernier *N* which reads usually to 10″. The **horizon-**

⌜ Since about 1918, position-finding by wireless telegraphy has been increasingly employed, and it is not unlikely that this method may supersede the astronomic ones.

mirror H is attached to the frame in such a position that when the index reads zero the horizon-glass and index-glass are parallel. The horizon-glass is of about twice the height of the index-glass, and only the lower half is silvered, the upper half remaining transparent. At E is a small telescope directed to the horizon-glass.

The sextant may be used for measuring any angular distance, but its most common use is for finding the altitude of the Sun. With the index near zero, the observer holds the instrument so that the telescope points to the Sun, of which two images appear, one seen directly through the clear half of the horizon-glass and the other by reflection, first from the index-mirror to the horizon-mirror, and then from the latter to the eye. Then, keeping the doubly reflected image in the field of view by moving the index-arm with his left hand, the observer lowers the line of sight of the telescope until he sees the horizon through the horizon-glass and adjusts the index-arm until, as the instrument is rotated slightly about the axis of the telescope, the arc in which the Sun appears to swing comes tangent to the horizon; the reading of the index is then the angular distance of the Sun's limb from the apparent horizon. To get from this the true altitude of the Sun's center, corrections must be applied for dip of the horizon, atmospheric refraction, parallax, and the Sun's semi-diameter.

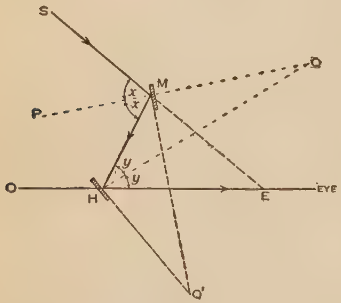

FIG. 50. PRINCIPLE OF THE SEXTANT

That the measured altitude is twice the angle $HQ'M$ (Fig. 50) between the mirrors, which is the angle passed over by the index from zero, is demonstrated as follows:

Let PQ and HQ be the normals to the index-glass and horizon-glass, respectively; OHE the line of sight from the horizon; and $SMHE$ the ray of sunlight reflected by the two mirrors to the eye. By the law of reflection the angles marked x are equal to each other, and the same is true of those marked y. But $2x$ is exterior to the triangle HME, and therefore the angle E, which is the apparent altitude of the Sun's limb, is equal to $2x - 2y$. Similarly, since the lower x is exterior to the triangle HMQ, the angle $Q =$

$x - y$. Hence $E = 2Q$. But $Q = Q'$ since the sides of Q are perpendicular to those of Q'; therefore $E = 2Q'$. Q.E.D.

The **magnetic compass** consists of a steel bar, or set of bars, strongly magnetized, and attached to a graduated circular card which is pivoted at its center in a hemispherical bowl. The bowl is swung on gimbals so as to maintain a horizontal position however the ship may roll or pitch, and on the inside are painted two black lines that mark the direction of the ship's keel; the one toward the bow is called the **lubber's line.** The magnetized bar being held in the magnetic meridian by the force of the Earth's magnetism, the graduation of the card indicated by the lubber's line is the magnetic bearing of the ship's course. As it is only in a few parts of the Earth's surface that the lines of magnetic force are due north and south, this usually differs by some degrees from the true azimuth; but the compass variation has now been determined in most parts of the Earth, and is given for different localities by charts. The **gyroscopic compass,** which depends for its action on the directive effect of the Earth's rotation upon the axis of a rapidly rotating, delicately balanced, massive wheel, gives directly the true azimuth of the ship's keel.

There are many types of **log** for measuring the speed of a ship. The simplest and most primitive, which is still used on some sailing vessels, consists essentially of a wooden "log chip" with a knotted line attached to it, which is thrown overboard. The chip being left behind, the line is drawn across the rail and the speed is determined by counting the knots that pass over in an interval of time that is measured by a watch or sand-glass.

The unit of speed is the **knot,** which is a speed of one **nautical mile** per hour. A nautical mile is the length of a minute of arc measured on the Earth's meridian; on account of the Earth's oblateness this length varies from 6,046 feet at the equator to 6,108 feet at the poles. For practical purposes it is taken as 6,080 feet. The distance between the knots tied in the log-line is such that the number that pass the rail while the sand of the sand-glass is running out equals the speed of the ship in knots.

Latitude by Meridian Altitude.—Latitude is determined at sea most simply by measuring the meridian altitude of the Sun.

A few minutes before apparent noon, the navigating officer brings the image of the Sun's limb into coincidence with the horizon in the field of his sextant, and keeps it there by slowly moving the index-arm until the altitude ceases to increase. This moment of maximum altitude is apparent noon and the Sun is on the meridian. The reading of the sextant, corrected for dip, refraction, parallax, and semi-diameter, is the Sun's true meridian altitude, and subtracting this from 90° gives its zenith distance. In Fig. 51, let the semicircle represent the visible half of the meridian, O the observer, S the Sun, Z the zenith, P the pole, E the intersection of the meridian and the celestial equator, and H and N the south and north points of the horizon, respectively. The ship's latitude, being equal to the altitude of the pole, is represented by the arc PN; the zenith distance of the Sun is SZ; and its declination is ES. The arc EZ, the declination of the zenith, is equal to PN and therefore to the latitude; for each equals 90° minus the arc PZ. But $EZ = ES + SZ$ or, in the customary symbols,

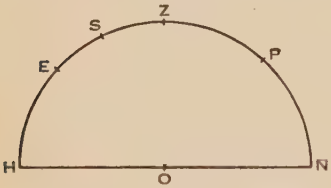

FIG. 51. LATITUDE BY MERIDIAN ALTITUDE

$$\phi = \zeta + \delta,$$

where ϕ denotes the ship's latitude, ζ the zenith-distance of the Sun, and δ the Sun's declination. This latter quantity is given for every day of the year in the nautical almanacs published by the principal governments, having been computed from the elements of the Earth's orbit. If the Sun be south of the equator, its declination is negative, and the equation still holds; and if we agree that zenith distances shall be + when the Sun is south of the zenith and – when it is north, the equation becomes of universal application. A star or other object whose declination is known may be and often is substituted for the Sun.

Time by Altitude Observation and Solution of the *ZPS* Triangle.—The moment of noon cannot be sharply determined by observing the Sun's altitude because the altitude is nearly the same during several minutes before and after noon. Time is determined at sea by observing the Sun or other heavenly body when it is several hours east or west of the meridian. The apparent altitude is measured with the sextant and the usual corrections applied, giving the quantity h in the ZPS triangle (page 68). The declination δ is given by the *Nautical Almanac* and, if the latitude ϕ is known by a previous observation, the hour-angle t may be computed. From the formulæ on page 68 may be obtained by a rather long transformation the formula that is most convenient:

$$\sin^2 \tfrac{1}{2}t = \frac{\sin \frac{1}{2}[\zeta + (\phi - \delta)] \sin \frac{1}{2}[\zeta - (\phi - \delta)]}{\cos \phi \cos \delta}.$$

The object usually observed for time is the Sun, and the resulting hour-angle is the apparent solar time. Since the latitude observation is made at noon and the time observation several hours before or afterward, the change of the ship's latitude must be allowed for by the observations with log and compass, a process known as **dead reckoning.** If a star is observed for time, its right ascension must be added to the hour-angle found, giving sidereal time.

Longitude by Chronometer.—Every sea-going vessel carries one or more **chronometers,** large, accurately made watches which are mounted on gimbals to prevent their partaking of the rolling of the ship and which are regulated to mean solar time. Before leaving port the error of the chronometer on Greenwich mean time, and the rate of change of that error, are carefully determined; and since the advent of wireless telegraphy it is possible to keep account of the error throughout the voyage by the time signals broadcast from various stations. To determine the longitude, the chronometer reading is noted at the moment the time observation is made. The apparent solar time found by the observation is reduced to mean solar time by applying the equation of time given in the almanac; and the difference between this local mean time and the Greenwich mean time given by the chronometer is the required longitude.

In the early days of navigation, no portable timepiece that could be depended on throughout a long voyage had been produced, and the accurate determination of longitude was impracticable. Had Columbus been able to determine his longitude, he would not have mistaken America for the East Indies, but would have recognized it as a new land. Long after Columbus's time, a shipmaster sailing from Liverpool to Rio de Janeiro, for instance, would sail in a generally southwesterly direction until he reached the latitude of Rio, and then sail westward until he sighted land, having in the meantime no dependable knowledge of his distance from port.

Methods of Sumner and Marcq St. Hilaire.—The method of navigation just described, in which the latitude and longitude are found from separate observations, has been largely superseded by a method named for Captain Sumner of Boston, who first used it in 1837. The Sun's zenith distance, given by the sextant observation, determines a **circle of position** on the surface of the Earth, at some point of which the ship is situated. Observation of a second heavenly body, or a later one of the Sun, gives a second circle of position, and the intersection of the two circles, if the ship has not moved between the two times of observation, is the position of the ship. If the ship has moved, her motion may be allowed for by dead reckoning. A little reflection will show that the center of the circle of position, called the **sub-solar point**, is the point where the Sun is at the zenith; the longitude of the sub-solar point is the Greenwich apparent time, which may be found from the chronometer reading by applying the equation of time; and its latitude is the Sun's declination. From these data, the circles of position might be plotted on a terrestrial globe; but a globe of sufficient size would be inconvenient, and a Mercator chart[1] is used instead, only a small portion of the circle of position being plotted, and this portion being drawn as a straight line.

Sumner's method has been further improved by Marcq St. Hilaire, a captain in the French navy. For the details of the two methods the reader must be referred to works on Navigation and Nautical Astronomy.

The Loxodrome or Rhumb Line; Great Circle Sailing.—It might at first sight be supposed that, to sail from San Francisco to Yokohama, which is in nearly the same latitude, the shortest route would be straight westward, along a parallel of latitude. This is not the case, because the shortest distance between two points on the surface of a sphere is an arc of a great circle, while the parallel of latitude passing through either of the above cities is a small circle. A course which has throughout its length the same azimuth (that is, makes the same angle with every meridian crossed) is called a **loxodrome** or **rhumb line.** On a Mercator chart it is represented by a straight line, but due to the convergence of the meridians it is in reality a spiral, and a ship that followed such a course indefinitely would (unless the angle with the meridian were 0 or 90°) approach the pole as a limit, making around it an infinite number of turns. Modern vessels on voyages between distant ports usually follow arcs of great circles. The great-circle course from San Francisco to Yokohama leaves the former port in almost a northwesterly direction, reaches a point as far north as Seattle, and bends southward again, arriving at Yokohama in a southwesterly direction; but it is only 4,536 miles long, whereas the Mercator course is 4,799.

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[1] A Mercator chart represents the meridians and parallels of latitude as straight lines, the parallels intersecting the meridians at right angles. Since the terrestrial meridians really converge to the poles, the map is badly distorted in the polar regions. The name Mercator is the Latinized form of the surname of Gerhard Kramer, a Flemish mathematician employed by Charles V of Spain to make maps for the use of his sailors.

Measurement of the Size of the Earth.—The first intelligent estimate of the size of the Earth of which we have any record is that of Eratosthenes of Alexandria (*c.* 250 B.C.), who used a method of which the underlying principle is essentially that of the best modern determinations. He found that, at Syene, a place almost directly south of Alexandria, at noon of the day of the summer solstice, the Sun cast no shadow in the bottom of a well and was therefore in the zenith; while at the same time its zenith distance as measured at Alexandria was 7° 15′ or one-fiftieth of a circumference. Since the Sun is so distant that its rays reaching the two places are sensibly parallel, this angle, θ, is equal to the angle between vertical lines drawn at the two cities, as is evident in Fig. 52. If the Earth is spherical, as Eratosthenes believed it to be, these lines meet at its center and the angle is measured by the arc *SA*, which is therefore one-fiftieth of the circumference of the Earth. Using the value 5,000 stadia as the distance between the two cities, Eratosthenes calculated that the Earth's circumference was 250,000

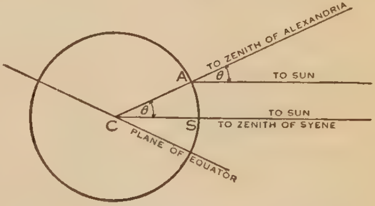


FIG. 52. ERATOSTHENES' DETERMINATION OF THE SIZE OF THE EARTH

stadia, a value that we are not in position to dispute because we do not know the length of his stadium.

The angles made by the verticals *CA* and *CS* with the plane of the equator are, by definition, the astronomic latitudes of the two places of observation and hence the angle *ACS* is the difference of these latitudes. In modern measurements of the Earth, the latitudes of the two stations are determined with the greatest possible precision, usually by the employment of the zenith telescope; and the length of the arc of the Earth's meridian included between their parallels of latitude is determined by the process of **triangulation.**

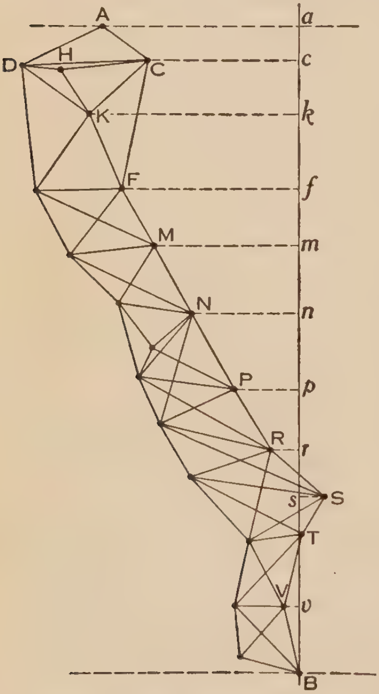


FIG. 53. A TRIANGULATION

Suppose that the latitudes of the points *A* and *B* (Fig. 53), which may be several hundred miles apart, have been accurately measured. To determine the length of a degree of the Earth's surface, and from this the size of the Earth, it is necessary to know the distance *aB*, in miles or kilometers, between the parallels of latitude of the two stations. The direct measurement of so great a distance would involve great difficulties, but in the method of triangulation the only direct measurement of length that is necessary is that of a comparatively short **base line,** as *HK*. This line is laid off on carefully leveled ground, its ends are indicated by permanent marks on stone or concrete, and its length is carefully measured. In the recent work of the United States Coast and Geodetic Survey, the measurement of the base line is made with steel tape of known temperature, and the distance between two marks ten miles apart is determined with an error not exceeding a few millimeters. In the remaining operations, points *C*, *D*, *F*, etc., are chosen which are each visible from at least two other points of the series and which are as definitely marked as are *H* and *K*. The lines joining these points thus form a network of triangles joining *A* and *B*, the angles of which, and also the azimuths of many of the lines, are measured with accurate theodolites. For example, the theodolite is set up over the point *K* and its telescope directed first to *H* and then to *C*, when the difference of the readings of the horizontal circle gives the angle *HKC*. The azimuth of *KC* is 180° plus the difference of the circle readings when the telescope is directed, first to *C* and then to Polaris at the time of its meridian passage. The angles *KHC* and *HCK* are measured by setting the theodolite over the points *H* and *C* respectively. Having the length *HK* and the angles of the triangle *HKC*, the sides *HC* and *KC* are computed by trigonometry; these then serve as known lengths for solving the triangles *HAC* and *CKF*, and so on until the chain is completed. The desired length *aB* is the sum of the projections *ac*, *ck*, *kf*, etc., of the lines *AC*, *CK*, *KF*, etc., upon the meridian, the length of each projection being that of the corresponding line multiplied by the cosine of its azimuth. The distance *aB* divided by the number of degrees in the difference of latitude between *A* and *B* is the length of a degree; and, on the assumption of a spherical Earth, this number multiplied by 360 is the Earth's circumference.

The method of triangulation was first applied in 1617 by Snell, who made a series of measurements in the flat country of Holland from which he deduced a length of the degree of about sixty-seven miles, a value later changed to about sixty-nine miles by one of his pupils, who found an error in the original calculations. In 1671 Picard, from measurements made near Paris derived a length of about sixty-nine miles, and it was this determination that enabled Newton to verify the law of gravitation (page 222).

The Form of the Earth.—It was inferred by Newton from the fact of the Earth's rotation and has been proved by pendulum observations (page 64), that the Earth is not spherical, but oblate. The mean surface of the Earth (neglecting such minor irregularities as mountains and valleys) is a *level* surface—that is, a surface which is at every point normal to the direction of gravity; hence, vertical lines at two places near the equator must converge to a point C_1 (Fig. 54) which lies between the surface and the Earth's center, while verticals near the pole meet at C_2, beyond the center. For a difference of latitude of one degree, therefore, the distance between two stations near the pole must be greater than that between two stations near the equator; and this is what is found by actual measurement.

In the early part of the eighteenth century, Cassini was of the opinion that the opposite was true and that the Earth was prolate; but Picard's arc was extended to 9° in France, and other arcs were measured by French astronomers in Peru and Lapland, and by 1745 these measures had established beyond a doubt that the degree of latitude increases in length from the equator to the pole. In the last century extensive arcs have been measured in many different countries and the form and size of the Earth determined with great accuracy.

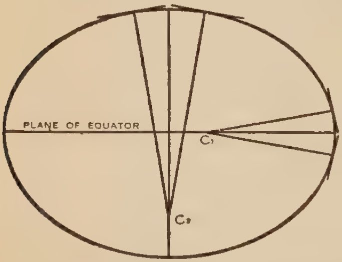


FIG. 54. LENGTH OF A DEGREE ON THE SURFACE OF A SPHEROID

From the way in which the length of a degree varies in different parts of the Earth's surface the precise form and size of the Earth are computed, but the process is too complicated to be described here. The most reliable measurements do not fit exactly into any mathematical figure. The equator at sea level is almost, but not quite, a circle, and each meridian is very nearly, but not quite, an ellipse, the departures from these curves being much less than a mile. The results of the very extensive measurements of the United States Coast and Geodetic Survey are summed up in Hayford's *Spheroid of 1909*, from which are derived the following dimensions of the Earth:

Equatorial Radius, $a = 6{,}378.388$ km. $= 3{,}963.34$ mi.
Polar radius, $b = 6{,}356.909$ km. $= 3{,}949.99$ mi.
Oblateness, $(a - b)/a = 1/297$

and the following values of the length of a degree in various latitudes:

Latitude	Length of a Degree
0°	68.708 miles
15	68.757
30	68.882
45	69.056
60	69.231
90	69.407

The Theory of Isostasy.—It was suggested by British geodesists in the middle of the nineteenth century, and has been established by Hayford from measurements in the United States, that the departures of the figure of the Earth from a perfect oblate spheroid are due to the fact that the outer portion of the body of the Earth is not homogeneous, and that the higher portions, such as the continents, are composed of and underlaid by material of lesser density, while the depressed portions, such as the floors of oceans, are composed of denser stuff. In other words, the continents rise above the level of the ocean because they are floated. This condition is known as one of **isostasy** or isostatic adjustment.

EXERCISES

1. What are the approximate dip and distance of the sea horizon as seen from a ship's deck 50 feet above the water? From an airplane 10,000 feet above the water?

Ans. 7′, 8⅔ miles; 1° 40′, 122 miles

2. What angle is made by the celestial equator with the horizon in latitude 40°?

Ans. 50°

3. What angle is made by the ecliptic and the horizon, in latitude 40° north, at sunset on March 21? At sunrise on March 21?

Ans. 73°.5; 26°.5

4. Prove that, in latitudes greater than 48°.5, twilight lasts all night on the shortest night of the year.

5. Trace on a terrestrial globe the great-circle course of an airplane flying from New York to Paris; from Santiago, Chile, to Cape Town, South Africa; from Nome, Alaska, to Archangel, Siberia.

6. In latitude 40° north, what are the altitude, azimuth, hour angle, and declination of the zenith? Of the north celestial pole?

7. Toward what point of the celestial sphere is the observer carried by the Earth's rotation?

Ans. The east point of the horizon

CHAPTER IV

THE ORBITAL MOTION OF THE EARTH

The Sun's Apparent Motion Explained by a Real Motion of the Earth.—In Chapter I (page 23) we described the apparent annual motion of the Sun among the stars, and remarked that this apparent motion is really due to an orbital motion of the Earth. This view was held as long ago as 280 B.C. by Aristarchus of Samos, but his successors, Apollonius, Hipparchus, and Ptolemy, rejected it in favor of the stationary Earth, and their ideas were held almost universally until the time of Copernicus (1473–1543), and pretty generally until at least a century later.

That it is at least possible that the apparent motion of the Sun is produced by a real motion of the Earth around the Sun is shown in Fig. 55. Late in September, for example, the Earth is in such a position that the Sun appears in the direction of the stars of the constellation Virgo; Libra, on one side, and Leo, on the other, are too nearly in the Sun's direction to permit their stars to be easily seen; Scorpius is visible only in the early evening, being just east of the Sun, and Cancer may be seen just before dawn, west of the Sun; while at midnight, the stars of Pisces, near the vernal equinox, are on the observer's meridian. As the Earth moves in its orbit, with perfect silence and smoothness so that we are unaware of its motion except by observation of the heavenly bodies, the direction of the Sun from us changes, and by January it hides the stars of Sagittarius, Scorpius has apparently emerged upon the west side of the Sun and can be seen before dawn, Aquarius is low in the west at twilight, and Gemini is on the meridian at midnight. The ecliptic, or Sun's apparent path, is thus explained as the projection of the Earth's orbit upon the celestial sphere as seen from the Sun, and, if we are to adopt the belief that the Earth moves, since the ecliptic is known to be a great circle, we must admit that the Earth moves always in the same plane and that this plane passes through the Sun.

The arc of the ecliptic, measured eastward from the vernal equinox to the apparent position of the Sun as seen from the Earth, is, according to the definition in Chapter I (page 25), the **geocentric longitude** of the Sun. Since, for an observer on the Sun, the Earth would appear exactly at the opposite point of the celestial sphere, the **heliocentric longitude** of the Earth differs from the geocentric longitude of the Sun by just 180°. About March 21, for example, the Sun appears at the vernal equinox and its geocentric longitude is therefore 0°, while at the same time the Earth's heliocentric longitude is 180°.

Proofs of the Earth's Motion.—That, relative to landmarks among the stars, it is the Earth and not the Sun that moves in a great orbit is shown by three different phenomena, the discovery of which required exact measurement with instruments that were not available until long after the time of Copernicus. These are the aberration of light, discovered by the English astronomer Bradley in 1727; the annual parallactic displacement of the nearer stars, first detected by Bessel in Germany in 1837; and the annually periodic variation in the radial velocities of stars, which has been observed since the first work of Sir William Huggins on stellar spectra in 1864. It will be convenient to consider these proofs in the reverse order of their discovery.

The Annual Variation in the Radial Velocities of Stars.—By the **radial velocity** of a star or other object is meant the rate at which the distance between the object and the observer is changing. If this distance is increasing, the radial velocity is said to be positive; if diminishing, negative. In such remote objects as the stars, the change of distance produced even in many centuries by any known velocity is not sufficient to make any perceptible change of brightness or appearance; but the radial velocity can be readily detected and measured by observations on the stars' spectra, as explained in the discussion of the Doppler-Fizeau principle in Chapter VII (page 167). In this way, the radial velocities of many stars and nebulæ are determined to within a small fraction of a kilometer per second.

From a consideration of Fig. 55 it may be seen that, if the Earth revolves around the Sun, it will at some time of the year be traveling directly toward any given star on the ecliptic, while six months later it will be moving directly away from the same star. In October, for example, the orbital motion must carry us toward the stars of Gemini and away from those of Sagittarius, while in April the case must be reversed. Observation shows that, although the stars have motions of their own in various directions, the radial velocity of those in Gemini is about sixty km./sec. greater, and of those in Sagittarius about sixty km./sec. less (having due regard to sign) in October than in April and that similarly, in other parts of the zodiac, there is a variation of radial velocity in harmony with the idea that the Earth revolves yearly in a nearly circular orbit and with a velocity of about thirty kilometers (eighteen miles) a second.

This annual variation of radial velocity is of course less for stars that are not on the ecliptic, since they are not directly in the plane of the Earth's motion. It is strictly proportional to the cosine of the star's latitude, and vanishes at the ecliptic poles. Among the stars are many **spectroscopic binaries** (page 338), which have orbital motions of their own around companion stars and so periodically recede from and approach the Earth; but in every such case the radial velocity due to this motion of the star is superposed upon that due to the orbital motion of the Earth, and the two effects can readily be disentangled.

The Annual Parallactic Displacement of the Stars.—It was remarked by Aristotle that, if the Earth traveled in a great orbit as Aristarchus believed it to do, we should be brought into different regions of the stars at different times of year, and this would change the appearance of the constellations. Not being able to perceive any such changes, Aristotle concluded that the Earth was immovable. His fallacy lay in his failure to appreciate the enormous distances of the stars. The fact is that the Earth's motion does produce changes of apparent position, but the stars are so remote that these changes are too minute to be detected except by careful measurements with the telescope.

Suppose for a moment that a star were fixed at the point *A* in Fig. 55 and that the other stars were all much more remote. If the Earth did not move, the star *A* would remain always in the same direction from us and would appear fixed among the constellations; but if the Earth revolves in the orbit shown, the star *A* must, in October, appear among those of the constellation Gemini and in April among those of Taurus. This apparent annual shift of a near star among its remote fellows is called its **parallactic displacement.**

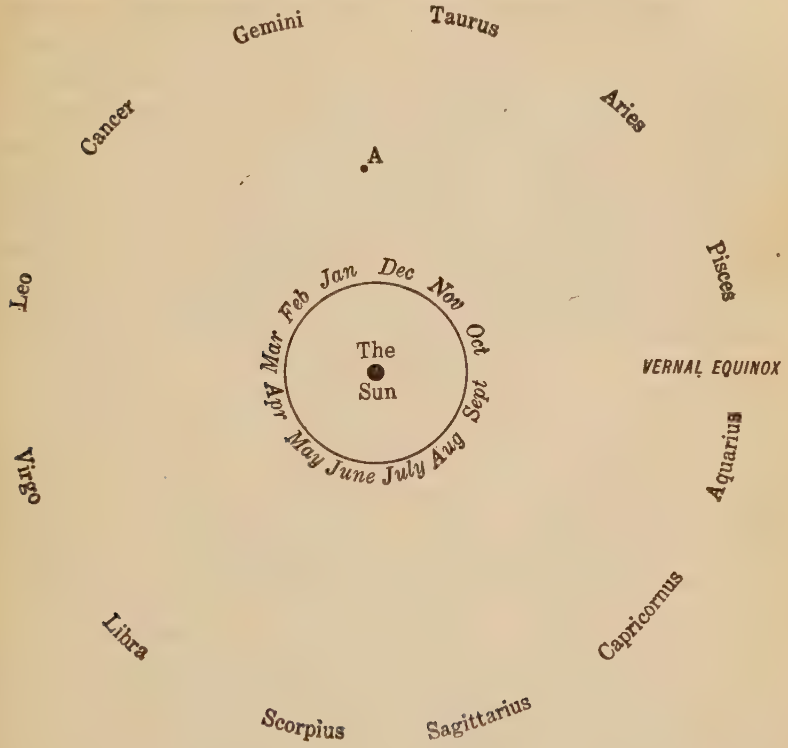

Gemini
Taurus
Cancer
Aries
A
Leo
Pisces
Jan
Dec
Feb
Nov
Mar
Oct
The
Sun
Apr
Sept
May
June
July
Aug
VERNAL EQUINOX
Virgo
Aquarius
Libra
Capricornus
Scorpius
Sagittarius

FIG. 55. ANNUAL APPARENT MOTION OF THE SUN

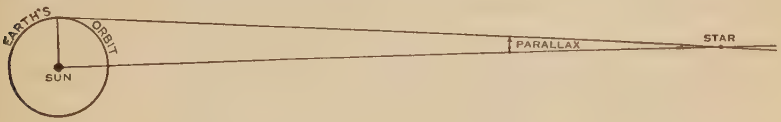

EARTH'S
ORBIT
SUN
PARALLAX
STAR

FIG. 56. HELIOCENTRIC PARALLAX

The **heliocentric parallax** is the maximum value of the parallactic displacement, counted from the star's mean position, and is the angle subtended at the star by the semidiameter of the Earth's orbit (Fig. 56). The heliocentric parallax is inversely proportional to the distance of the star and its measurement affords the most direct means of determining that distance (page 309). Instead of being at the order of distance suggested in the figures, even the nearest star is so remote that its heliocentric parallax is very minute—only 0".78—and to draw Fig. 56 in correct proportion would necessitate drawing a triangle more than a mile long, the diameter of the circle representing the Earth's orbit being a half-inch. All other known stars have still smaller parallaxes, and the vast majority of stellar parallaxes are too small for measurement even by the most modern methods. Enough, however, have been accurately measured to leave no doubt of their existence or of the motion of the Earth.

The Aberration of Light.—Suppose that rain is falling vertically and that it is desired to catch a raindrop in a narrow, straight tube in such a manner that the drop will enter the center of the top and fall upon the center of the bottom without touching the side. If the tube be stationary, this can be accomplished by simply holding it in an upright position, as at *A* in Fig. 57. But suppose the tube to be held by a person who is walking toward our right. It is now necessary to incline the tube forward, as at *B*, at an angle α, such that, while the raindrop is falling through the height V of the tube, the bottom moves a distance u to the point vertically below the position that was occupied by the top of the tube at the moment the raindrop entered it. The tube must thus form the diagonal of a rectangle of which the vertical and horizontal sides V and u are proportional to the velocities of the raindrop and tube respectively. The relation connecting the three quantities is

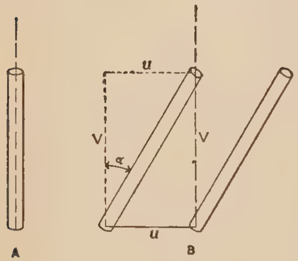


FIG. 57. ABERRATION

$$\tan \alpha = \frac{u}{V}$$

or, if the person be walking very slowly so that u is very small compared with V,

$$\alpha = \frac{u}{V} \times 206{,}265''.$$

A similar effect, called **aberration,** is produced by the orbital motion of the Earth upon the apparent direction of the light that comes to us from a star. The place of the tube in the above illustration is supplied by a telescope in which it is desired to catch the light-waves from a star in order that an image of the star may be formed upon the cross-wires or seen in the eyepiece. It is, in fact, necessary to direct the telescope *ahead* of the star's true position by an angle α which is equal to $u/V \times 206{,}265''$, where u is the velocity of the Earth in its orbit and V is the velocity of light. If the Earth were fixed among the stars, this would not be the case, and if it moved forever uniformly in a straight line we should be unaware of the existence of aberration, since the displacement would be constant and we should be ignorant of the star's true position. If, on the other hand, the Earth revolves in a closed orbit, the amount and direction of the effect of aberration on any given star must be different at different times of the year, and observation shows this actually to be the case.

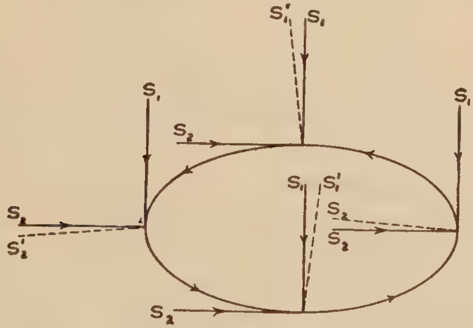


FIG. 58. ABERRATION IN DIFFERENT PARTS OF THE SKY

The effect of aberration upon the apparent positions of stars in different parts of the sky may be studied in Fig. 58, which represents the Earth's orbit in perspective as if seen from a point outside the plane of the ecliptic, with the Earth in four positions, A, B, C, and D. Rays of light are represented as coming from two stars: S_1, at the pole of the ecliptic, and S_2, on the ecliptic itself. When the Earth is at A, S_1 is displaced toward the left to S_1', but S_2 is not displaced at all because, as the Earth is going straight to meet its light, the effect is the same as if the tube, in the raindrop analogy, were carried endwise upward. At B, the Earth is moving at right angles to both light rays, and both stars are displaced, by the same amount, in the direction of the imaginary onlooker. At *C*, the conditions are the reverse of those at *A*: S_1 is displaced toward the right, and S_2 not at all. At *D*, the conditions at *B* are reversed. A little thought will show that a star on the ecliptic, as S_2, seems to oscillate in a straight line through its true position, and that one at the pole of the ecliptic, as S_1, is always displaced, but in an ever-changing direction, so that, in the course of a year, it appears to describe a small, closed orbit on the surface of the celestial sphere. Stars between the ecliptic and its poles describe closed aberrational orbits which vary in form with the celestial latitude, as shown in Fig. 59.

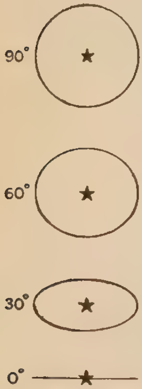


FIG. 59. ABERRATIONAL ELLIPSES

The apparent displacement of a star, produced by aberration when the Earth, traveling at its mean speed, is moving at right angles to the star's light, is called the **constant of aberration.** Observation shows that its value is very nearly $20''.5$. The mean orbital velocity of the Earth is therefore

$$u = \frac{20.5 \times V}{206,265}$$

or slightly less than one ten-thousandth of the velocity of light.

Determination of the Velocity of Light.—To find out the Earth's orbital velocity in miles or kilometers per second from the observed value of the constant of aberration, it is necessary to know *V* in similar units; and, as will be shown in Chapter VII (page 169), the velocity of light is needed also for the determination of the radial velocities of stars by spectroscopic observations.

That light does not travel instantaneously was demonstrated in 1675 by Olaus Römer in Denmark by a careful study of the times of the eclipses of Jupiter's satellites. Three of the bright satellites discovered by Galileo in 1610 are eclipsed (Fig. 139) regularly at every revolution, and so by observing the interval between eclipses during a given season Römer could predict the times of future eclipses. He found that, if the predictions were based upon observations made when the Earth was on the side of its orbit nearest Jupiter, the eclipses that occurred during the succeeding months came increasingly later than the predicted times until the Earth was on the opposite side of its orbit; then, as the Earth overtook Jupiter again, the error of the prediction diminished, and when the two planets were again at their shortest distance the eclipses were once more on time. Römer correctly attributed this apparent delay of the eclipses to the time required for the light reflected by the satellites to overtake the Earth, and concluded from his observations that light requires about 600 seconds to travel the distance from the Sun to the Earth, a figure that has been changed by subsequent investigations to 499 seconds.

This study is complicated by the fact that the motions of the satellites are disturbed by their mutual attractions, and is also rendered inaccurate by the slowness with which the disappearance and reappearance of the satellites take place which makes it impossible to observe the exact times of the eclipses. Moreover, to determine the velocity of light in this way one must know the distance from the Earth to the Sun in miles or kilometers, and this distance is not known so accurately as is the velocity of light as determined by modern experimental methods; and so the problem is now reversed and the times of the Jovian eclipses are used for finding with greater accuracy the distance of the Sun.

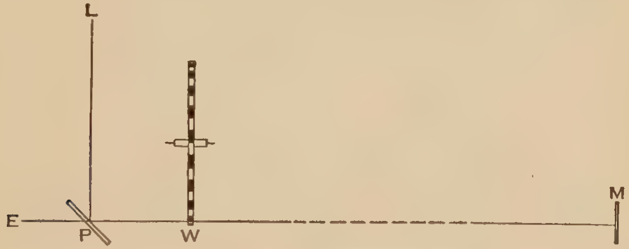


FIG. 60. FIZEAU'S METHOD OF DETERMINING THE VELOCITY OF LIGHT

The experimental methods of determining *V* are based upon two devices, known as the methods of Fizeau and of Foucault, having been first used by those French physicists in 1849 and 1850. The essential element in the former is a toothed wheel, and in the latter a rotating mirror. The principle of Fizeau's method is illustrated in Fig. 60. Light from a bright lamp at *L* is partly reflected and partly transmitted by the thinly silvered glass plate *P*. The reflected portion proceeds between two teeth of the wheel *W* to a distant mirror *M*, is returned to *P*, where it is again divided, and the transmitted portion of the returning ray is received by the observer's eye at *E*. When the wheel is rotated, the light is interrupted and proceeds in flashes which, however, unless the speed of the wheel is very low, are too brief to be perceived, and the eye has the impression of a continuous beam. This is seen until the speed of the wheel becomes so great that, while the light travels from *W* to *M* and back, a tooth has moved to the position of the space through which the light passed and so stops it on its return, in which case no light is seen. If the speed is made twice as great, the flash that escaped through one space returns through the next and the light reappears. Knowing the number of teeth on the wheel and also the speed when the light first disappears, the experimenter computes the time taken by a tooth in replacing a space, which is the same as the time taken by light in traveling the double distance between *W* and *M*.

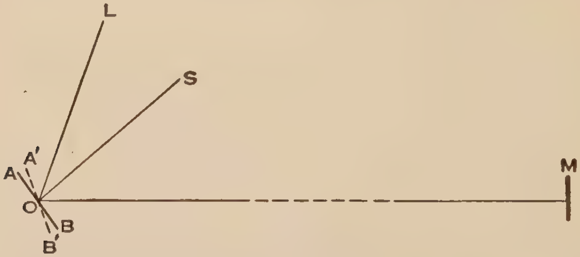


FIG. 61. FOUCAULT'S METHOD OF DETERMINING THE VELOCITY OF LIGHT

In Foucault's method, light from *L* (Fig. 61) falls upon a mirror *AOB*, which may be rotated about an axis parallel to its own plane, and is reflected to the distant mirror *M* which returns it to *O*. Unless the rotating mirror occupies the position *AOB*, the light will not be reflected by it to *M*, but in some other direction; if it does occupy that position, the returning beam will be reflected back to *L*. Suppose the mirror to be rotated slowly. A flash will be sent to *M* at each rotation, and will return to *O* and be reflected very nearly to *L*. Suppose the speed to be increased so that, while the flash has traveled from *O* to *M* and back, the mirror's position has changed to *A'OB'*. The returning beam will then be reflected to *S*, and by measuring the angle *LOS*, the angle *AOA'* through which the mirror has turned (which is, by the law of reflection, just half as great as the angle LOS) can be determined; and then, knowing the speed of the mirror in rotations per second, the experimenter can determine the time taken by it to turn through the angle AOA' and hence the time taken by light to traverse twice the distance OM.

In the best determinations, the distance WM (Fig. 60) or OM (Fig. 61) is several miles, and the strength of the beam is reinforced by means of condensing and collimating lenses or mirrors. During the summers of 1921 to 1926, Michelson used a rotating octagonal mirror on Mount Wilson, which reflected a beam of light from a powerful arc lamp to Mount San Antonio, 22.5 miles away, whence it was returned by a mirror and caught by the adjacent face of the octagon to be reflected to the eye. The distance between the mirrors was determined by triangulation by the United States Coast and Geodetic Survey with an error not exceeding one part in two million (about a centimeter). The octagonal mirror, mounted on jeweled bearings, was made to rotate by blasts of air which played upon metal vanes attached to its upper surface, and the speed was such (about five hundred rotations per second) that the impact of the air produced a shrill scream that could be heard over a distance of a quarter of a mile; and yet the speed of light is so great that it traveled forty-five miles while this mirror made one-eighth of a turn!

Michelson's result, which he believes to be correct within one part in 20,000 (14 km./sec.), is

$$V = 299{,}796 \text{ kilometers per second,}$$

or

$$V = 186{,}284 \text{ miles per second.}$$

The Orbital Velocity of the Earth and the Distance and Dimensions of the Sun.—Calculations of the Earth's orbital velocity from the constant of aberration by the formula $u = V \tan \alpha$, and from the observed annual variation in the radial velocities of stars, agree on a mean value of 29.8 kilometers per second (18.5 miles per second).

The circumference of the orbit may be obtained by multiplying this number by the number of seconds required for the Earth to complete one revolution—that is, the number of seconds in the sidereal year, which is 31,558,150. Assuming the orbit to be circular, which is very near the truth, its radius, or the distance from the Sun to the Earth, may be found by dividing by 2π. The result is

$$R = 149{,}500{,}000 \text{ kilometers} = 92{,}900{,}000 \text{ miles.}$$

This number, which is exceedingly important in astronomy, has been confirmed to about one part in a thousand by many independent methods of measurement. The distance is so great that to traverse it an aviator, flying at a speed of 150 miles an hour, would have to fly continuously for seventy years. Sound travels in air at a speed of a fifth of a mile a second; but, if interplanetary space were filled with air and an explosion occurred on the Sun that was loud enough for the sound to reach the Earth, it would be fifteen years before we could hear it. Light, which has the greatest of known velocities, travels the distance in a little over eight minutes. The Sun's mean equatorial horizontal parallax (page 16) is 8″.80.

The mean apparent diameter of the Sun being half a degree, or, more exactly, 1920″, its real diameter is $\frac{1920 \times 149{,}500{,}000}{206{,}265}$ kilometers = 1,391,000 kilometers or 864,000 miles. This is 109.1 times the diameter of the Earth, and since the volumes of spheres are proportional to the cubes of their diameters, the volume of the Sun is 109.1^3, or about 1,300,000, times that of the Earth. Thus, great though the Earth is, it fills less than a millionth of the space occupied by the Sun.

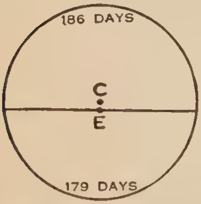


FIG. 62. ORBIT OF THE SUN ACCORDING TO HIPPARCHUS

The Form of the Earth's Orbit.—As long ago as 120 B.C. it was noticed by Hipparchus that the time occupied by the Sun in its apparent motion from the vernal to the autumnal equinox was 186 days, whereas to go from the autumnal to the vernal required only 179; and, as it was contrary to his sense of the fitness of things that a *heavenly* body should move otherwise than with uniform speed in a *perfect* curve (*i.e.*, a circle) he inferred that the Earth was placed eccentrically within the orbit of the Sun. The unequal division of the circle by the straight line passing through the Earth and the equinoxes explained satisfactorily the observed difference of seven days, as may be seen in Fig. 62.

The investigations of Kepler, seventeen centuries after

Hipparchus, showed that the orbit of each of the planets was an ellipse instead of a circle, that the Sun occupied the focus of the ellipse, and that the motion of the planet was not uniform; and Newton proved that these facts were necessary consequences of the attraction of the Sun according to the Law of Gravitation. Observational evidence of the form of the Earth's orbit may be obtained by measuring the apparent diameter of the Sun in different longitudes, for, there being no reason to suppose that the Sun's real diameter changes with the position of the Earth, any apparent change in its diameter must be attributed to a variation of the distance of the Earth.

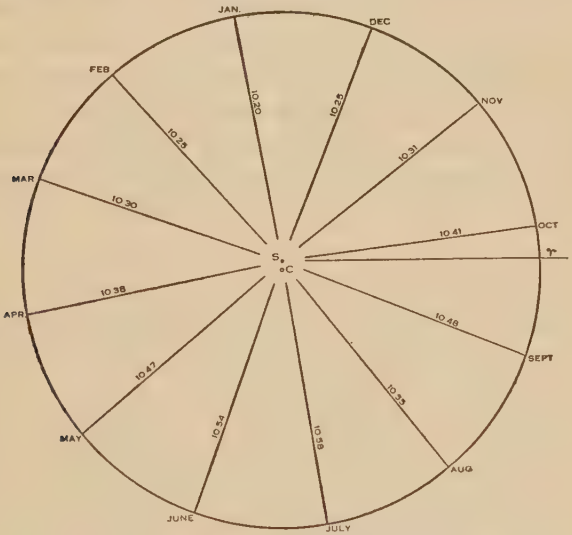


FIG. 63. TRUE FORM OF THE EARTH'S ORBIT

The form of the Earth's orbit is shown in Fig. 63, which was constructed as follows: The Sun's center is represented by the black dot *S*, and the line *S* ♈ is taken as the direction of the vernal equinox. From *S* are drawn lines which make with the line *S* ♈ angles equal to the heliocentric longitude of the Earth at the first of each month, which is just 180° different from the geocentric longitude of the Sun given in Column 2 of Table 3 (page 100). Column 3 of the table gives the apparent semidiameter of the Sun for the same date, and Column 4 gives the result of dividing 10,000, an arbitrarily chosen number, by the numbers in Column 3. Since the apparent diameter increases as the distance diminishes, the numbers in Column 4 are therefore proportional to the distance from the Sun to the Earth. The radiating lines of Fig. 63 are cut off to lengths proportional to the numbers of Column 4, and the curve drawn through their extremities therefore has the form of the Earth's orbit. It is so nearly circular that the eye cannot distinguish it from a circle, but its center, which is at the point *C*, is very obviously not at the Sun. The appropriate mathematical treatment of the relation between the Earth's distance and its heliocentric longitude shows that the orbit is an ellipse having the Sun at one focus.

Definitions Relating to the Earth's Orbit.—An **ellipse** may be defined as a curve, every point of which is so situated that the sum of its distances from two points within, called the **foci,** is a constant. This definition affords a simple means of drawing an ellipse, which is illustrated in Fig. 64. Two pins are thrust through the drawing paper into a board, a loop of thread is tied loosely around them, and the thread is then kept stretched tight by a pencil, while the pencil is drawn around

TABLE 3

Date	Sun's long., λ	Apparent semidiameter, s	$10{,}000/s$
January 1	280°	978″	10.20
February 1	311	975	10.25
March 1	340	970	10.30
April 1	11	962	10.38
May 1	40	954	10.47
June 1	70	948	10.54
July 1	99	945	10.58
August 1	128	947	10.55
September 1	158	953	10.48
October 1	187	960	10.41
November 1	218	969	10.31
December 1	248	975	10.25

the pins. The sum of the distances of the pencil from the pins is thus kept constant, being equal to the whole length of the loop minus the distance between the pins, and so the pencil traces an ellipse which has the pins as foci.

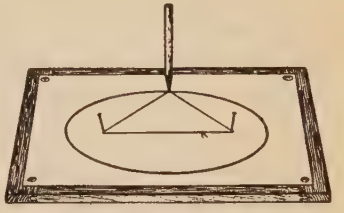


FIG. 64. HOW TO DRAW AN ELLIPSE

In Fig. 65, the curve represents an ellipse of which the foci are at F and S. The point C, midway between them, is called the **center.** The longest diameter, AP, which is the one that passes through the foci, is the **major axis,** and the shortest diameter, BD, which is at right angles to the major axis, is the **minor axis.** The semi-major axis, CP, is often denoted by the letter a, the semi-minor axis by b, and the distance, CS, from center to focus, by c. The **eccentricity** of an ellipse is defined as the ratio

$$e = \frac{c}{a}$$

or the ratio of the distance between the foci to the major axis. The eccentricity of an ellipse is never greater than 1, and the less the eccentricity the more nearly circular is the ellipse. A circle is an ellipse of eccentricity zero. The eccentricity of the Earth's orbit is 0.016.

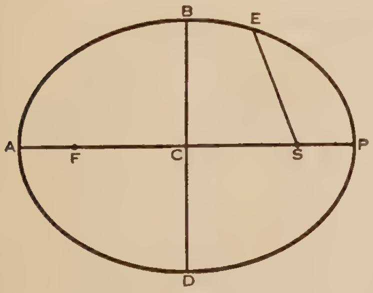


FIG. 65. AN ELLIPSE

Definitions relating to the Earth's orbit may perhaps be better illustrated in Fig. 65, in which the eccentricity is exaggerated, than in the more accurate Fig. 63. Since the Sun is at one focus, let it be represented by S. The point of the orbit nearest the Sun, which is the end P of the major axis, is called the **perihelion;** the most distant point, A, the **aphelion.** As may be seen in Fig. 63, the Earth is at its perihelion about January 1 and at its aphelion about July 1.

Either the perihelion or the aphelion is sometimes referred to as an **apse,** and the line of indefinite length that passes through them and through the foci, of which the major axis is a segment, is known as the **line of apsides.**

The line SE, connecting the Sun with the Earth at any point of its orbit, is called the Earth's **radius vector,** and the angle PSE, made by the radius vector with the line of apsides, is its **true anomaly.**

Variation of the Earth's Speed. The Law of Areas.—The orbital speed of the Earth is not uniform, but varies in accordance with a famous principle which was discovered by Kepler and which is known as the **law of areas.** This law is: *The radius vector of the Earth passes over equal areas in equal intervals of time.* If the Earth move from A to B (Fig. 66) in a given length of time, say a week, then the distances CD and EF, over which it passes in a week when in other parts of its orbit, are such that the areas ASB, CSD, and ESF are all equal. Thus, the orbital speed is greatest at perihelion and least at aphelion.

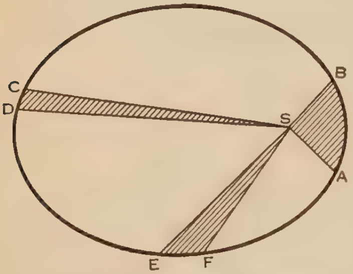


FIG. 66. THE LAW OF AREAS

The **angular velocity** of the Earth, which is the rate of increase of its heliocentric longitude or of its true anomaly, is related in a simple manner to the length of the radius vector. The area described by the radius vector in a short interval of time is very nearly equal to that of the triangle bounded by the two limiting radii vectores and their chord. Let θ be the angle CSD in the slender triangle formed by the radii vectores CS and DS. Taking DS as the base of this triangle, its altitude is $CS \sin \theta$ and its area is $\frac{1}{2}$ $DS \times CS \sin \theta$. If the interval be very short, say one second (the Earth will have traveled only eighteen miles in that time), the sine of θ will be practically equal to the value of θ in radians, and the two radii vectores will be practically equal in length. Call their length r. The area passed over in a second is then equal to $\frac{1}{2} r^2\theta$. Since, according to the law of areas, this quantity is constant for all parts of the orbit, we have the general principle, *The product of the square of the radius vector and the angular velocity is a constant.*

The law of areas makes it possible to compute the length of the radius vector and the value of the true anomaly when the time that has elapsed since perihelion passage is known; but the rigorous solution of this problem, which is known as **Kepler's problem,** involves mathematical principles of some difficulty.

The Seasons and the Climatic Zones.—With the exception of the internal heat of the Earth, which has but slight effect at the surface, our sole appreciable source of heat and light is the Sun. If this supply were permanently cut off from the whole Earth, as on the rare occasions of total eclipses of the Sun it is cut off for a few minutes from a small region, the temperature would fall very rapidly and all activity and life would cease within a few days. The question is sometimes asked, Why do we not have the warmest weather in January, when the Earth is nearest the Sun? The fact is that the Earth as a whole does receive the most heat in January, but its orbit is so nearly circular that the difference between the perihelion and aphelion distances is too small—about 3,000,000 miles—compared with the mean distance of 92,900,000 miles to make any obvious difference in the temperature.

The changes of season which the Earth experiences are due mainly to the fact that the axis about which the planet rotates is not placed at right angles to the plane in which it revolves. It will be remembered that the ecliptic and celestial equators are the traces upon the celestial sphere of the planes of the Earth's orbit and equator, respectively, and that the angle between these circles is $23^\circ\!.5$; therefore, since the Earth's axis makes a right angle with the plane of the equator, it must make an angle of $66^\circ\!.5$ with the plane of the orbit. Since the equinoxes are, throughout the year, practically fixed among the stars, the axis must remain practically parallel to the position that it had at the beginning of the year. These relations are shown in Fig. 67.

The position of the Earth in relation to the rays of the Sun in June is shown in Fig. 68, and in December in Fig. 69. In the northern hemisphere, the weather is warmer in June than in December for two main reasons. These may be understood by considering the point *O*, which may represent a locality somewhere in the United States. The first reason is that in June the rotation of the Earth keeps *O* a longer time in the sunlight than in the shadow, while in December these conditions are reversed. In summer, therefore, more heat is received from the Sun every twenty-four hours than can be radiated away, and so the temperature continues to rise until some weeks after the summer solstice, when the loss of heat equals the inflow. The second reason is that, in June, the rays of sunlight fall more nearly normally upon the surface and a beam of sunlight of a given cross-section is spread over a smaller area of the soil than the same beam can cover in December, when it falls more obliquely. This may readily be seen by noting the arc of the Earth's surface near *O* that is included

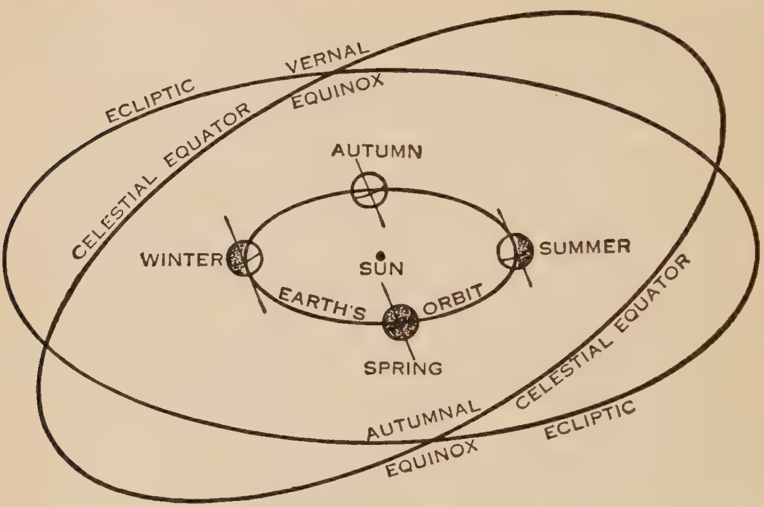
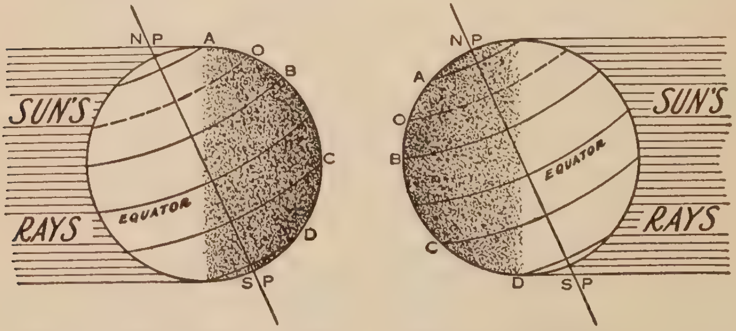

VERNAL
EQUINOX
ECLIPTIC
CELESTIAL EQUATOR
AUTUMN
WINTER
SUN
SUMMER
EARTH'S
ORBIT
SPRING
CELESTIAL EQUATOR
AUTUMNAL
EQUINOX
ECLIPTIC

FIG. 67. THE SEASONS


N P
A
O
B
SUN'S
C
EQUATOR
RAYS
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S P
N P
A
O
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SUN'S
EQUATOR
RAYS
C
D
S P

FIG. 68. THE EARTH IN JUNE

FIG. 69. THE EARTH IN DECEMBER

between two of the parallel sun rays in the two figures. A point in the southern hemisphere, at the same distance from the equator as that of *O*, has similar changes of season, but at opposite times of year, the warm weather occurring in January and the cold in July.

The existence of climatic zones is due to the same cause as the change of seasons. The circles *A* and *D*, which are the parallels of latitude 66°.5 from the equator, are known respectively as the **arctic** and **antarctic circles,** and inclose the **frigid zones,** at every point of which, on at least one day of the year, the Sun does not set and at the opposite time of year does not rise. The parallels *B* and *C*, 23°.5 from the equator, are the **tropics of Cancer** and **Capricorn,** between which lies the **torrid zone,** where, at some time of the year, the Sun may be seen at the zenith. Between the torrid and the frigid zones are the **temperate zones.**

The Calendar.—The solar day as a unit of time has already been discussed, and it was seen that this unit was divided artificially into hours, minutes, and seconds, subdivisions that are commensurable with the day. For the designation of long periods of time, however, the day is inconveniently short, and for a longer unit we naturally adopt the interval between successive recurrences of a given season. Since the seasons are determined by the position of the Earth's axis with reference to the Sun, which also determines the position of the equinoxes, the natural unit thus forced upon us is the **tropical year,** defined as the interval between successive arrivals of the Sun at the vernal equinox. Its length is very nearly 365.24219 mean solar days, but the fraction is not exactly known, and probably the exact value could not be expressed by a finite number of digits following the decimal point. This circumstance, the incommensurability of the day and the year, makes the measurement of time more complicated than other kinds of measurement, such as that of length, weight, or value, in which the larger units are defined arbitrarily so as to contain an exact number of the smaller; a foot, for example, being exactly twelve inches, and a dollar exactly one hundred cents.

The calendar, or system of keeping account of time over long intervals, is in most nations involved with the celebration of religious festivals. In Christian countries, the calendar now used is that known as the Gregorian, having been made the official calendar of the Roman Catholic Church by Pope Gregory XIII in 1582. This calendar superseded the Julian calendar, instituted in Rome by Julius Cæsar. The ancient Romans had used a system of time-reckoning in which the principal unit was the lunar month, and, since the year does not contain an integral number of months, they were obliged frequently to insert an "intercalary" month to keep the seasons in their places. By the advice of the astronomer Sosigenes, Cæsar decreed that the Roman year should consist ordinarily of 365 days, but that every fourth year should contain 366, an arrangement which would work perfectly if the tropical year contained exactly 365.25 days, since the quarter of a day would accumulate to exactly one day in four years. The tropical year, however, is really nearly 0.008 of a day shorter than this, and so in the course of a thousand years the Julian calendar falls into error by nearly eight days. In 1582 the accumulated error was about thirteen days; but instead of shifting back to the time of Julius Cæsar (45 B.C.), Gregory made the date of the vernal equinox the same as in the year A.D. 325, the year of the celebrated Council of Nicæa which decided upon the method of reckoning Easter as well as many other abstruse matters. He accordingly dropped only ten days from the Julian reckoning, and decreed that the rule of adding an extra day every fourth year should be followed *except* in the case of those century years whose number is *not* divisible by 400. Thus, A.D. 2000 will be a leap year, but the years 1700, 1800, and 1900 were not. In four hundred years, the Julian calendar had one hundred leap years, but the Gregorian has only ninety-seven. The error of the Gregorian calendar amounts to only about a day in 3,000 years.

The change to the Gregorian calendar was not immediately accepted in countries where the Pope's authority was not recognized. In England and her colonies it was made in 1752. We celebrate George Washington's birthday on February 22, but he was born on February 11, 1732 (Old Style). Russia, Greece, and other countries in which the Eastern orthodox church held sway adhered to the Julian calendar until 1923, when they adopted a leap-year rule which is even more accurate than the Gregorian, namely, that century years shall be leap-years only in case their numbers when divided by 9 give a remainder of 2 or 6. The average length of the year thus determined is within three seconds of the true length of the tropical year, while the average Gregorian year is about twenty-four seconds in error. The Eastern and Gregorian calendars will agree, however, until A.D. 2800, when the difference will be one day, the Gregorian year being a leap year and the Eastern an ordinary one.

Non-Christian peoples have calendars of their own, some of which, like the Jewish and Mohammedan calendars, are based on the lunar month. The years are numbered from some historical or legendary event. The Mohammedan era began with the Hegira, or flight of Mohammed, in A.D. 622, the Jews reckon from the traditional date of the creation of the world, which they put in the year 3761 B.C., and the ancient Romans numbered their years from the legendary date of the founding of Rome (A.U.C.). The Christian era began with the date of the birth of Jesus as determined in the sixth century by Dionysius Exiguus, but subsequent investigations, based on the computed date of an eclipse that occurred at the time of King Herod's death, show that this is in error, and that, according to our adopted reckoning, Jesus was born in the year 4 B.C.

The **Julian period,** which is often used in astronomical calculations, is a period of 7,980 years, the least common multiple of three cycles that were much used in Roman chronology, and are called the Roman Indiction, the Solar Cycle, and the Lunar Cycle. The current Julian period began January 1, 4713 B.C., on which date all three cycles began together. The name of the period has nothing to do with Julius Cæsar, but was applied in 1582 by its inventor, Joseph Scaliger, in honor of his father. In using this device, it is customary to indicate dates by the total number of days that have elapsed since the beginning of the period, making no reference to years at all. The **Julian Day** (abbreviated by J.D.) thus determined is understood by every astronomer, regardless of his nationality or religion, and the system is especially useful for obtaining the interval in days between two observations, such as those of the brightness of a variable star.

The Months and the Days of the Week.—Two serious faults of both the Julian and the Gregorian calendar are the irregularity of the lengths of the months and their absurd nomenclature. Both may be traced directly to the changes made by the Cæsars. Julius Cæsar, in his reform of the Roman calendar, changed the beginning of the year from March to January, and established the simple system by which the odd months should each have thirty-one days and the even ones thirty, except the second, which was to have thirty in leap years only and twenty-nine in common years. However, he introduced confusion in the names of the months by appropriating the month Quintilis to himself, for which reason we call it July, and retaining for the months to which they no longer applied, the numerical names Sextilis, September, etc., so that the name September, for instance, which means seventh, has for two thousand years been applied to the ninth month. After Julius came Augustus, who, in correcting an error that had been made by the pontiffs in carrying out the Julian decree, named the month Sextilis for himself and, to make it as long as that of Julius, stole another day from February and added it to August. Many proposals have been made for correcting the resulting muddle, but so far without success.

The origin of the week is not known with certainty, but the names of the days are almost certainly of astronomic origin. The ancient Chaldeans named the hours of the day for the seven heavenly bodies which they noticed moving in the zodiac, in what was then supposed to be the order of their distance from the Earth, viz.: Saturn, Jupiter, Mars, the Sun, Venus, Mercury, the Moon. The day was then named identically with its first hour. Since in naming the twenty-four hours the seven names were used three times with three names over, the order of the days became that of Saturn, the Sun, the Moon, Mars, Mercury, Jupiter, Venus. These names, or the names of corresponding local deities, came to be used by the Jews and Romans, and, after them, by other European nations, as the following table shows:

NAMES OF THE DAYS OF THE WEEK

Celestial origin	*Latin*	*Saxon*	*English*	*French*	*German*
Sun	Dies Solis	Sunnan-daeg	Sunday	Dimanche	Sonntag
Moon	" Lunæ	Monan-daeg	Monday	Lundi	Montag
Mars	" Martis	Tues-daeg	Tuesday	Mardi	Dienstag
Mercury	" Mercurii	Wodens-daeg	Wednesday	Mercredi	Mittwoche (mid-week)
Jupiter	" Jovis	Thors-daeg	Thursday	Jeudi	Donnerstag
Venus	" Veneris	Friga-daeg	Friday	Vendredi	Freitag
Saturn	" Saturni	Saeter-daeg	Saturday	Samedi	Samstag, Sonnabend

Precession.—While it is true that, during a year, the Earth's axis remains very nearly parallel to its initial position, thus causing the seasons, its direction does change slowly and in the course of many years the change becomes conspicuous. The motion of the Earth is somewhat like that of a boy's top, which, set spinning with its axis inclined to the vertical, moves so that the axis slowly describes a vertical cone. So the Earth's axis gyrates in a cone while keeping its inclination to the ecliptic practically unchanged, but the gyration is so slow that one complete circuit occupies 25,800 years. This motion of the Earth is called **precession.**

Since the celestial poles are the points where the Earth's axis meets the celestial sphere, this conical motion of the axis results in a circular motion of the poles of rotation around the poles of the ecliptic, which for the north pole is illustrated in Fig. 70. The radius of the circle is equal to the obliquity of the ecliptic, 23°.5. At present, the north "pole star" is α Ursæ Minoris, but some 4,000 years ago it was α Draconis, and in about 5,000 years more it will be α Cephei. Since the circles of perpetual apparition and occultation are at a distance from the poles equal to the observer's latitude, precession makes a very marked difference in the constellations that are visible at a given place. For example, in the year 3000 B.C. the Southern Cross, which is now invisible throughout the United States, could be seen north of the site of Quebec.

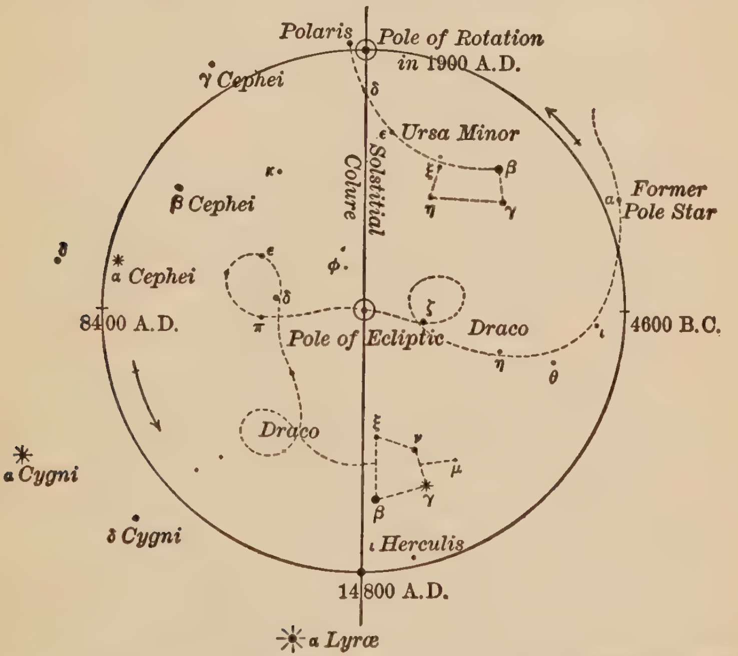
Polaris
Pole of Rotation
in 1900 A.D.
γ Cephei
δ
ε
Ursa Minor
κ
ζ
β
η
γ
β Cephei
Solstitial
Colure
α
Former
Pole Star
δ
α Cephei
ε
φ
δ
8400 A.D.
π
Pole of Ecliptic
ζ
Draco
4600 B.C.
η
θ
ι
Draco
ξ
ν
μ
γ
β
α Cygni
δ Cygni
ι Herculis
14800 A.D.
α Lyræ

FIG. 70. PRECESSIONAL PATH OF THE CELESTIAL POLE (FROM YOUNG'S *Manual of Astronomy*)

Since the celestial equator and the other circles of the equator system of co-ordinates are fixed with respect to the poles of rotation, precession causes them all to move with respect to the stars. As the equator is a great circle, the equinoxes, or points where it cuts the ecliptic, must remain opposite one another, and any motion of the equator necessitates a motion of the equinoxes. They move westward along the ecliptic, at a nearly uniform rate of 50″.2 a year. Right ascension and longitude being reckoned from the vernal equinox and declination from the equator, these three co-ordinates of a star are all affected by precession, but latitude, which is counted from the ecliptic, is not so affected. The longitudes of all stars continually increase, while the right ascensions and declinations are affected differently in different parts of the sky. The results of precession are illustrated in Fig. 71. As the pole of rotation moves through the short distance from PR_1 to PR_2, the equator tilts about the points midway between the equinoxes so that the vernal equinox moves from V_1 to V_2 and the autumnal from A_1 to A_2. The right ascension, declination, and longitude of the star shown are thus increased. The declination of a star at the opposite point of the sphere would be diminished but the other two co-ordinates would still be increased.

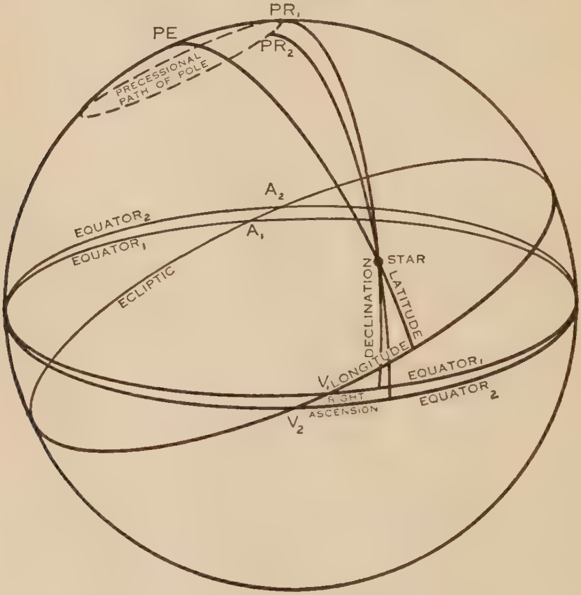
PR₁
PE
PR₂
PRECESSIONAL PATH OF POLE
A₂
A₁
EQUATOR₂
EQUATOR₁
ECLIPTIC
STAR
DECLINATION
LATITUDE
V₁
LONGITUDE
EQUATOR₁
EQUATOR₂
RIGHT ASCENSION
V₂

FIG. 71. PRECESSION

Precession was discovered in 125 B.C. by Hipparchus by comparing the length of the year determined by the dates of the **heliacal risings** of certain stars (when the stars could first be seen in the dawn after the Sun had passed them in its annual motion) with its length determined by the dates when the shadow of a vertical post or **gnomon** was at its average length. The former is the **sidereal year,** or interval between successive arrivals of the Sun at a given place among the stars, and has a length, as we now know, of 365.25636 mean solar days; the latter is the tropical year, already defined (page 105), with a length of 365.24219 days. The relative shortness of the tropical year is due to the motion of the equinoxes, which go to meet the Sun. The origin of the word precession, which is perhaps a little far-fetched, is found in the fact that the position of the equinox at any epoch *precedes* its positions of earlier epochs as the celestial sphere is carried westward by the diurnal motion.

Since the time when the signs of the zodiac (page 25) were named for the twelve zodiacal constellations, the equinoxes have moved more than the length of a sign; and so the vernal equinox, which is even yet called the "first point of Aries" by some writers, lies actually in the constellation of Pisces.

The intersection of the celestial equator with the galactic circle, which is now used as the origin of galactic longitudes (page 25), is of course also displaced by the precessional motion of the equator. The reasonable suggestion has been made by Stroobant that the point of the galactic circle where the equator crossed in 1900 be in future taken as the origin and that its position be defined by means of the galactic longitude of the star Deneb which is situated near the galactic circle and which has a very small proper motion.

Nutation.—The cause of precession, as will be explained in Chapter X (page 235), lies in the attraction of the Sun and Moon for the protruding material at the Earth's equator. The Moon's share of this would, if acting alone, cause a conical motion of the Earth's axis around the pole of the Moon's orbit instead of the pole of the ecliptic; but, due to the regression of the Moon's nodes (page 118), the pole of its orbit itself moves around the pole of the ecliptic in a small circle with a period of nineteen years. The result is that, superposed upon the precessional motion, there is a small oscillation of the pole of rotation to a distance of about 9″ from the position that it would occupy if precession acted alone. This motion is called **nutation** (nodding) since it causes the pole of rotation to "nod" toward and from the pole of the ecliptic.

Reduction of Star Places.—Star catalogues have been published which give the right ascensions and declinations of thousands of stars. For the sake of convenience, these co-ordinates are always given as reckoned for some fixed and designated

epoch, for example, the beginning of the year 1900. As the observations could not all have been made at that time, it was necessary for the authors of the catalogue to correct the observed apparent place of each star for the changes due to precession, nutation, and aberration that took place between the time of observation and the epoch of the catalogue. Moreover, the astronomer who uses the catalogue must make similar corrections in order to obtain accurately the place of the star at any subsequent date.

Fig. 72 shows graphically the change in the declination of the star α Persei during a period of twenty years. Precession alone would cause the declination to increase at a nearly uniform rate; but superposed upon the straight line that designates this change are the nineteen-year sinuosity of nutation and the annual wave of aberration. For the nearest stars, a slight further complication is added by parallactic motion, and most stars have a small **proper motion** (page 324) of their own besides.

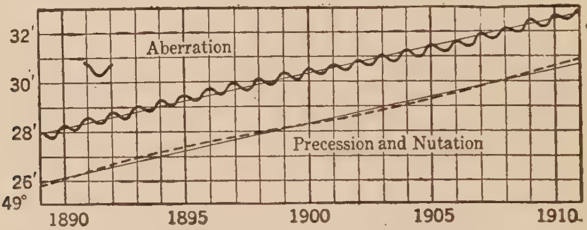

FIG. 72. CHANGE OF DECLINATION OF α PERSEI
(DRAWN BY L. S. FLINT)

Cause of the Change of the Equation of Time.—The equation of time has been defined (page 72) as the difference between mean and apparent solar time, the latter being the hour angle of the true Sun and the former the hour angle of the mean sun, which moves with uniform angular speed in the celestial equator. It may equally well be defined as the difference of the right ascensions of the mean and the true suns, for

$$\text{mean time} = \text{sidereal time} - \alpha_m,$$
$$\text{app. time} = \text{sidereal time} - \alpha_t,$$

where α_m and α_t signify the right ascension of the mean and the true Sun, respectively; and subtracting the second equation from the first gives

$$\text{equation of time} = \alpha_t - \alpha_m.$$

The equation of time varies throughout the year from the combined effect of two principal causes: the varying speed of the Earth due to the eccentricity of its orbit, and the obliquity of the ecliptic. To make the action of the first cause clear, let us define an **intermediary sun** as an imaginary body which moves in the *ecliptic* with the mean speed of the true Sun, and which coincides with the true Sun when the Earth is at perihelion. The apparent motion of the true Sun is as if it moved in an ellipse and obeyed the law of areas with respect to the Earth situated at the focus E (Fig. 73). Let its position at different times of year be denoted by S_1, S_2, S_3, . . . Since, near perihelion, its angular velocity is greater than the mean, the true Sun forges ahead of the intermediary, and when the true Sun is at S_1 the intermediary is at I_1, the right ascension of the former is greater, and the equation of time is positive. The angular distance between the two "suns" increases until the speed of the true Sun has fallen to its mean value, which occurs near the end of the minor axis of the orbit, when the equation of time is a maximum; then the true Sun slows up, and at aphelion A, where half the circumference and also half the area of the ellipse have been described, the two are together. From aphelion to perihelion the true Sun lags behind the intermediary. The portion of the equation of time which is due to eccentricity is thus zero at perihelion and aphelion, is positive during the first half of the year, and is negative during the second half. Its maximum value is about seven minutes, and its changes are as shown in the dotted curve of Fig. 75.

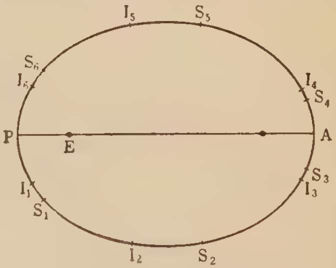

FIG. 73. EQUATION OF TIME DUE TO ECCENTRICITY

To explain the action of the second cause, let the intermediary sun, moving uniformly in the ecliptic, coincide with the mean sun, moving with equal angular speed in the celestial equator, when they are at the vernal equinox (Fig. 74). After a brief interval, the intermediary sun will have moved to I_1 and the mean sun an equal distance to M_1; the longitude of the former will equal the right ascension of the latter; but the right ascension of the intermediary sun, being the projection of its longitude upon the equator, will be less than that of the mean sun, and so the portion of the equation of time that is due to this cause will be positive. At the summer solstice each will have traveled 90° and their right ascensions will again be equal since the sixth hour circle passes through the solstice. Between the summer solstice and the autumnal equinox the intermediary sun will have the greater right ascension because of the convergence of the hour circles as they recede from the equator, but the two will coincide again at the autumnal equinox, each having traveled 180°, though by different routes; and the changes will be repeated in the second half-year. (A celestial globe will be of great assistance in visualizing these relations.) The equation of time due to obliquity thus passes through zero four times a year—at the equinoxes and solstices. Its maximum numerical value, which is twice positive and twice negative during the year, is about ten minutes, and its changes are as exhibited in the broken curve of Fig. 75.

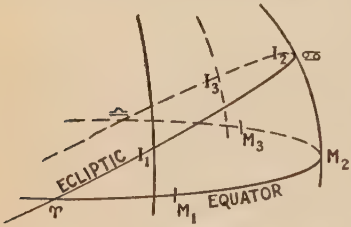

Fig. 74. Equation of Time Due to Obliquity

To obtain the combined effect of the two causes for any date, it is necessary simply to add together the ordinates of the two curves. The actual value of the equation of time, obtained in this way, is shown in the continuous curve of Fig. 75. It is zero about April 16, June 15, September 1, and December 25, and reaches a maximum positive value of $+14^m\ 28^s$ on February 11 and a maximum negative value of $-16^m\ 21^s$ on November 3.

Reckoned by apparent time, the length of the morning—*i.e.*, from sunrise to noon—is, neglecting the slight daily change of the Sun's declination, equal to that of the afternoon; but reckoned by mean time, noon does not generally thus occur midway between sunrise and sunset, and the change of the equation of time renders the resulting inequality of morning and afternoon different at different times of year. To make this clear, let $2L$ be the length of the day (from sunrise to sunset); then the apparent time of sunset will be L (P.M.) and that of sunrise, $12^h - L$ (A.M.). Let r and s be the *mean* times of sunrise and sunset, respectively, and E the equation of time; then,

$$r = 12^h - L + E,$$
$$s = L + E.$$

The effect is especially noticeable from about December 10 to January 5, for then E is increasing rapidly while L is changing but little; and so both r and s increase, the afternoon lengthening at the expense of the morning.

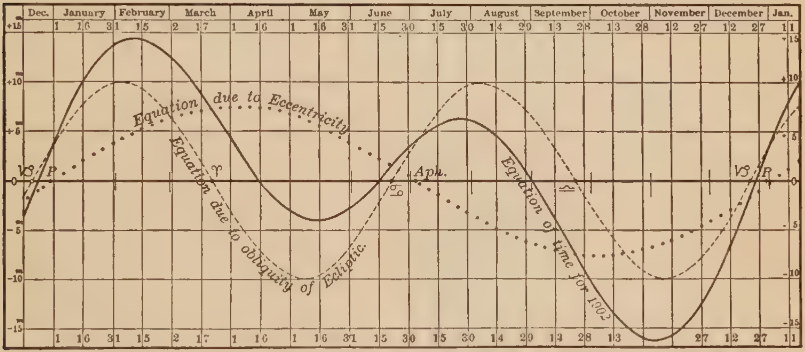

FIG. 75. THE EQUATION-OF-TIME CURVE (FROM YOUNG'S *Manual of Astronomy*)

NOTE. Authorities differ as to the sign used for the equation of time. The *American Ephemeris* defines it as apparent time minus mean time, thus giving it the opposite sign to that used in this chapter.

EXERCISES

1. At what times of year is the parallactic displacement of a star at the vernal equinox a maximum? When is its aberrational displacement a maximum?

2. To draw an ellipse with major axis 4 inches and eccentricity 0.5, what must be the distance between the pins? The length of the loop?

Ans. $2c = 2$ inches; $a + c = 3$ inches

3. Draw ellipses of eccentricity 0.2, 0.5, 0.9.

4. What is the eccentricity of an ellipse in which the minor axis is three-fifths as long as the major axis?

Ans. 4/5

5. What effect on the seasons would be produced if the obliquity of the ecliptic were reduced to zero? If the Earth's axis were made to lie in the plane of the ecliptic?

6. Is precession now increasing or diminishing the declination of the Pleiades? Of Arcturus? Of Antares? Of Deneb? (See star charts.)

7. Is the right ascension of any star being diminished by precession? If so, where is the star?

8. The star charts in this book (Maps 1–8) have the epoch 1950 A.D. What displacement of the equinoxes on the page would be necessary to change to epoch 2000 A.D.?

9. Toward what point of the celestial sphere is the Earth moving on June 22?

Ans. The vernal equinox

10. Toward what point of the celestial sphere is the Earth moving on September 22?

CHAPTER V

THE MOON

General Remarks.—The Moon is the satellite of the Earth and accompanies the latter on its journey around the Sun, at the same time revolving around the planet as a gull may circle about a moving ship. Because it is our nearest neighbor, it is given perhaps an undue amount of attention by mankind, for as seen from any other body than the Earth it would seem insignificant. It resembles the Earth in its spherical shape and solid body and in having its surface diversified by mountains, valleys, and plains, but its diameter is only about a fourth that of the Earth and it has very little, if any, atmosphere or water. Some of the important facts about the Moon are:

Mean distance from Earth,	60 Earth-radii, or 239,000 miles
Greatest distance,	253,000 miles
Least distance,	222,000 miles
Diameter,	2,163 miles
Mass,	1/81.5 of Earth's mass
Mean density,	3.39 × that of water
Surface gravity,	1/6 that of Earth
Mean albedo,	0.07
Period of revolution (sidereal),	27⅓ days
Period of rotation,	Exactly the same as that of revolution

Apparent Motion of the Moon.—Even the most inattentive observer of the Moon must notice that it rises and sets a little later each day than on the preceding day. The average delay is about fifty-one minutes. This means that it does not keep up with the general diurnal motion of the celestial sphere, but, relative to the stars, has a motion of its own of about half a degree an hour (13° a day) toward the east. By carefully noting the Moon's position among the stars on successive nights, its apparent path may be determined, and it is found to be very nearly a great circle that lies near the ecliptic, intersecting the latter at an angle of about 5°. This angle is called the **inclination** of the Moon's orbit, and the points of intersection are known as the Moon's **nodes,** the one where the Moon crosses from the south side of the ecliptic to the north being called the **ascending** node and the other the **descending** node.

Observations of the Moon's path among the stars in different years show that it changes markedly, the nodes moving westward along the ecliptic, while the inclination remains about the same. This change, known as the **regression of the nodes,** carries each node completely around the sky in about nineteen years. It is very similar to the precessional change of the celestial equator, but takes place much more rapidly.

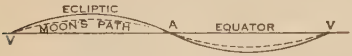

FIG. 76. MOON'S PATH BETWEEN ECLIPTIC AND EQUATOR

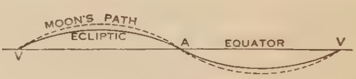

FIG. 77. ASCENDING NODE AT VERNAL EQUINOX

The regression of the nodes produces a very noticeable change in the *diurnal* path of the Moon in different parts of the nineteen-year cycle; for, when the descending node coincides with the vernal equinox (Fig. 76), the Moon's path lies entirely between the ecliptic and the equator, and the maximum declination is only 23°.5 − 5° = 18°.5; but nine years later the Moon's orbit is shifted halfway around so that the nodes are interchanged (Fig. 77), the Moon's path lies entirely outside the space bounded by the ecliptic and the equator, and the declination reaches 23°.5 + 5° = 28°.5. The total range of declination, and therefore of meridian altitude, is in the one case from − 18°.5 to + 18°.5, or 37°, while in the other it is from − 28°.5 to + 28°.5, or 57°.

The difference of longitude between the Sun and the Moon is called the Moon's **elongation.** Since the Moon moves among the stars about 13° a day while the Sun moves only 1°, the Moon overtakes the Sun about thirteen times a year. When this occurs, the elongation is zero and the Moon is said to be in **conjunction;** when the elongation is 90° east or west, the Moon is said to be in **quadrature;** when 180°, in **opposition.** Either conjunction or opposition is sometimes referred to as **syzygy.** At syzygy, the Sun, Moon, and Earth are nearly in the same straight line and would be exactly so if the inclination of the Moon's orbit were zero. Actually, the three bodies are in the same straight line only on the rare occasions when syzygy occurs at one of the nodes.

The Month.—The word *month* is etymologically related to the word moon, and the month as a unit of time had its origin in the use of the Moon as a timekeeper. The months of the calendar, although arbitrarily made unequal in length, are approximately the time of a revolution of the Moon. Astronomers recognize a number of different kinds of month. The **sidereal month** is the interval between two successive arrivals of the Moon at a given apparent place among the stars and averages 27.32166 mean solar days, but varies about three hours because of the perturbations of the Moon's motion (page 236). The **synodic month** is the time of a revolution with respect to the apparent place of the Sun—that is, from conjunction to conjunction. Suppose the Sun, Moon, and a star at a given moment to be in the same direction from the center of the Earth. After a sidereal month the Moon will have arrived again at the star, but the Sun will in the meantime have moved forward about 27°, and the Moon must travel two days longer to overtake it. The average length of the synodic month is, in fact, 29.53059 days.

If we let M be the number of days in the sidereal month, then $360/M$ will be the average number of degrees passed over by the Moon (relatively to the stars) in a day—its **mean daily motion.** Similarly if E be the number of days in a year, $360°/E$ will be the mean daily motion of the Earth, or, apparently, of the Sun; and if S be the number of days in the synodic month, $360°/S$ will be the mean daily gain of the Moon upon the Sun. Hence,

$$360/M - 360/E = 360/S,$$

or, more simply,

$$1/M - 1/E = 1/S,$$

a relation which expresses the length of the synodic month in terms of the sidereal or *vice versa*. The synodic month is the more natural as a unit of time, for in this period the Moon passes through its cycle of phases (page 123).

In computations relating to eclipses of the Sun and Moon, the **nodical month** is useful. It is the time of the Moon's revolution with respect to either node, and averages 27.2122 days in length.

The Form of the Moon's Geocentric Orbit.—The Moon's apparent diameter varies from 1764″ when it is farthest from the Earth to 2013″ when it is nearest.

These numbers represent the **geocentric** diameter—that is, the diameter the Moon would appear to have if viewed from the distance of the center of the Earth. From the surface of the Earth, the distance is a little less, the observer being brought nearer the Moon as the Moon approaches the zenith. The resulting increase of the Moon's apparent diameter, called the **augmentation,** may easily be computed and allowed for. When the Moon is rising or setting, the distance of the observer (*MR* or *MS*, Fig. 78) is very nearly the same as that of the Earth's center *C*; but when the Moon is at the zenith, the observer *O* is nearer it by one Earth-radius, or 1/60 of the whole distance, than is the Earth's center. The maximum augmentation therefore amounts to 1/60 of the whole apparent diameter, or about 30″.

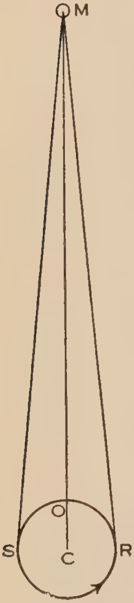

FIG. 78. THE AUGMENTATION OF THE MOON'S DIAMETER

By following the method that has already been described (page 99) for determining the form of the Earth's orbit around the Sun, and drawing a "spider," the legs of which have lengths proportional to the reciprocal of the measured diameter and make angles with a fixed line equal to the Moon's longitude, the form of the Moon's geocentric orbit may be determined. Like the Earth's orbit, it turns out to be a nearly circular ellipse, but of greater eccentricity—averaging about 0.056. The Earth is at the focus, and the Moon's radius vector describes equal areas in equal intervals of time.

The point of the lunar orbit that is nearest the Earth is called the **perigee** and the point most remote is called the **apogee.** Other terms relating to the orbit have the same significance as in the case of the Earth's orbit.

Distance and Dimensions of the Moon.—Of many methods which may be used to measure the Moon's distance, one of the simplest consists of a triangulation from two points of observation on the Earth. Suppose that two observers *A* and *B* (Fig. 79), situated on the same terrestrial meridian but widely separated in latitude, observe the zenith distance of the Moon with meridian circles. Neglecting for simplicity the oblateness of the Earth (it can be accurately allowed for), these observations give the angles *ZAM* and *Z′BM*. The angle *ACB* is the algebraic difference of the latitudes of the observers. In the triangle *ABC*, the sides *CA* and *CB* are radii of the Earth, and from these and the angle *ACB* may be computed trigonometrically the angles *CAB* and *CBA* and the distance *AB*. In the triangle *ABM* the base *AB* is now known, and the angles *BAM* and *ABM* may be found by subtracting $ZAM + CAB$ and $Z'BM + CBA$ from 180°; hence *AM* and *BM* may be computed. Finally, in the triangle *CAM*, the sides *CA* and *AM* are known, and the angle *CAM* is the supplement of *ZAM*, hence *CM*, the geocentric distance, may be computed in terms of the radius *CA* or *CB*. The Moon's mean distance is thus found to be 60 Earth-radii, or about 239,000 miles. Its distance at perigee is 222,000 miles, and at apogee 253,000 miles.

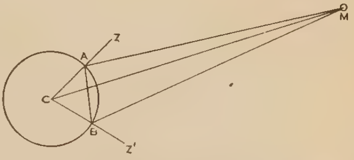

FIG. 79. DETERMINING THE MOON'S DISTANCE

From the Moon's distance and its apparent diameter its real diameter may be found at once (page 9), and it turns out to be 2,163 miles, a little over a quarter of the Earth's.

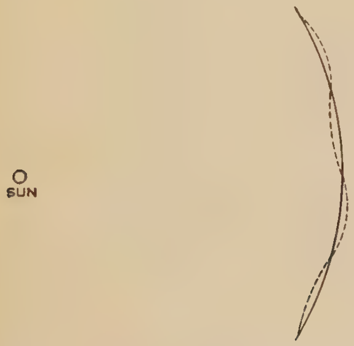

FIG. 80. THE MOON'S HELIOCENTRIC PATH

Form of the Moon's Heliocentric Orbit.—Since the Earth moves continually in its orbit around the Sun while the Moon circles around the Earth, the Moon's path around the Sun is a sinuous line as suggested in Fig. 80, where the continuous circle represents the path of the Earth and the dotted line that of the Moon. The proportions of the figure, however, are far from correct, and it would not be practicable to represent them correctly for, the Moon's distance being only about 1/400 of the Sun's, if we represent the Earth's orbit by a circle having a radius as great as two feet, the departures of the Moon from that circle would be only about 1/16 of an inch. These departures are so small that the Moon's orbit is everywhere concave to the Sun.

Mass, Density, and Surface Gravity of the Moon.—As the Moon revolves, its gravitational attraction sways the Earth slightly from the position it would otherwise occupy, the two bodies revolving around their common center of mass (page 227). This monthly motion of the Earth causes a slight but observable parallactic displacement of the nearer planets, from which the distance of the center of the earth from the center of mass may be computed.

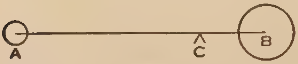

Fig. 81. Center of Mass of Two Spheres

The **center of mass** of two homogeneous spheres is a point on the line joining their centers such that the product obtained by multiplying its distance from the center of either sphere by the mass of that sphere is the same as the corresponding product relating to the other sphere. For example, if m and M be the masses of the two spheres A and B (Fig. 81), then their center of mass, C, is such a point that $m \times AC = M \times BC$. The center of mass of two bodies near the surface of the Earth may be determined as the point about which they balance when joined by a light, straight rod.

The distance of the Earth's center from the center of mass is found to be 2,880 miles, so that the latter is about a thousand miles within the surface of the Earth. This distance is about 1/82.5 of the distance from the Earth to the Moon, and therefore the Earth's mass must be about 81.5 times the Moon's.

Since the volumes of spheres are as the cubes of their diameters, the Earth's volume is $(\frac{7920}{2163})^3$, or about 49 times the Moon's. The mean density of a body is its mass divided by its volume, and so the Moon's density is 49/81.5 times the Earth's or, since the Earth's density as compared to water is 5.53, that of the Moon is about 3.4.

By the **surface gravity** of a heavenly body is meant its attraction for bodies at its surface. It can easily be computed when the body's mass and radius are known (page 225), and in the case of the Moon turns out to be about one-sixth that of the Earth. This means that a body that weighs six pounds on the Earth would weigh but one on the Moon. A given upward force would shoot a projectile six times as high there as here.

A ballet dancer who weighs 120 pounds on the Earth would weigh only 20 on the Moon, and could leap six times as high.

The Phases of the Moon.—The most obvious phenomenon shown by the Moon, and one which must have excited the admiration of mankind from the earliest times, is its apparent change of shape from a narrow crescent to a full circle and back to the crescent form. This change of shape, or of **phase,** is due to two circumstances: first, the Moon shines only by reflected sunlight, and second, as it revolves around the Earth, different portions of its sunlit side are presented to our view. This may be understood by a study of Fig. 82, or, better, by holding a tennis ball at arm's length in the light of a single lamp and turning slowly on one's heel. When the Moon is at conjunction, its unilluminated side is turned toward us and for this reason and also because of the dazzling light of the Sun, the Moon is invisible. The phase is then that of **new moon.**

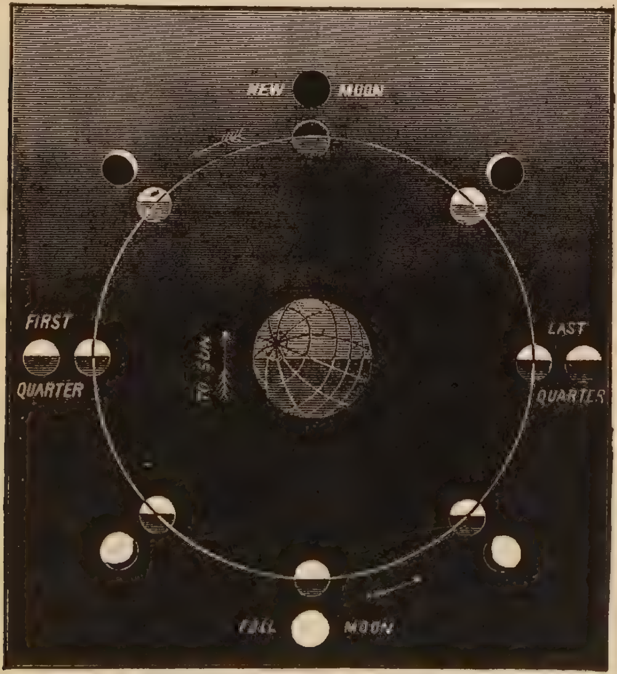

FIG. 82. EXPLANATION OF THE MOON'S PHASES (FROM YOUNG'S *General Astronomy*)

Two or three days later, it has moved several degrees east of the Sun and is visible in the western sky soon after sunset, but still only a little of the illuminated side is presented to us and it appears as a thin crescent. The crescent grows wider on succeeding nights until east quadrature is reached,[1] when we see half of the illuminated half, the Moon appears as a semicircle, and the phase is called **first quarter.** From east quadrature to west quadrature more than a quarter of the surface is visible, and the Moon is said to be **gibbous,** except at opposition, when the illuminated half is turned full upon the Earth, and we have **full moon.** After full moon the succession of phases is reversed, passing through **last quarter** at west quadrature to new moon again at the next conjunction.

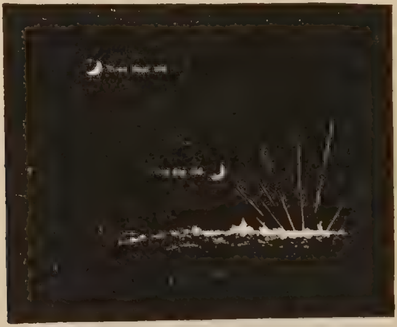

FIG. 83. THE YOUNG MOON
(FROM *Dante and the Early Astronomers,* BY M. A. ORR)

The line which separates the dark half of the Moon from the sunlit half—the sunrise or sunset line—is called the **terminator,** and always has the form of a semi-ellipse because it is a circle seen edgewise. At its extremities, the **cusps** of the Moon, it is tangent to the Moon's circular outline, or **limb.**

From Fig. 82 it may be easily seen that the cusps, or "horns" of the crescent Moon are always turned away from the Sun, and that therefore they are never pointed downward when the Sun is below the horizon—a fact sometimes disregarded by artists. In northern latitudes, moreover, the visible half of the ecliptic lies south of the zenith, and so the horns of the young Moon, when it is visible after sunset, are always turned upward and toward the left, as in Fig. 83.

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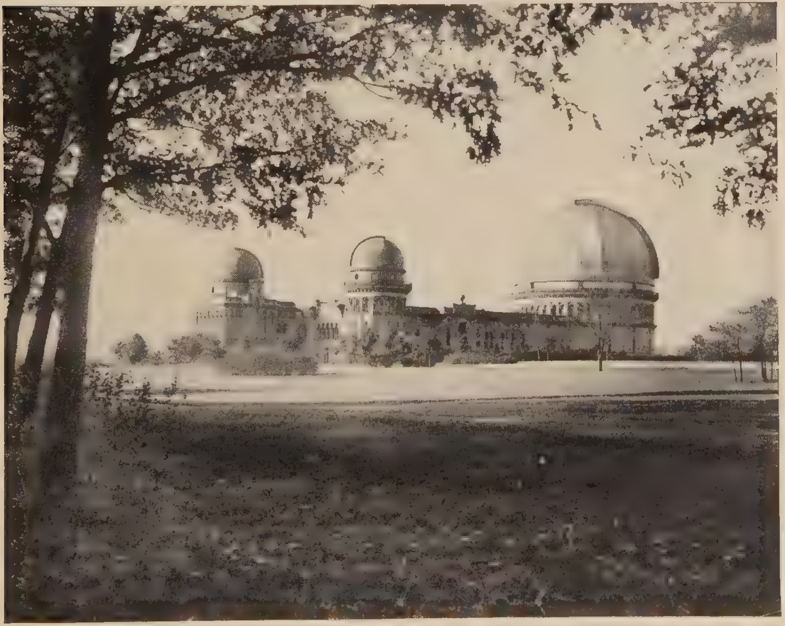
[1] More precisely, until the angle Sun-Moon-Earth is just 90°; but this occurs only a few minutes before quadrature.

Lick Observatory from the East



Yerkes Observatory from the Northeast

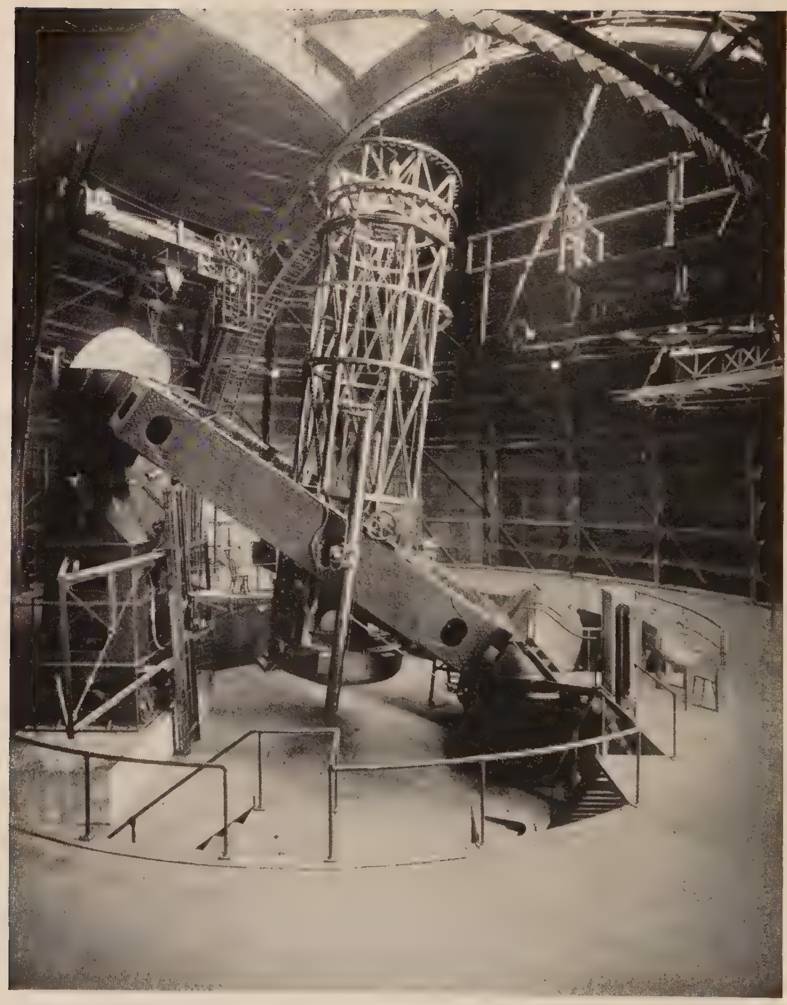


PLATE 6. THE 100-INCH HOOKER REFLECTOR OF THE MOUNT WILSON OBSERVATORY

Earthshine.—When the Moon is near conjunction and so appears to us as a crescent, an observer on the Moon would see the Earth as a great bright body, about two degrees in diameter, in the gibbous phase. The Earth at that time thus lights up the side of the Moon that is turned away from the Sun, and it is often easy to see this earthlit lunar landscape, which is fancifully called the "old Moon in the new Moon's arms."

Rotation and Librations of the Moon.—Observations of the markings on the Moon's surface show that the Moon rotates on an axis which is so placed that the plane of its equator makes with the plane of its orbit an angle of 6°.5. The intersection of the two planes always coincides with the line of nodes, and the plane of the ecliptic lies between them, so that the Moon's equator makes an angle of only 1°.5 (6°.5 − 5°) with the plane of the ecliptic. Very remarkably, the rotation and revolution take place in the same direction, from west to east, and *in precisely the same period*, so that one side of the Moon is always turned toward the Earth, while the opposite side, although nearer us than are any of the planets, is an undiscovered country wholly inaccessible to observation.

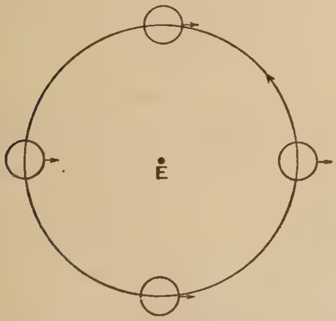


Fig. 84. Revolution without Rotation

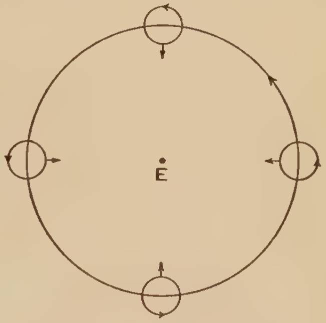


Fig. 85. Revolution and Rotation in the Same Period

At first thought this may not seem to be a rotation at all, but a consideration of Figs. 84 and 85 will show that it is. Suppose the orbit is circular and that an arrow is erected perpendicularly upon the Moon's surface. If there were no rotation, the arrow would be directed always to the same point among the stars, as in Fig. 84; whereas, if it points always to the center of the orbit, as in Fig. 85, its direction must sweep over a whole circle and coincide again with its initial direction at the end of one revolution. thus completing a rotation and a revolution in the same time.

The motion of the Moon is not, however, as if it were attached to the Earth by a rigid bar, for it has apparent balancing movements, or **librations,** which render about 18 per cent of its surface alternately visible and invisible. The principal librations are three in number, and are known as the librations in latitude and in longitude, and the diurnal libration.

The cause of the libration in latitude is the inclination of the Moon's equator to the plane of its orbit, and is exactly analogous to the cause of the Earth's change of season. Just as the Sun shines alternately over the Earth's north and south poles in the course of a year, so we on the Earth may look over the Moon's poles alternately in the course of a month.

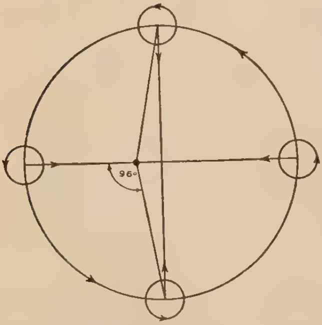


Fig. 86. Libration in Longitude

The libration in longitude is due to the fact that, while the Moon's rate of rotation is uniform, that of its revolution is variable, obeying the law of areas. Suppose an arrow erected perpendicularly on the Moon on the line that joins the centers of the Earth and Moon when the latter is at perigee (Fig. 86). From the direction of the Earth's center, it will be seen at the center of the Moon's disk. After a quarter of a sidereal month, the Moon's rotation will have changed the direction of the arrow just 90°, but the Moon's direction from the Earth will have changed 96°, so that the arrow will appear a little toward the east of the center of the disk, and we may see a little of the western side of that portion of the Moon which, at perigee, was concealed. After half a month, the radius vector has described half the area of the orbit, the Moon's direction has changed just 180°, and, as the arrow has been turned through an equal angle by the uniform rotation, it again points to the center of the Earth. At the end of the third quarter, the rotation has proceeded 270° and the revolution only 264°, and we then see a little of the eastern rim of the back side of the Moon. At the end of the month, the initial conditions are restored.

The diurnal libration is due to the change of the point of view of the observer as he is carried around by the Earth's rotation. When the Moon is on the horizon, the observer's line of sight makes an angle of about 1° with the line of centers, and this enables him to see about 1° beyond what would be the limb if the Moon were in the zenith.

Because of the small inclination of the Moon's equator to the ecliptic, it can have no perceptible change of season, and over the whole surface of the Moon the day is always very nearly equal to the night, each being about two weeks long.

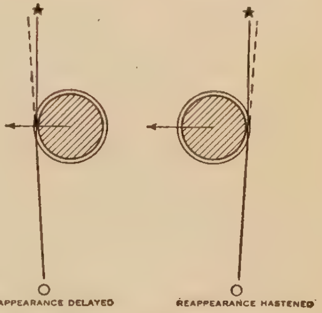


FIG. 87. EFFECT OF A LUNAR ATMOSPHERE

Physical Libration.—The Moon is not quite spherical; the diameter which lies nearly in line with the Earth is longest, the equatorial diameter at right angles to this is about a third of a mile shorter, and the polar diameter is about a mile shorter still. There is thus a slight protuberance extending toward the Earth, and the librations in longitude and latitude cause the direction of the Earth's gravitational attraction for this protuberance to change slightly, producing a real but very slight irregularity in the Moon's rotation which is known as the **physical libration.** It amounts to only about a mile on the Moon's surface.

Absence of an Atmosphere.—It is certain that the Moon does not possess an atmosphere which is at all comparable to that of the Earth. No clouds float over its surface, and the mountains and craters show no erosion by weather. More conclusive still, when the Moon passes between us and a star, or **occults** it, the star disappears and reappears with extreme suddenness and there is no evidence of any refraction of light at the Moon's limb. The refraction of the Earth's atmosphere deviates a horizontal ray by more than half a degree (page 60), and if the Moon were surrounded by gases of equal density a ray of starlight which passed the surface tangentially would be deviated by twice this amount, being refracted both on entering and on leaving the lunar atmosphere. Even a rare atmosphere should bend starlight slightly, delaying the disappearance and hastening the reappearance of the star as illustrated in Fig. 87; but the observed duration of an occultation is always equal within a fraction of a second to that which may be computed from the known diameter of the Moon and its rate of motion. From this fact it is certain that the Moon's atmosphere, if any exists, has a refractive effect less than 1/2000 that of air. Since gases consist of molecules in rapid motion, it is doubtful that the Moon, with its low superficial gravity, could retain an atmosphere if it ever had one.

If there is no atmosphere on the Moon, no liquid water can be there, for it would evaporate in the sunshine of the long lunar day and form an atmosphere of water vapor. Water might, perhaps, exist there in the form of ice or snow in crevices where sunshine enters but little.

Lunar Topography.—To the unaided eye there appear upon the bright surface of the Moon only certain vague, dark areas which suggest to the fancy such objects as the round face of a man, the profile of a lady, or a very long-eared donkey. In a telescope of low magnifying power these dark areas appear as smooth plains, so uniform that Galileo and his contemporaries thought that they might be seas and accordingly called them **maria,** a name still used by selenographers. Other parts of the lunar surface, even in a telescope such as Galileo's with its power of only thirty diameters, appear to be very rugged, with numerous mountains and other formations which, under favorable illumination by the Sun, cast long black shadows. With larger telescopes it is found that the smoothness of the maria is only relative, and that they too present a wealth of detail.

The principal features of the lunar surface are the maria, mountains, craters, "rills," and "rays." The nomenclature used on lunar maps is that of the Italian astronomer Riccioli, who mapped the Moon in the seventeenth century, naming the craters for astronomers or other men of science and giving the maria fanciful names, such as Mare Tranquillitatis, Sinus Iridum, and Oceanus Procellarum. A few of the mountain ranges retain names given earlier by the German Hevelius, who took them from terrestrial ranges. The most prominent are the Apennines, Caucasus, and Alps, which are situated north of the center of the visible disk.

The mountains are seen mostly in chains or groups, although on the surface of the Mare Imbrium are several isolated peaks of which a conspicuous one is Pico, about 8,000 feet high. The Apennine range consists of about 3,000 peaks and extends about 400 miles. Its loftiest peaks rise about 18,000 feet above the Mare Imbrium. The Alps are noteworthy for a great gorge, 80 miles long and 4 to 6 miles wide. All these features are best seen near the time of the first or last quarter. The highest lunar mountains, which rise to 25,000 feet or more, are situated in ranges near the limit of the visible side of the Moon, where they are seen in profile when the conditions of libration are favorable.

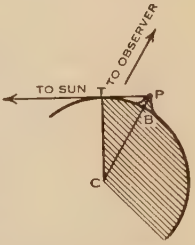


Fig. 88. Measuring the Height of a Lunar Mountain

The heights of lunar mountains are determined from the length of their shadows and a knowledge of the altitude of the Sun as it would appear from that point of the Moon, which can be obtained from the Moon's heliocentric position; also by a method used by Galileo, which depends upon the fact that the Sun rises earlier and sets later on a high peak than on lower mountains or the surrounding valleys. This may be understood by reference to Fig. 88. Suppose the Moon were perfectly smooth except for a mountain of which the base is at *B* and peak at *P*, and suppose the Sun were just shining on the peak. To an observer on the Earth, the Moon's terminator would appear at *T*, where the sunlight is tangent to the lunar surface, while the illuminated peak would appear as a detached point of light on the dark side of the Moon—not an unusual appearance. The distance *TP* may be found with the micrometer in terms of the Moon's apparent diameter, which gives two sides, *TP* and the radius *TC*, of a right triangle, whence the hypotenuse, which is the sum of the Moon's radius and the height of the mountain, may be computed.

The **rills** (German *Rille*, a ditch or furrow) are narrow crevices from ten to over three hundred miles long and less than two miles wide. Their bottoms are generally invisible and their depth consequently unknown. They are inconspicuous because of their narrowness, but over a thousand have been mapped. A remarkable example is the great Serpentine Cleft near the crater Herodotus.

The **rays** are narrow streaks which are lighter in color than their surroundings, and which radiate from certain prominent craters, notably Tycho, Copernicus, and Kepler. They are inconspicuous or invisible under a low Sun, but stand out very prominently near the time of full Moon. They can be neither ridges nor furrows, for they cast no shadows. The rays that center upon Tycho extend many hundreds of miles across the surface of the Moon, regardless of craters, mountains, and seas.

The **craters** are by far the most numerous of the lunar formations. Upward of 32,000 are shown on a map published in 1878 by Schmidt of Athens, and many others may be seen with present-day telescopes and on photographs made with them. The craters range in size from such "walled plains" as Maurolycus and Schickard, each of which is 150 miles in diameter and of greater area than the State of Maryland, down to the smallest dot discernible in the greatest telescope, which is about a tenth of a mile across. Several are so large that, from the centers of their floors, their walls would be concealed by the spherical surface of the Moon. They are depressions, the floor being usually lower than the surrounding plain, and are roughly circular in shape. Many contain lofty mountains which rise from their floors, and many large craters have numerous smaller ones superposed upon their floors or walls.

Some of the more important craters, with their diameters in miles, are listed below:

Proclus, 18; very bright, at edge of Mare Crisium
Linné, 6; has been suspected of change
Aristotle, 60
Eudoxus, 40
Eratosthenes, 38; at east end of Apennine range
Copernicus, 56; very conspicuous, surrounded by rays. Group of peaks in center
Archimedes, 50
Piazzi Smyth, 6
Ptolemy, 115; near center of visible disk
Alphonsus, 83; Arzachel, 66; linked together
Tycho, 54; center of great ray system
Gassendi, 55
Schickard, 134
Wargentin, 54; floor level with brim
Grimaldi, 150
Langrenus, 90; Vendelinus, 50; Petavius, 100; Furnerius, 80—forming chain near W limb

Plato, 60; floor very dark in color
Kepler, 22; surrounded by rays
Aristarchus, 29; brightest object on the Moon
Catherina, 70; Cyrillus, 60; Theophilus, 64—forming three links of a chain
Maurolycus, 150

The Best Time to Observe the Moon.—For observing the mountains, craters, and all other elevations or depressions on the Moon, the most favorable conditions are obtained when the object is near the terminator, for then it is brought into strong relief by the shadows. The maria and the ray systems are seen to the best advantage near full Moon.

Theories Regarding the Origin of Lunar Craters.—The craters on the Moon bear some resemblance to extinct volcanoes, and a theory which has been very generally held attributes their origin to volcanic action. An objection is found in the enormous size of some of the craters. Owing to the small superficial gravity of the Moon, a given eruptive force would have much greater effect there than on the Earth, but this fact is not sufficient to explain so great a difference in size as exists between the largest terrestrial volcano and Maurolycus or Schickard. Moreover, there are no lava flows on the Moon, and most of the lunar craters differ from terrestrial volcanoes in having their floors lower than the surrounding plain.

An alternative theory supposes the craters to have been made by the impact of huge meteorites upon the Moon, possibly at an exceedingly remote time when the Moon's surface was plastic. To explain the lack of similar craters on the Earth, it is recalled that the Earth's atmosphere protects it from all but the largest meteorites (page 276), and greatly diminishes the speed of even these; and it is further argued that, during many millions of years, the surface of the Earth has been eroded by the weather and the ancient scars may thus have been obliterated, while no such action has taken place on the airless Moon. The meteoric explanation derives some support from the appearance of Meteor Crater in Arizona (page 276). Airplane photographs of holes that were formed by the explosion of bombs dropped upon the ground from airplanes resemble lunar craters, and lend some support to a view suggested by Ives, that the craters were formed by explosions due to the heat generated by the impact of meteors rather than to the splash made by the impact directly.

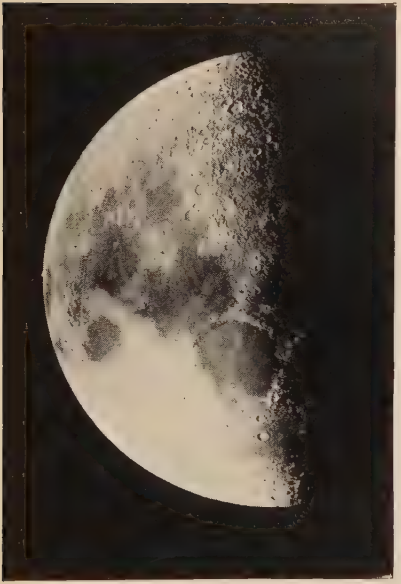
The Light Received by the Earth from the Moon.—Comparisons of the brightness of the Moon with that of the Sun are difficult and uncertain. Probably the most reliable value is that deduced by Russell from the work of Herschel, Bond, Zöllner, and others, which makes the light given the Earth by the full Moon 1/465,000 that given by the Sun (stellar magnitude of full Moon, −12.55). The brightness is much less at other phases, partly because of the smaller area of the visible illuminated surface, and partly on account of the presence of shadows; and it is less after full Moon than at the corresponding phase before full. The photographic brightness is less than the visual, showing that moonlight is yellow. It is, in fact, known to be somewhat yellower than sunlight, although this would not be suspected from the work of certain artists, who paint moonlight scenes in blue.

The mean **albedo,** or reflecting power of the Moon, is given by Russell as 0.07; which means that the Moon reflects this fraction of the sunlight which falls upon it, absorbing the remainder. This albedo is the same as that of rather dark terrestrial rocks. The reflecting power of different parts of the Moon ranges, according to Russell, from that of slate to that of white sand.

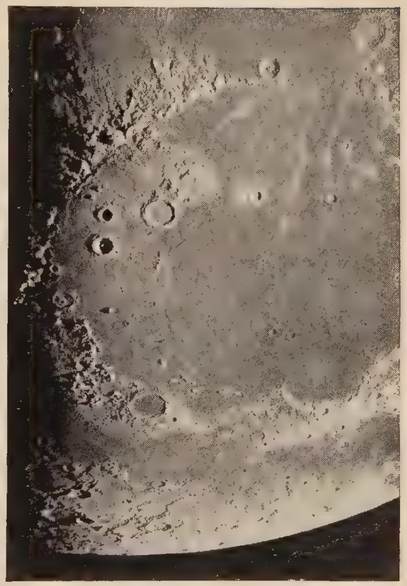
Temperature of the Moon.—Since the surface of the Moon is exposed to the rays of the Sun continuously for two weeks without the protection of an atmosphere, and then deprived of sunlight for an equal length of time, a great range of temperature is to be expected. Measurements of the heat which the Moon sends us indicate, for the sunlit side, a temperature well above 100° C. and for the dark side extreme cold—near the absolute zero of temperature (−273° C.). At the time of a lunar eclipse, the temperature falls very rapidly as the sunlight is withdrawn and rises with even greater rapidity soon after the surface emerges from shadow, showing that the Sun's heat does not penetrate deeply below the surface.

The Question of Changes on the Moon.—If any changes have occurred on the Moon since it was first accurately mapped, they

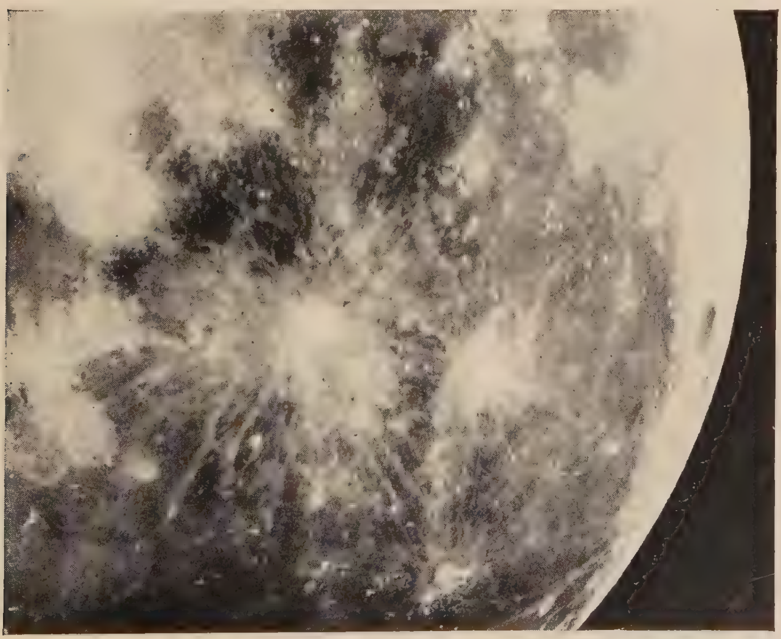
PLATE 7. PHOTOGRAPHS OF THE MOON



First Quarter; Sunrise on Tycho and the Apennines. 40-inch Refractor (Ritchey)

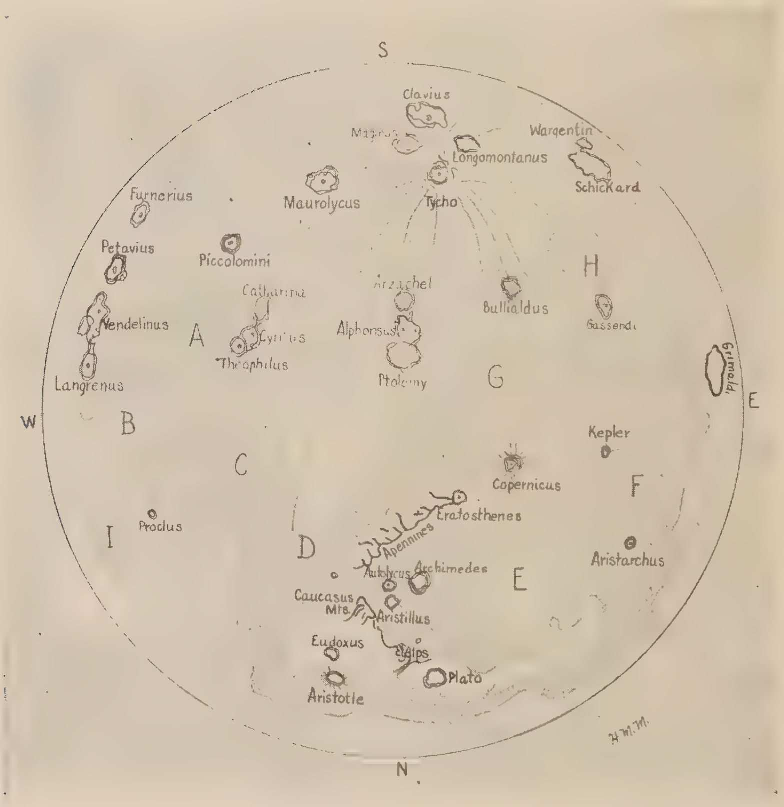


Sunset on Northern Mountains. 100-inch Reflector (Pease)



Copernicus, Kepler, and Aristarchus in full Sunlight. 100-inch Reflector (Pease)

PLATE 8. MAP OF THE MOON


S
Clavius
Maginus
Wargentin
Longomontanus
Schickard
Furnerius
Maurolycus
Tycho
Petavius
Piccolomini
H
Catharina
Arzachel
Bullialdus
Vendelinus
A
Cyrillus
Alphonsus
Gassendi
Theophilus
Grimaldi
Langrenus
Ptolemy
G
E
W
B
Kepler
C
Copernicus
F
Eratosthenes
Proclus
I
Apennines
D
Aristarchus
Autolycus
Archimedes
E
Caucasus Mts.
Aristillus
Eudoxus
Alps
Plato
Aristotle
H.M.M.
N

A—Mare Nectaris
B—Mare Fecunditatis
C—Mare Tranquilitatis
D—Mare Serenitatis

E—Mare Imbrium
F—Oceanus Procellarum
G—Mare Nubium
H—Mare Humorum

I—Mare Crisium

have been very slight. In the absence of any perceptible amount of air or water, no extensive erosion could take place, but the expansion and contraction of the rocks with the changing temperatures might indeed cause landslips on some of the many steep slopes. Unless such a change affected an area of a square mile or more, it would be extremely difficult to detect, and he would be dogmatic indeed who would say that no such changes occur; but we have no certain evidence of them. The little crater Linné has been suspected of change, for its present appearance is distinctly different from that shown by drawings and descriptions made by Mädler early in the nineteenth century; but the evidence has failed to be universally convincing. A few observers, chief among whom is W. H. Pickering, whose long experience and keen sight cannot be denied, have reported temporary greenish patches in Eratosthenes and some other craters which Pickering attributes to a low form of vegetation that passes through its life history in a synodic month. This would imply the presence in these craters of a slight amount of moisture and of some kind of atmosphere. Any observational evidence of changes on the Moon, however, must be accepted with caution because the appearance of certain parts changes strikingly with the direction of the Sun's rays and it is extremely difficult to secure two photographs or drawings under identical conditions of lighting.

If air and water are really totally lacking, life as we know it, either animal or vegetable, is of course impossible; and in the opinion of most astronomers the Moon is an arid, barren waste without life or sound or any change.

EXERCISES

1. What are the greatest and least inclinations of the Moon's path to the horizon of an observer in latitude + 40°?

Ans. 78½°; 21½°

2. In what part of the sky must the Moon be in order that, for such an observer, its daily retardation in rising shall be the least possible?

Ans. Near the vernal equinox

3. At what time of year is the Moon in this part of the sky at full moon?

Ans. In September (this is the so-called Harvest Moon)

4. Assuming Russell's value, 0.45, of the albedo of the Earth, how does full earthlight on the Moon compare with full moonlight on the Earth? *Ans.* Earthlight is about 90 times as bright as moonlight

5. Describe the diurnal motion of the stars, the Sun, and the Earth as seen from the Moon.

6. What interval of time must elapse between an occultation of Aldebaran and one of Antares (see star maps)? *Ans.* About nine years

7. What other bright stars may be occulted by the Moon?

8. Comment on Coleridge's lines:

"The hornéd Moon, with one bright star
Within the nether tip."

CHAPTER VI

ECLIPSES OF THE SUN AND MOON

Shadows of the Earth and the Moon.—Since the Earth and the Moon are opaque and are illuminated by sunlight, each is accompanied on its orbital motion by a shadow which is ordinarily invisible and which extends into space in a direction opposite that of the Sun. Occasionally the Moon passes into the Earth's shadow and is darkened by a **lunar eclipse;** at certain other times, its shadow falls upon the Earth, darkening the Sun for favorably situated observers, and so producing a **solar eclipse.** Evidently, a lunar eclipse can occur only at full Moon and a solar eclipse only at new Moon.

In Fig. 89 the formation of the shadows of the Earth and the Moon is illustrated diagrammatically, but it is impracticable to present the figure in correct proportion. For an Earth of the size depicted, the Sun should be nearly two feet in diameter, and about two hundred feet away, while the length of the shadows and the diameter of the Moon's orbit should be about six times as great as shown. The actual dimensions of the shadows may be found from geometric considerations as follows:

Let L be the length of the Earth's shadow, R the distance from the Earth to the Sun, d and D the respective diameters of the Earth and the Sun. The Earth's shadow is a cone, of which the cross-section HCK is bounded by the common tangents BK and AH drawn to the Earth and the Sun. On account of the Earth's great distance from the Sun as compared with the size of either body, these lines are nearly parallel (the angle between them is about half a degree, being slightly less than the apparent diameter of the Sun as seen from the Earth); hence the lines AB and HK, which join the points of tangency, very nearly coincide with the diameters D and d. In the similar triangles ABC and HKC we have $\frac{L}{R+L} = \frac{d}{D}$, whence $LD - d(R + L) = 0$, and $L = \frac{dR}{D - d}$. Now we have

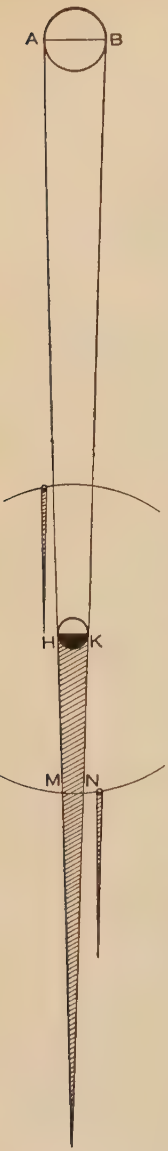


FIG. 89. SHADOWS OF THE EARTH AND THE MOON

seen (page 98) that the Sun's diameter D is about 109 times that of the Earth, and so $d/(D - d) = 1/108$. Therefore, since R is about 93,000,000 miles, the length L of the Earth's shadow is $1/108 \times 93{,}000{,}000$, or 857,000 miles. This length varies by about 28,000 miles with the changing value of R, but always far exceeds the distance of the Moon.

The length of the Moon's shadow, as found by a similar calculation, is about 232,000 miles on the average, and varies about 4,000 miles either way. Since the Moon's distance ranges from 222,000 miles at perigee to 253,000 at apogee (page 121), its shadow is sometimes long enough to reach the Earth, but more often falls short. In the former case, an observer situated on the line of centers of the Moon and the Sun may see a **total** solar eclipse; in the latter, the apparent diameter of the Moon is less than that of the Sun, and an observer on the line of centers may see a bright ring, or *annulus* of the Sun surrounding the black Moon; the eclipse is then said to be **annular.**

The diameter, MN, of the Earth's shadow at the place where the Moon crosses it may be computed from the similar triangles HCK and MCN. Thus, if Δ represents the Moon's distance from the Earth when it crosses the shadow, $\frac{MN}{L - \Delta} = \frac{d}{L}$. The mean value of Δ being 239,000 miles, this gives for MN about 5,700 miles; but its actual value varies by about 400 miles with the varying distance of the Moon from the Earth and of the Earth from the Sun. At its greatest, MN is about three times the diameter of the Moon, and, as the Moon travels a distance

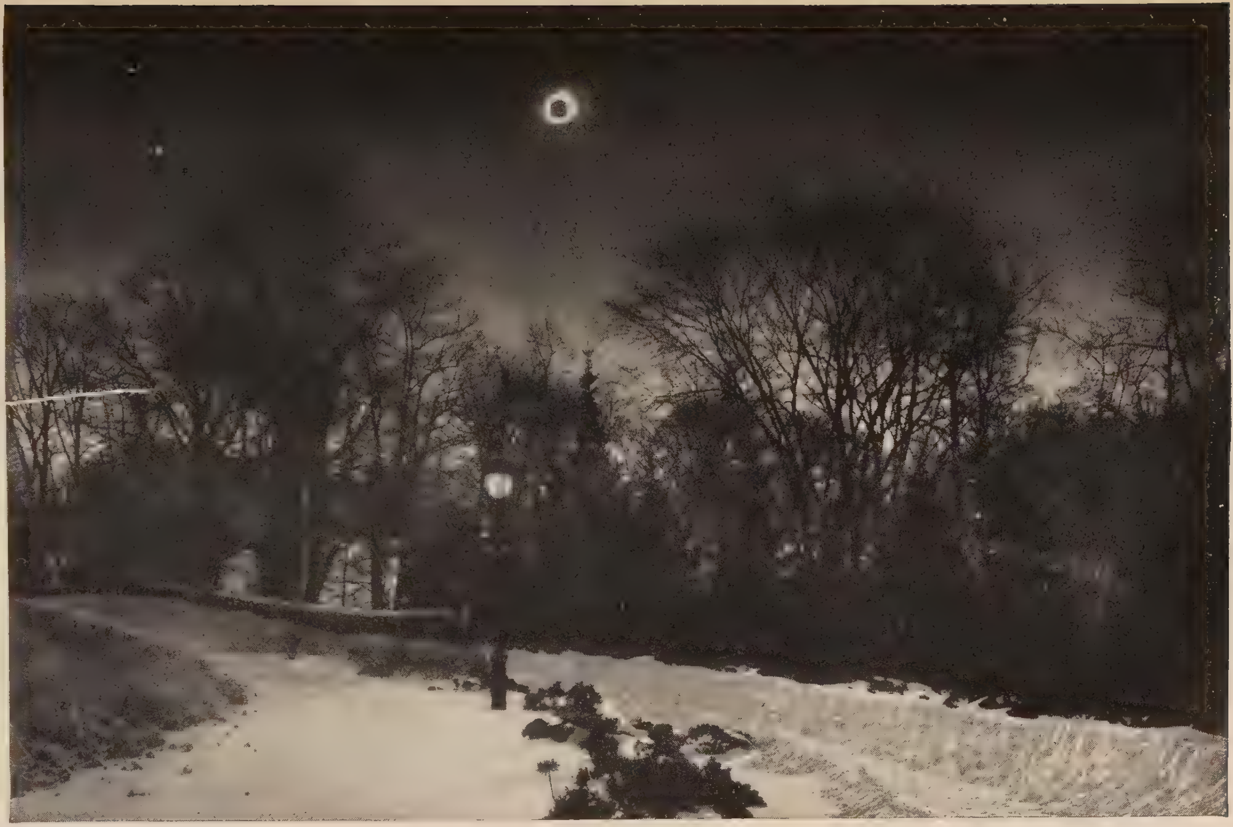
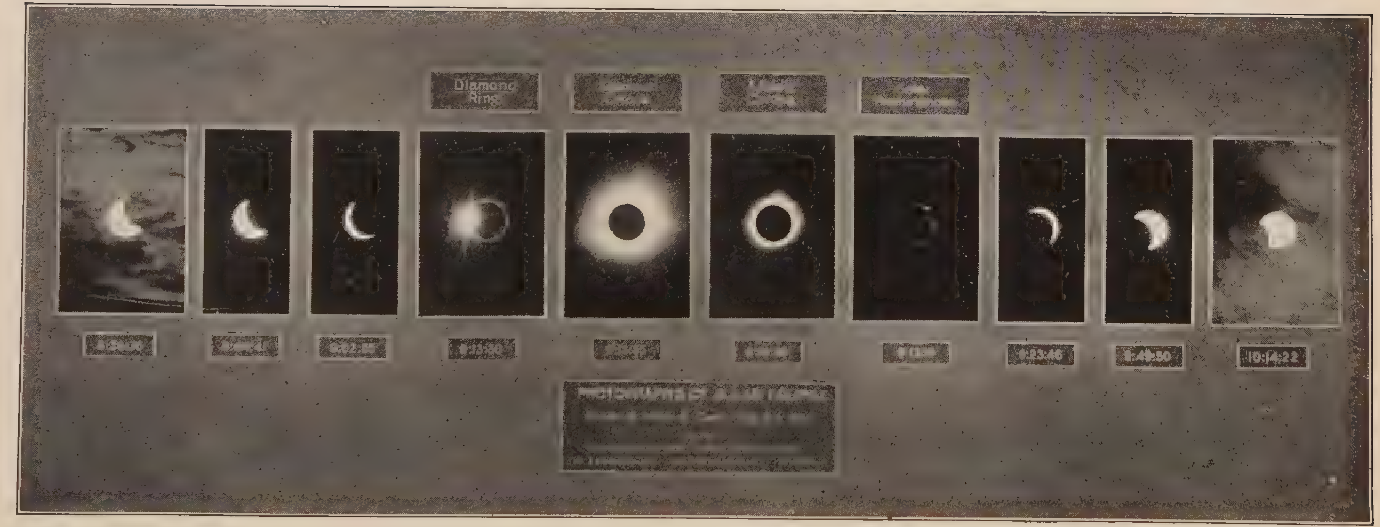


PLATE 9. TOTAL SOLAR ECLIPSE OF 1925 JANUARY 24, Photographed in Bronx Park, New York City, by A. Fasshender

Plate 10. The Solar Eclipse of 1925


Diamond Ring
10:14:22

Copyrighted by the Edison Electric Illuminating Company of Boston

Photographs made with graded exposures in both partial and total phases about equal to its own diameter in an hour, it may be wholly within the shadow (total lunar eclipse) about two hours, while the time from its first contact with the shadow until it finally leaves it may be as long as four hours.

The shadow cast by a luminous body of appreciable angular size, such as the Sun, consists of two parts: the shadow proper, or **umbra,** which is the part discussed above, and the **penumbra,** which is the region included between the prolongations of the tangents *AK* and *BH* (Fig. 90). Within the umbra, the direct rays of the Sun are completely excluded, although some sunlight, refracted by the air, is bent into the umbra of the Earth. Within the penumbra, there is direct illumination from a part of the Sun's face, but not the whole of it.

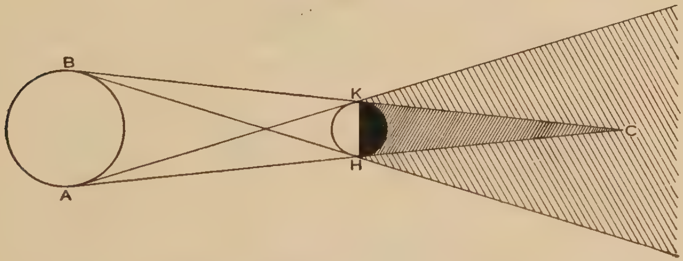


FIG. 90. SHADOW AND PENUMBRA

Effect of the Inclination of the Moon's Orbit.—If the Moon moved always in the plane of the ecliptic, there would be two eclipses, one of the Moon and one of the Sun, every month; but the plane of its orbit is inclined 5° to the ecliptic plane, and so the Moon usually passes either north or south of the line joining the Earth and Sun, and it is only when the Earth's radius vector nearly coincides with the Moon's line of nodes that an eclipse can occur. Fig. 91 represents the terrestrial orbit with the Earth and the lunar orbit in two positions, as seen from a point about 45° north of the plane of the ecliptic. The dotted portion of the Moon's orbit is south of the plane of the ecliptic, and must be thought of as lying behind the page on which the figure is drawn; while the remainder is north of the ecliptic, or above the page. In Position 1, the Moon passes above the Earth's shadow at full Moon (*B*) and below the

Earth's radius vector at new Moon (*A*), so that no eclipse occurs. In Position 2, the radius vector coincides with the line of nodes, the points *A* and *B* lie in the plane of the ecliptic, and, if the Moon is in the proper phase, an eclipse is inevitable.

Ecliptic Limits; Frequency of Eclipses.—For an eclipse to occur, the Earth's radius vector must be near the line of nodes, but the two lines need not coincide exactly. The greatest angular distance of the Sun (or of the Earth's shadow) from the nearer node which is compatible with an eclipse is called the **ecliptic limit**—lunar or solar, according to the kind of eclipse concerned. Fig. 92 represents a portion of the ecliptic and of the Moon's apparent path with the Earth's shadow and the full Moon in four positions. At *A*, both the Moon and the shadow are centered on the descending node. At *D*, the Moon just touches the shadow, and the distance *AD*, measured along the ecliptic, is the lunar ecliptic limit. Its value varies

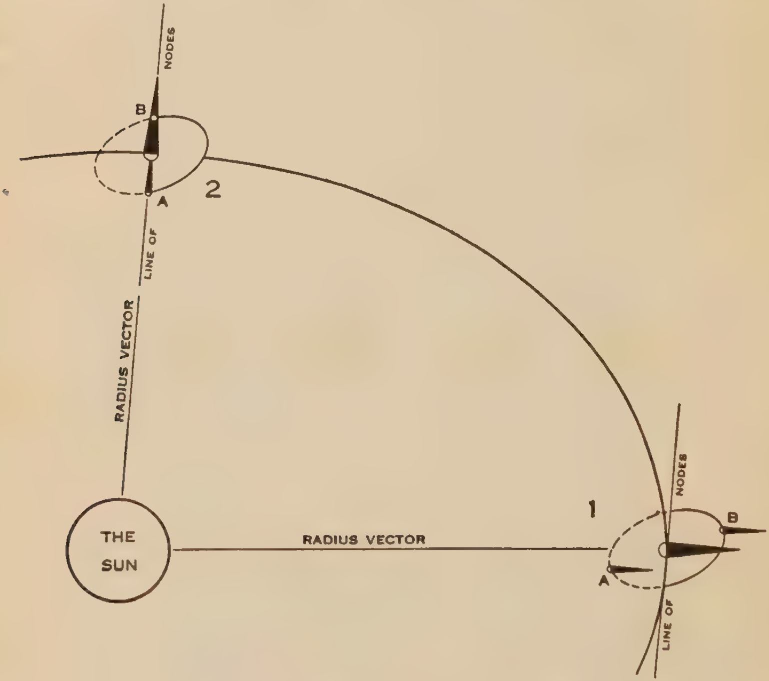


FIG. 91. WHY ECLIPSES DO NOT OCCUR EVERY MONTH

somewhat with the distance of the Moon, the dimensions of the shadow, and the inclination of the Moon's orbit. When the Moon crosses the shadow at the point where its diameter is three times that of the Moon (1°.5), the distance between their centers is about 1°, and, as the inclination is always near 5°, or one-twelfth of a radian, the distance *AD* is then about 12°. The maximum value of the lunar ecliptic limit is in fact 12°.3, while the minimum is 9°.5.

There is thus, under even the most favorable conditions, an arc of the ecliptic less than 25° long, extending to the major limit on either side of the node, within which the Sun must be situated at the time of full Moon if a lunar eclipse is to occur. The Sun moves along the ecliptic about a degree a day, and so will cross this region in twenty-five days; but the synodic month is more than twenty-nine days long, and so it is possible for the Sun to pass the node without our having a lunar eclipse at all. If a lunar eclipse does occur, even if it be just within the limit so that it is only a small partial one, before the Moon can be full again the Sun will have moved beyond the limit, and so only one eclipse of the Moon can occur at a given node passage. If the Sun passes one of the nodes early in January, it will pass the other in June and return to the first in December; for, on account of the regression of the nodes of about 19° a year, the interval between successive arrivals of the Sun at the same node, which is called the **eclipse year,** is only 346.62 days. The greatest possible number of lunar eclipses in a calendar year is therefore three, and a year may pass without any.

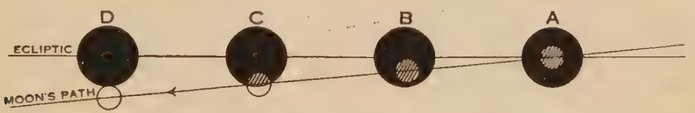


FIG. 92. LUNAR ECLIPTIC LIMIT

An eclipse of the Sun will be visible somewhere on the Earth if the Moon touches the boundary of the region *AKHB* (Fig. 89 or 93) which incloses the Earth and the Sun. The geocentric distance of its center from the Earth's radius vector is then the angle *SEM* (Fig. 93), which is the sum of the semidiameters *REM* and *AES* of the Moon and Sun, respectively, and the angle *REA*. This last angle is equal to the difference between the

Moon's geocentric parallax, *KRE*, and the Sun's parallax, *KAE*. The semidiameters of the Moon and Sun being each about 0°.25, while the maximum (horizontal) parallax of the Moon is about a degree and that of the Sun only 8″.8, the angle *SEM* has a maximum value of 1°.5. The solar ecliptic limit is therefore about 50 per cent greater than the lunar.

The solar ecliptic limit ranges from 15°.4 to 18°.5. There is thus a region at least 31° long in which the Sun may be situated at the time of a solar eclipse, and, as it requires longer than a synodic month to traverse this arc, one solar eclipse is inevitable and two are possible at a single node passage. If a (partial) eclipse occur early in January near the solar ecliptic limit west of the node, a second will occur in the same month; two solar eclipses may likewise happen at the other node in the middle of the year, and a fifth may occur in December; but a sixth is impossible in that year, for it must take place, if at all, about 346.6 + 29.5 = 376 days later than the first, which would place it in January of the following year. The greatest possible number of solar eclipses in a single year is therefore five, and the least is two.

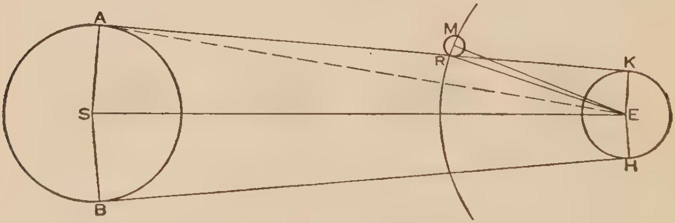


FIG. 93. SOLAR ECLIPTIC LIMIT

When two solar eclipses occur at a given node passage, a lunar eclipse always takes place between them. The greatest possible number of eclipses in a single year is seven, either two of the Moon and five of the Sun or three of the Moon and four of the Sun. The least possible number is two, both of the Sun.

Recurrence of Eclipses; the Saros.—As the two conditions necessary for an eclipse are the appropriate phase of the Moon and proximity of the Sun to the Moon's node, an eclipse must repeat itself after an interval that contains without a remainder both the synodic month and the eclipse year. The smallest interval which even approximately does this is that which consists of 223 synodic months, being 6,585.32 days (18 years, 11⅓ days—or 10⅓ days if the interval contains five leap-years), which is only 0.46 day less than 19 eclipse years. The recurrence of eclipses in this interval was discovered in prehistoric times by the Chaldeans, who named the interval the **saros,** signifying repetition. During the course of a saros there occur about twenty-nine lunar eclipses and forty-one solar ones, ten of the latter being total, and these eclipses are repeated approximately in the next saros, but are then visible in longitude 120° farther west, the Earth having made in the meantime 6,585 rotations and a third of a rotation over.

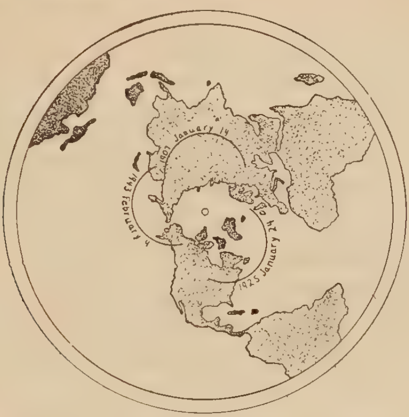

1907 January 14
1925 January 24
1943 February 4

FIG. 94. PATHS OF TOTAL SOLAR ECLIPSES AT INTERVALS OF A SAROS

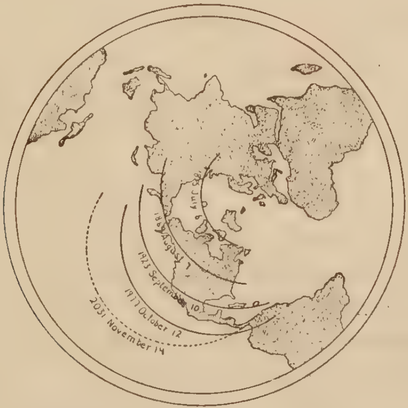

July
1869 August 7
1923 September 10
1977 October 12
2031 November 14

FIG. 95. PATHS OF TOTAL SOLAR ECLIPSES AT INTERVALS OF THREE SAROSES

Fig. 94 shows the paths of three total solar eclipses occurring at intervals of a saros. After three saroses, or fifty-four years, one month, the eclipses return to nearly the same longitude, but, owing to the slight difference between the saros and an integral number of eclipse years, the Sun has then a position a little farther west with respect to the node and the eclipses of the new set occur a little farther north if at the descending node or farther south if at the ascending. Fig. 95 shows the paths of a number of central solar eclipses occurring at intervals of three saroses. These eclipses take place at the ascending node, and so the series began at the north pole of the Earth and is sweeping slowly southward.

Eclipses of the Moon.—The entrance of the Moon into the Earth's penumbra makes no perceptible change whatever in its brightness, but as it approaches the edge of the umbra a darkening becomes noticeable at its eastern limb. This darkening is so slight, however, that the edge of the umbra, when the Moon encroaches upon it, seems black by comparison. In a telescope, even the deepest part of the shadow may usually be seen to contain some light, and when the Moon is completely immersed so that the eclipse is total it ordinarily remains easily visible to the naked eye; for it shines dimly with a dull reddish light which is sunlight, refracted into the shadow by the Earth's atmosphere as illustrated in Fig. 96, and tinged with sunset colors by selective absorption.

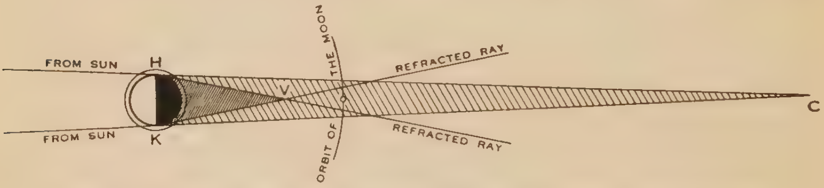


Fig. 96. Refraction of Sunlight by the Earth's Atmosphere

The angles *CHV* and *CKV*, between the limits of the geometric shadow-cone and the extreme refracted rays, are each somewhat over 1°, being twice the atmospheric refraction of sunlight at the horizon (page 60); hence the angle at *V*, which may easily be shown to be the sum of the angles *HCK*, *CHV*, and *CKV*, is about 2°.5, or five times the angle at *C*. The length of the cone *HVK*, from which sunlight is entirely excluded, is therefore about a fifth of the length of the geometric shadow or 170,000 miles—much less than the least distance of the Moon from the Earth. As seen from the Moon during a total lunar eclipse, the Earth would usually appear surrounded by a ring of brightly illuminated air; but if, as sometimes happens, the air throughout this ring contains dense clouds, most of this light is cut off. Under these circumstances, the Moon may, for a terrestrial observer, completely disappear.

Eclipses of the Sun.—A lunar eclipse, whether total or partial, can be observed simultaneously from every place on the terrestrial hemisphere that is turned toward the Moon. It is quite otherwise with eclipses of the Sun. A total solar eclipse can be seen only within the umbra of the Moon's shadow, which, at the point where the Earth's surface cuts it, is at most only 168 miles in diameter; an annular eclipse can be seen within a region which may be 230 miles wide; and a solar eclipse is partial anywhere within the Moon's penumbra, which at the Earth's distance is some 4,000 miles in diameter—sufficient to include about half of the exposed hemisphere. As the Moon moves in its orbit its shadow, like that of a vast bird, passes over the Earth's surface; and the solar eclipse is visible successively from all the points in its path.

The Moon and its shadow move eastward at a rate of about 2,000 miles an hour. A point on the Earth's equator is carried by the Earth's rotation, also eastward, at about half that speed. Hence, the Moon's shadow passes an observer at the equator with a velocity of about 1,000 miles an hour. In higher latitudes, where the observer's velocity is less, the shadow passes more rapidly, and if it falls obliquely, as it does when the eclipse occurs near sunrise or sunset, the relative velocity of the shadow and the observer may be as great as 5,000 miles an hour. The greatest possible duration of a total solar eclipse for a single observer is $7^m\ 30^s$; of an annular eclipse, about 12^m; and of the whole eclipse, from the first contact of the Moon's disk with the Sun until the last, a little over four hours. This maximum duration is achieved very rarely indeed, and a total solar eclipse that lasts five minutes is unusually favorable.

Because of its brief duration and the very limited area within which it is visible, a total eclipse of the Sun is a phenomenon which many persons never see, although, taking the Earth as a whole, such eclipses are not rare. According to

Rigge, only two such events could be seen at London and only three at Rome in the twelve centuries between A.D. 600 and 1800.

Astronomers go to remote parts of the Earth to observe them, often to be disappointed by cloudy weather after traveling thousands of miles. Probably no one has viewed the Sun's corona, which can be seen only while the eclipse is total, during an aggregate of thirty minutes.

Total Solar Eclipses.—Many excellent writers and competent observers have described total solar eclipses, giving accounts which are fascinating and memorable; but any one who has intelligently observed this most sublime of natural phenomena knows that all descriptions fail to convey an adequate impression of the reality.

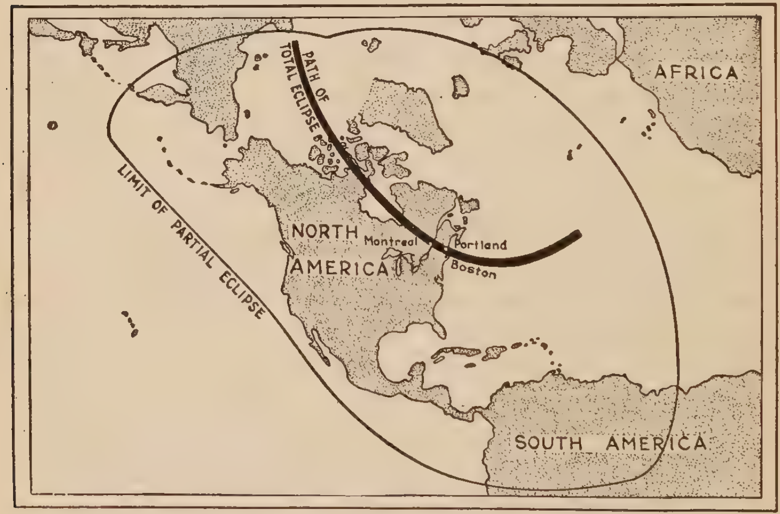


Fig. 97. Region of Visibility of Solar Eclipse of 1932 August 31

A moment after "first contact" the eastern limb of the black Moon may, with the protection of a smoked glass or overexposed photographic film, be seen on the western limb of the Sun. As the Moon moves steadily eastward the black area grows, but for some time the only effect apparent on the Earth is in the images of the Sun formed by such small apertures as the interstices between the leaves of trees, which change from their usual circular shape to narrowing crescents—an effect which may of course be seen at any partial eclipse. About half an hour before totality, the landscape becomes perceptibly darkened and both the Earth and the sky assume an indescribably weird color due to the light coming from near the Sun's limb which is of a different quality from the stronger radiance of the center of the disk. As the light fades and its unusual color becomes relatively stronger, it is noticed not only by men, but by other creatures; birds fly about and twitter excitedly, roosters crow, and dogs bark. Before the beginning of totality, fowls go to roost, dew or frost may form, and many flowers close their blossoms. Several minutes before "second contact" (the beginning of totality), ghostly **shadow bands** may be seen flitting along upon any exposed white surface. These are the shadows of waves in the Earth's atmosphere, rendered distinct by the narrowness of the remaining crescent of the Sun.

All this is but preliminary to the great spectacle of the total eclipse. Just before second contact a favorably situated observer may see the Moon's shadow in the air, looking like a vast thundercloud, but not so definitely limited, approaching from the west with a speed far exceeding that of any storm, being twenty miles a minute or greater. The thin crescent of the Sun's eastern limb breaks up into "Baily's beads," which are due to the Sun shining between irregularities at the limb of the Moon; while at the western limb there appears already the glow of light from the inner **corona,** the wonderful pale envelope of the Sun which can be seen only when the main body is covered by the Moon. In the eclipse of 1925 one of the "beads" persisted after the corona was distinctly visible all around the Moon, giving an appearance that was likened by thousands of delighted spectators to a diamond ring. The majesty of the spectacle at this point is such that it seldom fails to silence the chattering crowds of spectators.

The "beads" last but an instant, and then the full glory of the corona presents itself. Its form is never the same at two different eclipses, but it always consists of a bright ring around the Sun (and Moon), with irregular streamers extending several solar diameters away. In color it is pearly white. At its inner edge it is very brilliant, contrasting sharply with the dead-black Moon, while its outer streamers fade imperceptibly into the sky, which, if clear, is of a deeper blue than at other times, and on which are visible the brighter stars and planets. Often to the unaided eye and usually with a telescope, there may be seen rosy, flame-like **prominences** or **protuberances** extending from the red **chromosphere** which is visible at the disappearing or the reappearing limb of the Sun. Strangely enough, no account of the prominences was ever given before the eclipse of 1842, although they have been seen or photographed at most total eclipses that have been observed since.

The only changes to be noted during totality are those due to the motion of the Moon as it covers the inner corona and prominences at the eastern limb of the Sun and uncovers those at the western limb. From third contact, the end of the total phase, to fourth contact, when the Moon finally leaves the Sun's disk, the events of the earlier part of the eclipse are repeated in reverse order.

The spectacular features of the eclipse are sometimes heightened by the presence of a few clouds, and even if the sky is completely overcast the sudden coming of darkness at mid-day is exceedingly impressive. The darkness of an eclipse is never very deep, for the corona gives about half as much light as the full Moon, and the air and clouds forty or fifty miles away, where the Sun is only partially eclipsed, are still brightly illuminated.

A total solar eclipse affords a unique opportunity for many important investigations, such as:

(1) The study of the corona; visually, with long-focus cameras, and with the spectrograph, the photometer, and the polariscope.

(2) Photography of the spectra of the chromosphere and prominences and the "flash spectrum" (page 177).

(3) The search for new planets and other bodies near the Sun.

(4) Determination of the exact relative position of the Moon and Sun by observation of the contact times.

(5) Photography of the field of stars around the Sun to detect and measure the "Einstein displacement" due to the Sun's gravitational effect on light (page 239).

Prediction of Eclipses.—From a knowledge of the elements of the orbits of the Earth and the Moon and of the changes of these elements such as the regression of the line of nodes, it is possible to predict far in advance the circumstances of any eclipse and the time, within a few seconds, when it will occur. The calculation of a lunar eclipse is simpler than that of a solar one since the circumstances of the former are the same for all points from which it is visible. In a great work named *Canon der Finsternisse*, Oppolzer of Vienna has given the approximate data concerning all the eclipses that have occurred since 1207 B. C. or that will occur up to A. D. 2162, with maps showing, for central solar eclipses, the paths of the Moon's shadow on the surface of the Earth.

EXERCISES

1. What is the latitude of an observer who sees the Moon at the zenith at the middle of a total lunar eclipse on December 22?

Ans. 23½° north

2. At the total solar eclipse of 1918 June 8, the Moon's shadow swept from a point south of Japan across the Pacific Ocean, and across the United States from the State of Washington to Florida. Where and when will the eclipse be seen which follows at the end of a saros? Where and when was the eclipse seen which occurred one saros earlier?

CHAPTER VII

SPECTROSCOPY

The Analysis of White Light.—Although in ordinary speech the words *white* and *colorless* are often used as synonyms, it is a fact that white light, such as the light of the Sun, is in reality a mixture of light of all the different colors. This was first proved in 1666 by Sir Isaac Newton, who placed a triangular glass prism in the path of a beam of sunlight that he had admitted to a dark room through a hole in a window shutter, and found that the beam was not only deviated from its original path by the refraction of the prism, but that it was spread out into a band of light, red at the least refracted end and violet at the other. This band of light is called a **spectrum** and the separation of the colors by the prism is called **dispersion.** To show that the colors were not bestowed upon the light by the prism, Newton isolated from the spectrum a ray of a single color by passing it through a hole in a second screen, and placed in its path a second prism, when he found that, although the ray was further deviated, its color was no further changed; and he also recombined the colors of the spectrum by reversing the second prism, when the light again appeared white.

The branch of physical science that deals with the analysis of light, of which Newton's simple experiments were the beginning, is called **spectroscopy.** In the form into which it has been developed in the nineteenth and twentieth centuries, it has made possible the study of the chemical constitution, temperature, and motion of any source of light, whether it be a flame or electric spark in the laboratory or a distant body like the Sun or a star; and it is to spectroscopy that we owe much of the advance of astronomy since about 1860.

The Rainbow.—The rainbow, one of the most beautiful of natural phenomena, is a natural spectrum produced by the dispersion of sunlight by spherical raindrops. It appears as an arch of colored light having its center on the prolongation of the line from the Sun through the observer's eye, and therefore seems to move over the landscape if the observer moves. Most of the light of the rainbow is contained in the **primary bow,** which has a radius of about 42°; but there is also often seen a fainter **secondary bow,** of radius 51°, and sometimes certain very faint **supernumerary bows** near the inner edge of the primary and the outer edge of the secondary.

The complete explanation of these appearances cannot be entered upon here, but that of the primary bow is briefly as follows: A ray of sunlight, *SA*, falls upon a spherical drop of water at *A*, and is refracted and dispersed as at the surface of a prism. One ray, say the yellow, proceeds to *B*, where it is reflected at the inner surface of the drop, is refracted a second time at *C*, and emerges in the direction *CO*. It may be shown from the refractive index of water and the geometric relations involved that the direction, *CO*, along which yellow light emerges most copiously, makes an angle of 42° with the incident ray *SA*. An observer at *O*, therefore, receives yellow light from all raindrops that are situated upon a cone having his eye for vertex, the prolongation of the line from the Sun through his eye as axis, and a semi-angle of 42°. In a shower of rain, as each drop falls, its place is taken by another and the observer has the impression of a continuous arch of light which would be a complete circle but that the supply of raindrops ends at the horizon. It is obviously impossible to see a rainbow produced by the Sun when its altitude is more than 42°, and rainbows are most commonly seen in early morning or late afternoon. In both refractions at the surface of the raindrop, the red is deviated less than the yellow and the violet more, and so red light reaches the eye from drops outside the cone of yellow rays and violet from drops inside, and the colors of the spectrum are to be seen in the luminous arch, the red at the outer edge and violet at the inner. The secondary bow is formed of light that enters the drop at the lower instead of the upper side, is twice reflected, and emerges from the upper side. In it, the order of the colors is reversed so that red is at the inner edge.

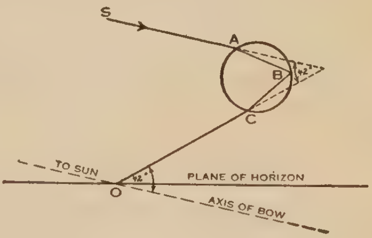


FIG. 98. OPTICS OF THE RAINBOW

The Prism Spectroscope.—When the source of light is a body of large angular size like the Sun, as in Newton's experiment and in the rainbow, the rays from different parts of the source strike the prism (or raindrop) at different angles and emerge at different angles, so that the colors in the resulting spectrum are mixed, the green light, for example, from one point of the source falling upon the red light from another. To obtain a *pure* spectrum with a prism from such a source it is necessary to pass the light first through a narrow slit placed parallel to the refracting edge of the prism. Even then the colors will mix unless all the rays of a given color be made to pass through the prism parallel to one another and to meet at a focus after leaving it.

An instrument for studying the spectrum visually is called a **spectroscope**; if arranged for photographing the spectrum it is called a **spectrograph.** In the most usual type of prism spectroscope the conditions necessary for producing a pure spectrum are met as follows: The light from the luminous body *F* (Fig. 99) first enters the **slit** *S* (only a few thousandths of an inch wide), the rays from different parts of the source crossing at the slit and diverging beyond. At a distance from the slit equal to its own focal length is placed the **collimating lens** *C*, which renders the rays parallel. This broad beam of parallel rays then passes through the **prism** *P*, which disperses the light into its component colors. On emergence, all the rays of a given color are parallel to one another, but are not parallel to rays of other colors. The light then passes through the **objective** *O*, which focuses the red light at R and the violet at *V*, the other colors being arranged between to form a spectrum, which may be photographed by placing a sensitive plate in the plane *RV*, or may be studied visually with an eyepiece which, with *O*, would form the **view telescope.** For studying the spectra of the heavenly bodies, the spectroscope is ordinarily attached to an astronomical telescope, the slit being placed in the focal plane of the objective, so that a bright image of the heavenly body is formed at the slit. The astronomical telescope thus becomes merely a collector of light. To secure high dispersion, the light is in some spectrographs passed successively through two or more prisms.

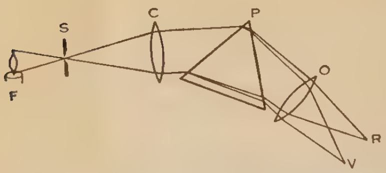
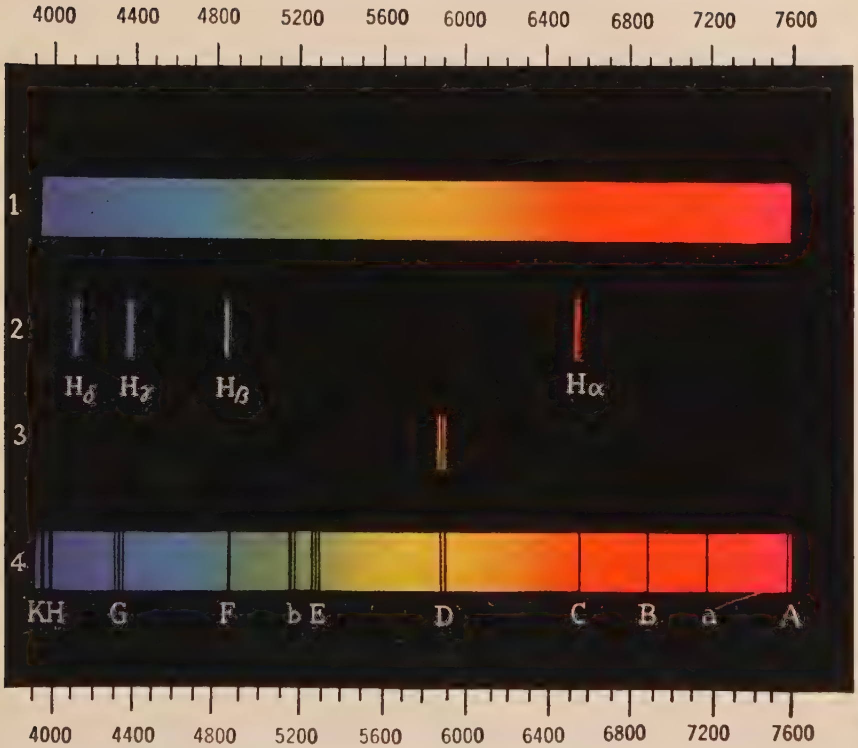


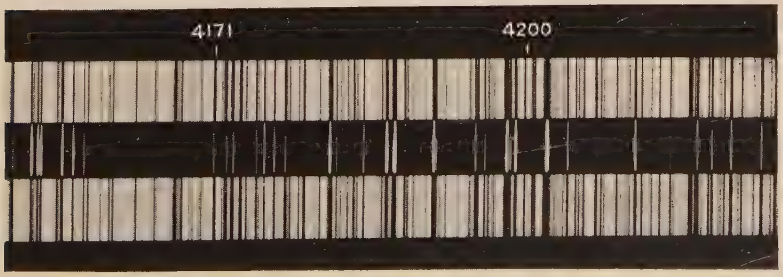
FIG. 99. THE PRISM SPECTROSCOPE

Plate II. Maps of Grating Spectra


4000
4400
4800
5200
5600
6000
6400
6800
7200
7600
1
2
H_δ
H_γ
H_β
H_α
3
4
KH
G
F
b
E
D
C
B
a
A
4000
4400
4800
5200
5600
6000
6400
6800
7200
7600

1. A continuous spectrum
2. The visible part of the Hydrogen spectrum
3. The visible part of the Sodium spectrum
4. The solar spectrum showing the lettered Fraunhofer lines

Below The spectrum of Iron with solar spectrum on either side. Photographed with high-dispersion spectrograph at Mt. Wilson Observatory



For a general inspection of the spectrum, a convenient form of spectroscope is one provided with a **direct vision prism,** which consists of a combination of prisms of flint and crown glass, usually arranged as in Fig. 100. A wide-angle prism of flint glass producing considerable dispersion is placed between two inverted crown-glass prisms. Since crown glass has a smaller dispersive power than flint, these rectify the deviation of the central ray of the spectrum without entirely destroying the dispersion. In small pocket spectroscopes intended for laboratory work, a direct-vision prism is used without a view telescope, since the emergent beam is already small enough to enter the pupil of the eye.

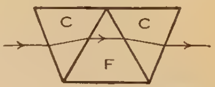


FIG. 100. A DIRECT-VISION PRISM

Many other modifications of the form of the spectroscope are made for special purposes.

Development of Spectroscopy.—In 1802 Wollaston, in England, improved upon Newton's experiment by admitting the light to his prism through a narrow crevice (one-twentieth inch wide), thus obtaining a spectrum of greater purity, and noticed that the spectrum of sunlight was crossed in a number of places by dark lines which did not appear in the spectrum of a candle flame. A few years later, the great German optician Fraunhofer further improved the spectroscope by using a narrow slit and a collimating lens and viewing the spectrum through the small telescope of a theodolite. He thus observed about seven hundred of the dark lines, which have since been known as Fraunhofer lines. He found that the dark lines occupied fixed places in the spectrum, and, although he was unable to explain their meaning, he realized their importance as points of reference and made a careful map of their positions, denoting the more prominent lines with the letters of the alphabet, beginning in the red with A.

In 1822, Sir John Herschel observed that a flame impregnated with certain metallic salts exhibited a spectrum consisting of isolated bright lines (each being an image of the slit), whose position was characteristic of the substance with which the flame was fed. Fraunhofer and others further noticed that the position of the single yellow line of which the spectrum of sodium seemed to consist was identical with that of the dark line in the spectrum of the Sun which Fraunhofer had lettered D. In 1859 Kirchhoff, by a brilliant series of investigations, showed that the spectra of different chemical substances which were brought to incandescence in a gaseous form were characteristic of the substances, no two being alike, and that dark lines were produced by the absorption of light by gases through which it passed. The work of Kirchhoff gave a great impetus to the study of spectroscopy and formed the basis of much of its application to astronomy.

The Production of Spectra.—Astronomical spectra are interpreted by comparing them with the spectra of sources of light artificially produced. These sources are principally flames, the electric arc, the electric spark, and vacuum tubes. Only a few elements, among which are sodium, lithium, and strontium, can be volatilized at so low a temperature as that of a flame. Their spectra may be obtained by placing small quantities of their salts in the flame of a Bunsen burner. All metals are sufficiently heated to yield their characteristic spectra when an electric **arc** is made between terminals composed of the metal, or when a **spark** from an induction coil is passed between such terminals. The spectra of permanent gases, such as hydrogen, helium, or argon, are best studied by causing the gas to shine in a **vacuum tube.** This is a tube of glass or quartz with a slender middle portion, in the ends of which are sealed metallic terminals for leading in an electric current. The tube is exhausted of air and a very little of the gas is introduced, of course at very low pressure, after which the tube is hermetically sealed. When the metal terminals are connected to an induction coil or static electric machine, the gas glows with a soft light that fills the tube.

Kinds of Spectra.—Spectra are generally classified as **bright-line, continuous,** and **dark-line** spectra. In the first kind, all the light of the spectrum is contained in separate bright lines, each an image of the slit, the spaces between the lines being completely vacant. A continuous spectrum is one in which the light is spread out in a continuous band without interruptions. A dark-line spectrum consists of separate dark lines which, to be detected, must be seen against a background of continuous spectrum. Such a spectrum is the reverse of a bright-line spectrum. Examples of these different types of spectrum are shown in Plate 11.

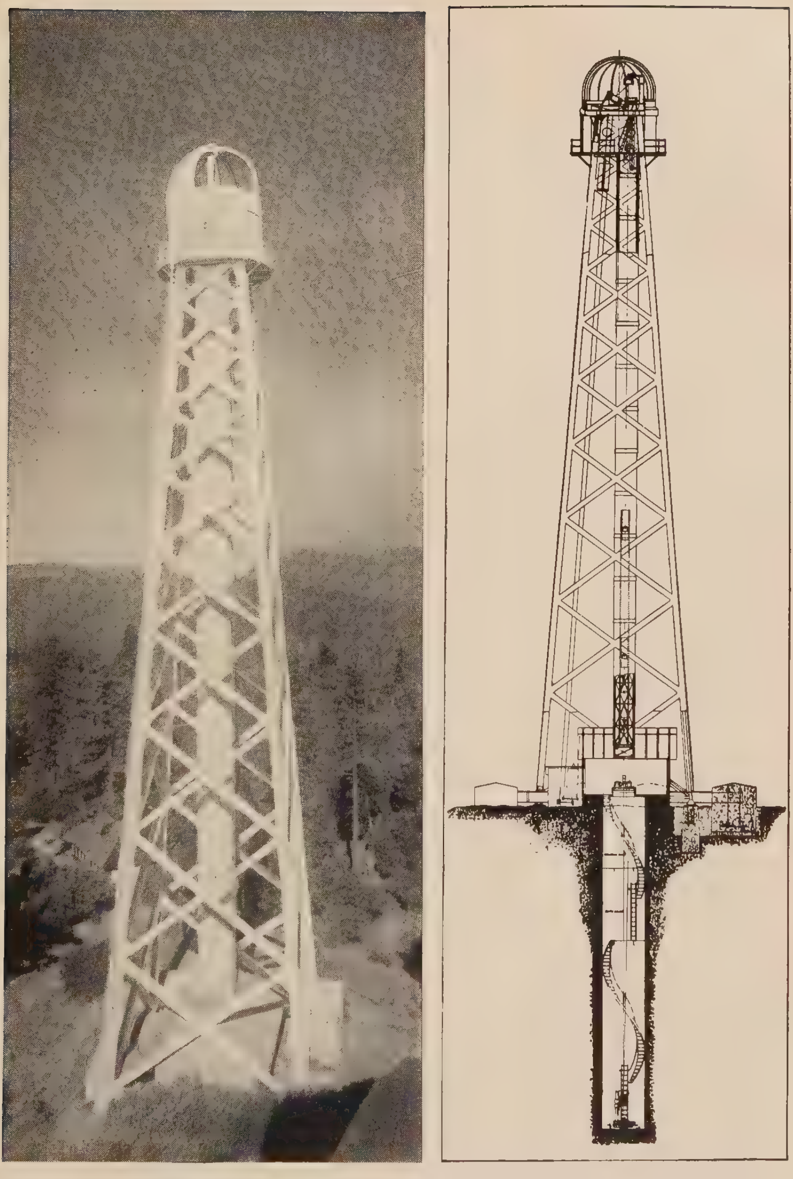
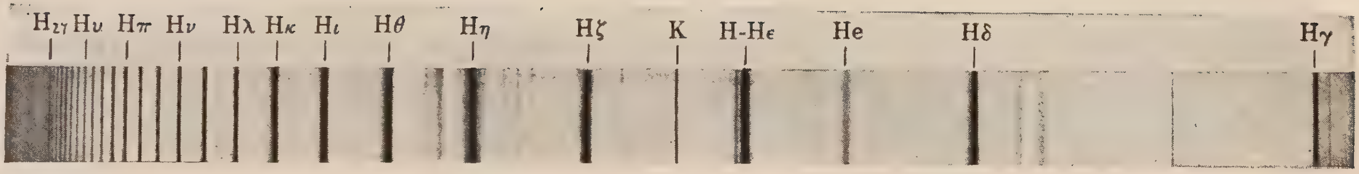
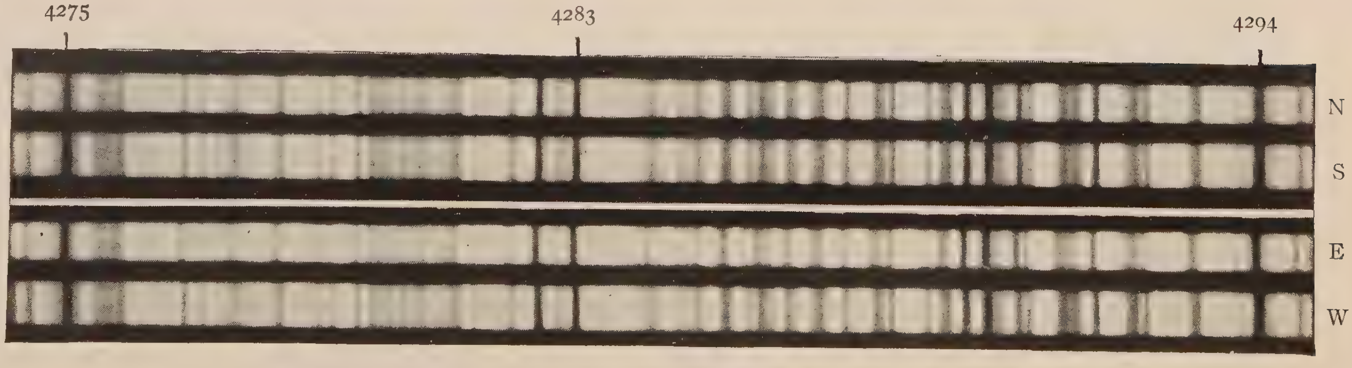


Plate 12. The 150-foot Tower Telescope and 75-foot Spectrograph on Mount Wilson

Plate 13


H27
Hυ
Hπ
Hν
Hλ
Hκ
Hι
Hθ
Hη
Hζ
K
H-Hε
He
Hδ
Hγ

Spectrum of the Star ζ Tauri, Showing the Balmer Series of Hydrogen Lines
Photographed by R. H. Curtiss, Detroit Observatory


4275
4283
4294
N
S
E
W

Spectra of North, South, East, and West Points of the Sun's Limb
Lower pair shows displacement due to the Doppler-Fizeau effect. Photographed by Adams, Mt. Wilson Observatory

The Principles of Spectral Analysis.—The work of Kirchhoff established the following principles, which, although there are exceptions, form a general basis for the analysis of sources of light by means of their spectra.

1. *An incandescent solid, liquid, or compressed gas gives a* ***continuous spectrum.*** Examples are found in the spectrum of red-hot or molten iron and in that of the glowing filament of an electric lamp. A rarefied gas also is known to yield a continuous spectrum under certain circumstances, as in the depths of some of the giant stars.

2. *An incandescent gas under low pressure gives a* ***bright-line*** *spectrum, the positions of the lines being characteristic of the chemical nature of the gas.* Bright-line spectra are often called **emission spectra.** Every known chemical element has more than one line in its spectrum, although in many cases two lines, as the two visible lines of sodium, which are yellow, are so close together that they appear as one in small spectroscopes. The spectra of many of the metals contain hundreds or thousands of lines. No two elements are known to have spectra with even a single line in common. Lines of many elements occur in what seems to the eye to be the same *color* but they always differ, though perhaps but slightly, in their *position* in the spectrum. For example, the spectrum of helium contains a prominent yellow line, the color of which the eye could not possibly distinguish from that of the sodium line; but when the two spectra are confronted in the same spectroscope, it is found that the helium line lies the farther toward the violet.

3. *When light from a source that gives a continuous spectrum shines through a gas whose temperature is lower than that of the source,* ***dark lines*** *appear in the spectrum in the positions characteristic of the bright lines belonging to the gas.* In other words, a gas absorbs from a beam of light just those rays that it emits, and no others. Dark-line spectra are often referred to as **absorption spectra.** The lines in such spectra are never totally *black*, but appear dark by contrast with the luminous background of continuous spectrum; the gas through which the light shines is emitting as much light as ever.

Lines in the emission or absorption spectra of gases which are composed of molecules instead of simple atoms are very numerous and are grouped closely together to form **bands,** which are usually sharply defined on one side and fade off gradually on the other. Common examples of such gases are the oxygen and nitrogen of the air and the carbon compounds which shine in the base of a blue gas-flame. The spectrum of the latter consists of bands which are sharp on the redward side and is called, from its earliest investigator, the **Swan spectrum.**

Selective absorption is sometimes produced also by liquids and solids, for example, by the Crookes glass used for special kinds of spectacle lenses, and by a solution of chlorophyll, the coloring matter of green leaves. This kind of absorption is manifested by dark bands that are broad and hazy, and not made up of fine lines.

The three principles may be illustrated as follows: Before the slit of a spectroscope is placed an incandescent electric lamp; the glowing filament is an incandescent solid, and a continuous spectrum is seen in the spectroscope, illustrating the first principle. The spectrum is brightest if an image of the filament is thrown upon the slit by means of a convex lens of which the ratio of aperture to focal length is about the same as that of the collimator, so that the collimator and prism are filled with light. Let an alcohol lamp be placed between the slit and the electric lamp, and let the latter be turned off. The blue alcohol flame is so nearly transparent that probably no light will be seen in the spectroscope, but if a little common salt (sodium chloride) be inserted on a wire at the base of the flame the sodium will be vaporized and the flame will be colored yellow by incandescent sodium gas; and in the spectroscope will be seen the two bright yellow lines of the sodium spectrum, exemplifying the second principle. Now let the electric lamp be turned on again, so that its light passes through the sodium flame; the continuous spectrum again appears, but this time it is crossed by a pair of *dark* lines in the yellow in the position that was occupied just before by the bright sodium lines. This illustrates the third principle.

The Solar Spectrum.—In Plate II, the colored dark-line spectrum represents that of the Sun as it appears in a grating spectroscope of moderate dispersion. The most prominent lines are marked with their Fraunhofer letters. From the third principle of spectral analysis, since the solar spectrum is a dark-line one, we may infer at once that the Sun consists of a central core of highly heated material in such a state as to give a continuous spectrum, and that this core shines out through an envelope or atmosphere of less intensely heated gases. Moreover, by comparing the solar spectrum with the spectra of terrestrial substances, the presence of many familiar elements in this gaseous envelope may be established. Plate 11 (below) shows a portion of the violet end of the spectrum of an electric arc that was formed between rods of iron and, on either side of this, the corresponding part of the spectrum of the Sun. In making the photograph, the middle of the slit of the spectrograph was illuminated by the arc while the ends were covered; then the middle was covered and the ends were illuminated by sunlight. The spectra are thus made to correspond exactly, and it is seen that each of the bright iron lines has a dark counterpart in the spectrum of the Sun. Some two thousand of these coincidences have been established, giving two-thousand-fold evidence that the atmosphere of the Sun contains iron. Since, in order to produce spectral lines, the iron must be so hot as to be not only melted, but vaporized, and since the core must be hotter still, this feature of the spectrum is alone sufficient to inform us that the Sun is intensely hot.

The principal Fraunhofer lines are as follows:

A. Band in the extreme red, sharply defined on the side toward the violet, due to oxygen, but belonging to the Earth's atmosphere instead of the Sun's as shown by its intensification when the Sun's altitude is low.[1]
a. Rather narrow band in the red, due to terrestrial water vapor.
B. Band in the red, similar to A, due to terrestrial oxygen.
C. Line in the red due to solar hydrogen; identical with Hα.
D. Double line in orange-yellow, due to sodium in the Sun.
E. Close group of lines in green, due to iron and calcium.
b. Close group in green, due mainly to magnesium.
F. Hydrogen line in blue-green, identical with Hβ.
G. Composite band in blue-violet.
H. Very broad calcium line in extreme violet.
K. Very broad calcium line, similar to H, in very extreme violet. K was not lettered by Fraunhofer, and can be seen only when very bright sunlight is used; but on photographs, H and K are the most prominent of all.

[1] Also by the absence of a Doppler-Fizeau displacement when the spectra of the receding and approaching limbs of the Sun are compared (page 169).

The Spectra of the Stars.—The great majority of the stars, like the Sun, show dark-line spectra, suggesting that their general architecture is the same as that of the Sun. Many star spectra are in fact practically identical with the solar spectrum, and it is certain that the stars are distant suns; but there is a very great variety in the arrangement and relative intensities of the lines in stellar spectra, corresponding to a vast range in the physical conditions of the stars. This subject will be discussed further in Chapter XIV.

The Spectra of Nebulæ.—Most of the nebulæ that are located in the Milky Way show bright-line spectra and are believed to be vast masses of luminous, tenuous gas. The lines shown include those of hydrogen and helium, and of oxygen and nitrogen in a highly ionized condition (page 167).

Most of the extra-galactic nebulæ that are bright enough for spectroscopic study exhibit dark-line spectra, in harmony with the theory that they are in reality remote aggregations of stars.

Dispersion by a Grating.—A **diffraction grating** is made by ruling fine, equidistant lines with a diamond point upon a polished surface of glass or metal. When white light is transmitted or reflected by such a ruled surface it is dispersed, not into a single spectrum, but into many, and the dispersion is the greater the closer the ruling. Gratings having 15,000 or more lines to the inch are often used in powerful spectrographs in place of prisms; but in astronomical spectroscopy their use is confined almost entirely to the study of the spectrum of the Sun, since only a fraction of the light is contained in any one spectrum and the light of the stars is too feeble to permit the use of such a wasteful instrument. In the grating spectrum the order of the colors is the reverse of that in the prismatic spectrum, the red being deviated most and the violet least.

In Plate 12 is shown the great tower telescope of the Mount Wilson Observatory. The tower is made double, each upright and cross-piece shown in the photograph inclosing a similar one that belongs to the inner tower. The outer tower supports the dome and protects the inner tower from the wind; the inner one supports at the top a **coelostat**, an arrangement of mirrors driven by clockwork so as to reflect the Sun's light vertically downward, and, just below this, a twelve-inch lens of 150 feet focal length, which forms a seventeen-inch image of the Sun at the foot of the tower. In the seventy-five-foot well beneath is placed a grating spectrograph having its slit in the focal plane of the lens so that the spectrum of any part of the Sun may be photographed with enormous dispersion. It was with this spectrograph that the photograph of the solar spectrum reproduced in the lower part of Plate II was made. The movements of the dome, coelostat, and spectrograph are, of course, controlled electrically.

Wave Theory of Light.—The action of the grating upon light is extremely important, for it shows that light consists of **waves,** the length of which is different in light of different colors, and enables the experimenter to measure the length of these waves.

A complete discussion of the grating is far beyond the scope of the present book, and for this the reader is referred to advanced works on physics; but some idea of the theory may be given as follows: In a homogeneous medium, waves spread out from a point-source with equal velocity in all directions, so that the wave-front is a sphere with the source at the center and the light is propagated in straight lines radially from the source. If the light encounters an obstacle, the waves bend slightly around the edge, and the limit of the shadow cast by the obstacle is not perfectly definite. This bending is called **diffraction.** The shorter the wave, the less the bending, and light-waves are so short that the diffraction is difficult to detect unless the source of light is very small and the edge of the obstacle very sharp. In the grating, the diamond-lines are effective obstacles with very sharp edges, the light being transmitted or reflected by the narrow spaces of polished glass between them. Let *A* and *B*, Fig. 101, be the upper edges of two adjacent spaces of a grating, seen edgewise and highly magnified, and let the grating be illuminated from the left side with light of a single wave-length. Waves of light will be diffracted and will proceed in all directions from *A* and from *B*. Let the light be focused upon a screen *CD* by means of a convex lens. Waves that proceed from *A* and *B* in a direction normal to the grating will meet at *E*, after having traveled equal distances. Hence, the crest of a wave arriving from *A* will meet the crest of one arriving from *B*, and the two will reinforce each other—there will be **constructive interference.** At a certain point *H* above *E*, and at another *H′* the same distance below *E*, the wave from one of the apertures will have traveled farther than the one from the other by a distance equal to half the wave length; there a crest from *A* will meet a trough from *B*, and the waves will neutralize each other—there will be **destructive interference.** At *K* and *K′* the difference of path, *BX*, is one whole wave-length, constructive interference again occurs,

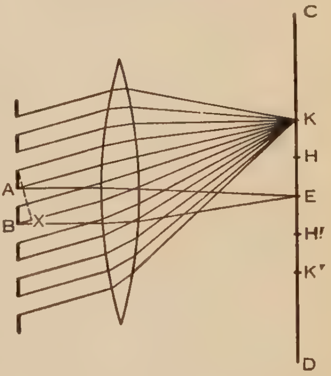


FIG. 101. ACTION OF THE DIFFRACTION GRATING

and a bright point like that at *E* is produced; and there will be a succession of these bright points where the difference of path from adjacent apertures is 0, λ, 2λ, 3λ, . . . , where λ is the wave-length of the light. If light of a different wave-length be used to illuminate the grating, its bright points fall between those that we have established, and so light of different wave-lengths is sorted out and a spectrum is formed. The greater the wave-length *BX*, the greater will be the distance *EK*, and from the distance *EK* and the distance of the screen from the grating it is easy to calculate the wave-length.

The grating thus sorts out the light according to wave-length, producing a series of bands, in each of which the light of greatest wave-length is deviated the most. What we actually see is a series of spectra, in which the red is deviated most; hence, color must be a matter of wave-length, and red waves must be the longest of those that impress the eye. It is customary to express the wave-length of light, which is always very small, in terms of what is called the **Angstrom unit**, which is one ten-millionth of a millimeter (10^{-8} cm.). Measurement with the diffraction grating shows the wave-length of the reddest visible light to be about 7,800 Angstrom units, while that of extreme violet light is about half as great.

It may be noted that in grating spectra, the distance between any two lines is proportional to their difference of wave-length. This is not true of spectra produced by prisms; there the dispersion is greater in the blue end than in the red.

Comparison of Light and Sound.—It is well known that sound consists of waves in the air and that the length of the waves determines the pitch of the sound, a high note being due to a short wave-length. The wave-length of middle C of the scale is about four feet, enormously greater than that of visible light. Ascending the scale one octave divides the wave-length by two, ascending two octaves divides it by four, and so on. Since the wave-length of violet light is about half that of red, it is evident that the eye is sensitive to but a single octave of light, while the ear detects sound over a range of some ten octaves.

The velocity of sound is about 1,090 *feet* per second, while that of light is, as we have seen, about 186,000 *miles* per second. The frequency of middle C is 256 waves per second. That of yellow light, having the wave-length of the D line of sodium, is about 500,000,000,000,000 (5×10^{14}) per second.

Sound-waves are longitudinal—that is, the motion of the particles of air is backward and forward along the line of advance of the wave. Light-waves are transverse. In ordinary light the wave-motion, or vibration, while always at right angles to the direction of propagation, takes place in all possible azimuths; but by certain means, as by passing the light through a Nicol prism made of a crystal of Iceland spar, the light may be **plane polarized,** when the vibrations all take place in the same plane.

The ear is provided with a special organ, called the organ of Corti, by which we recognize sounds of different pitch, even when they are produced simultaneously, as by an orchestra or a choir; but the eye has no such mechanism, and its place may be said to be taken by the spectroscope.

Light certainly does not consist, as sound does, of waves in the *air*; for it is transmitted freely through interstellar space and through artificial vacua. This fact has led physicists to postulate the existence of a medium called the **luminiferous ether,** a highly elastic, perfectly incompressible substance of great density which pervades all space and yet which is so imponderable that it offers no resistance to the motion of such material bodies as the planets and the stars. This is about equivalent to saying that empty space has properties other than those attributed to it in ordinary geometry, which enable it to transmit energy-carrying waves, the passage of which requires finite time.

Invisible Radiation.—Although the eye is not sensitive to light of wave-length shorter than about 3900 Angstroms, a photographic plate placed beyond the violet of the spectrum shows the existence of much shorter waves which are called **ultra-violet.** Light from the heavenly bodies and from terrestrial sources may be photographed down to about 2900 Angstroms, but the Earth's atmosphere is almost perfectly opaque to waves shorter than this, so that the extreme ultraviolet is of no consequence astronomically. Using reflection gratings inclosed with their sources of light in vacua, Millikan has detected lines in the spectra of certain elements due to waves as short as 200 Angstroms.

The wave-length of **X-rays** is of the order of one Angstrom. The finest artificial gratings are much too coarse to use in the study of X-ray spectra, and recourse is had to crystals, in which the natural orderly arrangement of molecules replaces that of the ruled lines of the artificial grating. The **γ-rays** emitted by radio-active substances are of smaller wave-length than X-rays, and the highly penetrating **cosmic rays** (page 365) recently investigated by Millikan, Kohlhörster and others are of smaller wave-length still—of the order of 0.001 Angstrom.

A delicate thermometer, or, better, a thermopile, shows that beyond the red of the visible spectrum lies a great range of wave-lengths belonging to what is called **infra-red** light. There is, in fact, no break in the series between these waves and the long waves produced at radio broadcasting stations, which are in some cases several miles long. The longest known ether wave is thus some 100,000,000,000,000,000 times as long as the shortest, a range of about sixty octaves, to only one octave of which the human eye is sensitive. Ultra-violet, visible, and infra-red waves are all included in the physicist's term **radiation.**

The Laws of Radiation.—All bodies, unless at the "absolute zero" of temperature (representing the complete absence of heat and assigned by physicists at −273° Centigrade) emit some radiation, although it may not be perceptible to the eye. If the body is "black"—that is, capable of completely absorbing radiation of all wave-lengths[1]—its spectrum is continuous and the relations of its temperature to the quantity and quality of its radiation are expressed by certain laws which have been well established by the experimental and theoretical researches of physicists. Although the stars probably do not radiate exactly like black bodies, these laws afford valuable means of investigating their temperature. The **effective temperature** of a star is defined as the temperature at which a black body of equal area would emit the same amount and kind of radiation.

The **Stefan-Boltzmann law** states that *the total rate of radiation of a black body (all wave-lengths combined) varies as the fourth power of the absolute temperature;* that is,

$$E = \sigma T^4$$

where E is the rate of emission of energy, T the absolute temperature, and σ a constant.

This law is by no means self-evident; for example, a doubling of the temperature might reasonably be expected merely to double the rate of emission of radiation, but in fact it would increase the rate of emission sixteenfold.

The intensity of the radiation emitted by a body is not the same at all wave-lengths, but is a maximum at a certain point in the spectrum whose position depends upon the temperature of the body. For cool bodies this point of maximum intensity of the continuous spectrum lies far in the infra-red, but with rising temperature it shifts to smaller wave-lengths and enters the visible portion when the temperature reaches about 4000° C. An obvious inference is that blue-white stars like Rigel are hotter than red stars like Antares. The precise relation is simply stated by **Wien's law:** *The wave-length of the point of maximum intensity in the continuous spectrum of a black body is inversely proportional to the body's absolute temperature;* or,

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[1] This technical use of the word black does not agree with the ordinary use unless the body is so cool as not to emit visible light. If heated to incandescence, the technically "black" body would appear white-hot.

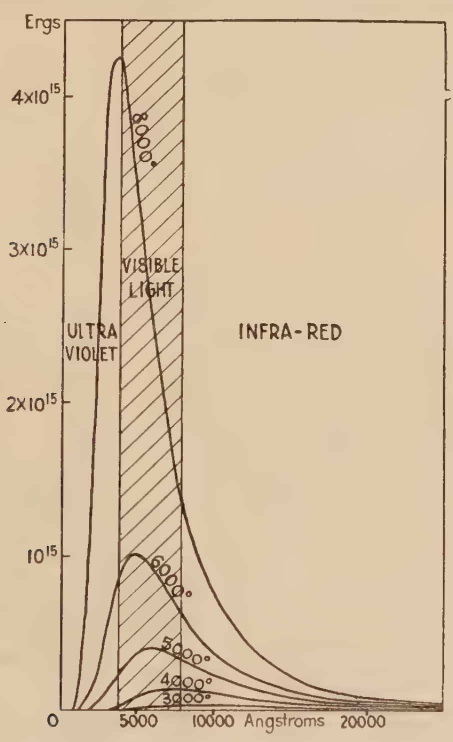


FIG. 102. DISTRIBUTION OF ENERGY IN THE SPECTRUM OF A BLACK BODY

$$\lambda_{max} = \frac{C}{T}.$$

Planck's Equation, which states the rate of emission, E_λ, of radiation of any particular wave-length λ, is more complicated:

$$E_\lambda = \frac{c_1{}^{-5}\ \lambda}{e^{\frac{c_2}{\lambda T}} - 1},$$

where e is the base of natural logarithms. Both the Stefan-Boltzmann law and the Wien law may be derived from Planck's equation.

If the rate of radiation is expressed in ergs per square centimeter per second, the wave-lengths in centimeters, and the temperatures in Centigrade degrees, the constants of the above laws have the following values:

$$\sigma = 5.72 \times 10^{-5}$$
$$C = 0.289$$
$$c_1 = 3.71 \times 10^{-5}$$
$$c_2 = 1.435$$

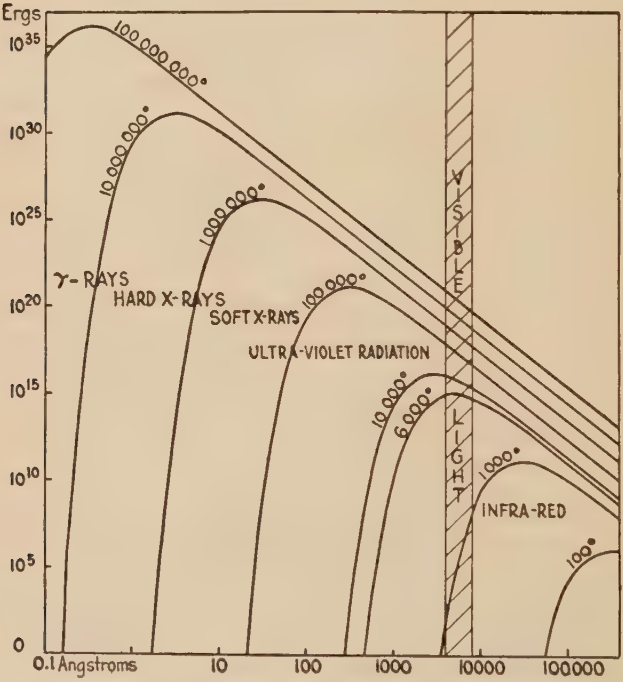


Fig. 103. Energy Curves of a Black Body

The laws of radiation may be better understood from a study of the curves in Figs. 102 and 103, which were drawn from data calculated by Planck's formula. In Fig. 102, the rate of emission of energy in ergs per square centimeter per second (ordinates) is plotted against wave-length in Angstroms (abscissæ) for values of the absolute temperature ranging from 3000° to 8000°. The ordinate of any point is Planck's function E^{λ}; the abscissa corresponding to the highest point of the curve is Wien's λ_{max}; and the area under the curve is the total emission E as given by Stefan's law. The shift toward violet of the top of the curve and the rapid increase of area which accompany increasing temperature illustrate Wien's law and the Stefan-Boltzmann law respectively.

The height of the curve increases so fast with increasing temperature that it is not practicable to plot curves in the usual way for the great range of temperatures that is believed to exist among the stars. In Fig. 103 use is made of a logarithmic scale, both the ordinates and abscissæ increasing in a geometric instead of an arithmetic progression, and it is thus possible to represent both large and small numbers in the same drawing. The curves show that, at either very high or very low temperatures, only a small fraction of the radiation of a star is sensible to the eye; and that at temperatures of millions of degrees, which are believed to exist in the interiors of stars, while the radiation of all wave-lengths is enormous the extremely short waves (X-rays) vastly predominate. However, such short waves are unable to penetrate any great thickness of the star's atmosphere, and so are imprisoned within the interior, to be transformed slowly into longer waves which as slowly filter out (page 359).

The Pressure of Radiation.—It was shown theoretically by Maxwell in 1873, and experimentally by Nichols and Hull and by Lebedew in 1900, that radiation exerts a pressure upon any surface on which it falls. This pressure is so feeble that it is very difficult to detect by experiment, but it is important in the study of the motion of fine particles near the Sun or the stars, such as occur in comets and in stellar atmospheres.

The radiation pressure upon a body is proportional to the body's area or (for a sphere) to the square of its radius. The Sun's gravitational attraction for the body is proportional to its mass or, for a given density, to the *cube* of its radius. Both are inversely proportional to the square of its distance from the Sun. Therefore, whatever the distance, the ratio of radiation pressure to gravitational attraction is inversely proportional to the first power of the radius; hence, there must be a certain critical diameter for a body of given density, for which the light-pressure equals the attraction. Bodies having a diameter less than this critical value (which, for unit density, is about 0.0015 mm.) will be repelled instead of attracted by the Sun. The pressure of radiation affords an adequate explanation of the behavior of the tails of comets and is of importance in considerations of the equilibrium of gases in the solar chromosphere and in the hotter stars.

Series of Lines in Spectra.—In the spectra of many elements, the lines are so placed as to form a series, in which the wave-lengths of the different lines fall into a simple formula. The most obvious of these series is that of hydrogen (Plates 11 and 13). A wide gap intervenes between the red line, Hα, and the green one, Hβ; a shorter gap between Hβ and the blue line, Hγ; and as we proceed into the ultra-violet the gaps become shorter and shorter until the lines are found to crowd closely together at about λ3646. The Swiss physicist Balmer showed in 1885 that the wave-lengths of the hydrogen lines could be expressed by the formula

$$\lambda = 3646 \frac{n^2}{n^2 - 4}$$

by giving to n the successive values 3, 4, 5, etc. Thus, if $n = 3$, we get $\lambda = 6563$, the wave-length of Hα; if $n = 4$, $\lambda = 4861$, the wave-length of Hβ; and so on. The hydrogen spectrum contains another series of lines in the far ultra-violet, and still another in the infra-red. Many other elements besides hydrogen exhibit series of spectral lines, but the hydrogen series is the simplest.

The Structure of the Atom and the Production of Light.—The investigations of many physicists, chief among whom is Bohr of Denmark, have shown that many phenomena, including those of spectral series, are best explained by supposing that the atoms of matter are miniature solar systems, the Sun being represented by the **nucleus** of the atom which consists of a number of indivisible units of positive electricity called **protons** sometimes combined with a smaller number of negative units called **electrons,** while other electrons revolve around the nucleus like planets around the Sun. Although the quantity of electricity in the electron is exactly equal to that in the proton, the mass of the proton is some 1,800 times that of the electron, and so the mass of an atom is concentrated principally in its nucleus. The diameter of the proton is estimated to be less than 10^{-13} cm., while that of the average electronic orbit is about 10^{-8} cm.—a hundred thousand times as large as the proton and of the order of the wave-length of X-rays. The atom is thus conceived to be mostly empty space.

The simplest atom, according to the Bohr theory, is the atom of hydrogen, and it consists of a single proton and a single revolving electron. The electron may revolve in any one of many orbits, at distances from the proton that are proportional to the squares of the natural numbers—that is, to 1, 4, 9, 16, etc. The orbits are usually elliptic, but may be circular, and the four inner orbits are so represented in Fig. 104. So long as the electron continues to revolve in any one of these orbits the atom neither emits nor absorbs light; but, contrary to the state of things in the Solar System, it is capable of jumping suddenly from one orbit to another. If it falls from an outer orbit to an inner one, the atom loses energy by a definite amount, which is called a **quantum,** and an ether disturbance of a definite wave-length is sent out. The length of this wave depends upon the distance fallen through by the electron; if it falls from the third to the second orbit, Hα-light is emitted; if from the fourth to the second, Hβ-light; if from the fifth to the second, Hγ-light; and so on. If the electron falls from any outer orbit to the first, a line of the ultra-violet series is emitted; if to the third, a line of the infra-red series.

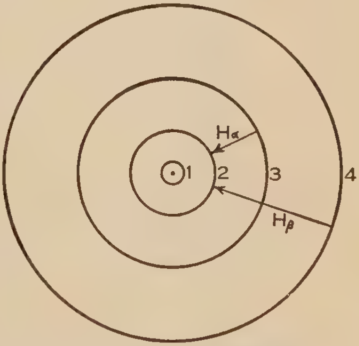


Fig. 104. Orbits of the Hydrogen Electron

The quantum of energy is proportional inversely to the wave-length and therefore directly to the frequency of the light-waves, and is equal in every case to $h\nu$ where ν is the frequency and h is a constant known as Planck's constant.

When energy is imparted to the atom, as by the absorption of radiation passing through the body of gas, an electron is expelled from an inner orbit to an outer. The atom is then said to be **excited.**

According to the Bohr theory, the atom of any chemical element in its normal state contains just as many electrons as protons; all the protons are in the nucleus, and if the latter contains electrons also, as in many elements it does, the protons must exceed the nuclear electrons by a number equal to that of the revolving electrons. The chemical elements differ from one another by the number of excess protons in the nucleus, and this is known as the **atomic number.** The helium nucleus is believed to consist of four protons and two electrons, while in the normal helium atom two additional electrons revolve in orbits that are mutually inclined at an angle of

TABLE 4

THE CHEMICAL ELEMENTS

Atomic Number	Name	Symbol[1]	Atomic Number	Name	Symbol[1]
1	Hydrogen	H	47	Silver	Ag
2	Helium	He	48	Cadmium	Cd
3	Lithium	Li	49	Indium	In
4	Beryllium	Be	50	Tin	Sn
5	Boron	B	51	Antimony	Sb
6	Carbon	C	52	Tellurium	Te
7	Nitrogen	N	53	Iodine	I
8	Oxygen	O	54	Xenon	Xe
9	Fluorine	F	55	Cæsium	Cs
10	Neon	Ne	56	Barium	Ba
11	Sodium	Na	57	Lanthanum	La
12	Magnesium	Mg	58	Cerium	Ce
13	Aluminum	Al	59	Praseodymium	Pr
14	Silicon	Si	60	Neodymium	Nd
15	Phosphorus	P	61	Illinium	Il
16	Sulphur	S	62	Samarium	Sm
17	Chlorine	Cl	63	Europium	Eu
18	Argon	A	64	Gadolinium	Gd
19	Potassium	K	65	Terbium	Tb
20	Calcium	Ca	66	Dysprosium	Dy
21	Scandium	Sc	67	Holmium	Ho
22	Titanium	Ti	68	Erbium	Er
23	Vanadium	V	69	Thulium	Tm
24	Chromium	Cr	70	Ytterbium	Yb
25	Manganese	Mn	71	Lutecium	Lu
26	Iron	Fe	72	Hafnium	Hf
27	Cobalt	Co	73	Tantalum	Ta
28	Nickel	Ni	74	Tungsten	W
29	Copper	Cu	75	Rhenium	Re
30	Zinc	Zn	76	Osmium	Os
31	Gallium	Ga	77	Iridium	Ir
32	Germanium	Ge	78	Platinum	Pt
33	Arsenic	As	79	Gold	Au
34	Selenium	Se	80	Mercury	Hg
35	Bromine	Br	81	Thallium	Tl
36	Krypton	Kr	82	Lead	Pb
37	Rubidium	Rb	83	Bismuth	Bi
38	Strontium	Sr	84	Polonium	Po
39	Yttrium	Y	85	———	——
40	Zirconium	Zr	86	Radon	Rn
41	Columbium	Cb	87	———	——
42	Molybdenum	Mo	88	Radium	Ra
43	Masurium	Ma	89	Actinium	Ac
44	Ruthenium	Ru	90	Thorium	Th
45	Rhodium	Rh	91	Protoactinium	Pa
46	Palladium	Pd	92	Uranium	U

[1] Derived from the Latin form of the name.

120°. The atomic number of helium is thus 4 − 2 = 2. The atomic number of lithium is three, that of sodium is eleven, that of iron is twenty-six, and that of uranium, the most complicated atom, is ninety-two. The names, symbols, and atomic numbers of all the known chemical elements are given in Table 4.

When an atom is acted upon by a powerful electric current or a very high temperature, one or more of the revolving electrons may be not only expelled to an outer orbit, but may be entirely lost to the atom, which is then left with a superfluous positive charge and is said to be **ionized.** Ionization is promoted by a decrease in the pressure of the gas. The spectrum of an ionized atom is entirely different from that of a non-ionized atom, and so, as a mass of gas is subjected to higher and higher temperatures and more and more atoms become ionized, the spectrum is modified by the emergence or strengthening of certain lines which are referred to as **enhanced lines.** The temperatures and pressures at which different elements become ionized are very different, and the relative intensity of the enhanced and ordinary lines of the various elements in the spectra of the stars gives a key to their temperatures, a fact first pointed out by Saha of Calcutta in 1921.

The ionized helium atom, like the hydrogen atom, has but one revolving electron, but its nucleus, as we have seen, is very different from the hydrogen nucleus. It might be expected that the spectrum of ionized helium would be similar to, but not identical with, that of hydrogen. It does, in fact, contain a series of lines very similar to the Balmer hydrogen series, which was first observed by E. C. Pickering of Harvard in 1896 in the spectrum of the star ζ Puppis and was long attributed to hydrogen and known as the Pickering series.

The Doppler-Fizeau Principle.—An observer who stands near a railroad track while a locomotive with a sounding bell passes may notice that the pitch of the sound drops suddenly at the moment when the engine is nearest. The reason of this may be seen from Fig. 105. Let the engine move along the straight line from right to left, and let the positions of the bell at the moments of emitting successive wave-crests be *A*, *B*, . . . *F*. The wave emitted at *A* spreads in the air in all directions with the velocity of 1,090 feet a second, and when the engine has arrived at *F* this wave is represented by the largest circle, which is centered at *A*. The crest emitted at *B* spreads in the next circle, which is centered at *B*, and so on. Hence, when the bell has arrived at *F*, the successive waves have the positions shown by the five circles in the figure, and are crowded together in front of the bell and drawn apart behind it. That is, the sound received by an observer in front of the locomotive must be of shorter wave-length, and hence of higher frequency and pitch than the sound received in the rear. A similar effect may be noticed by passengers on a rapidly moving train in the pitch of the sound of warning bells at grade crossings. Here the bell is stationary and the observer moves. The wave-length is not modified, but the frequency with which the waves are received by the observer is greater as he approaches the bell than as he recedes, and the effect on the pitch is the same as if the wave-length were changed.

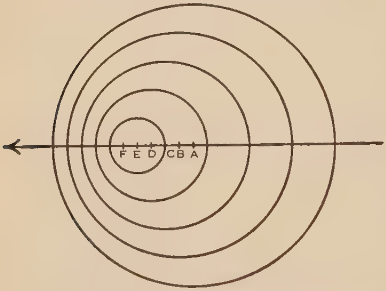


Fig. 105. The Doppler-Fizeau Principle

The effect of motion on the pitch of sound was first explained in 1842 by Doppler of Prague, who realized that a similar effect would take place in waves of light, and inferred that a rapidly approaching star would be blue and a receding one red. This inference, however, is incorrect, for the star differs from the bell in emitting waves having a great range in length and the effect of recession is to lengthen all the waves, so that the whole spectrum is shifted, a little of the visible portion being lost in the infra-red and a little of the ultra-violet being shifted over into the visible. Moreover, the change of wave-length due to any velocity as yet detected in the stars would not be sufficient to make a change in color that could be noticed by the eye.

The correct astronomical application of Doppler's principle was given by Fizeau in a paper read before a learned society in Paris in 1848 but not published until 1870. Although all wave-lengths are represented in the spectrum of a star, each *line* of the spectrum represents but a single wave-length, and so the effect of motion along the line of sight is to modify the wave-length of each line, and so to displace the line in the spectrum. The Doppler-Fizeau principle, which is one of incalculable importance in astronomy, may be stated as follows: *When the distance between an observer and a source of light is increasing, the lines of the spectrum lie farther to the red than their normal positions, and when the distance is diminishing they lie farther to the violet, the displacement being proportional to the relative velocity of recession or approach.* The formula for the change of wave-length of a spectral line is

$$\Delta \lambda = \frac{v}{V} \lambda,$$

where v is the relative velocity of the source and the observer along the line of sight (called the **radial velocity**; page 89), V is the velocity of light, and λ is the wave-length of the line. The highest velocity of any celestial object so far measured is only a small fraction of the velocity of light, and so the observed change of wave-length is small. The Doppler-Fizeau effect in the spectra of different parts of the Sun is illustrated in Plate 13. The Sun's rotation causes its west limb to move away from us and its east limb toward us, the relative velocity being about four kilometers per second. The lines of the spectrum of a point near the west limb are accordingly shifted to the red and those of a point near the east limb toward the violet, while the spectra of points near the north and south poles of the Sun are unmodified by its motion.

EXERCISES

1. Find, by Balmer's formula, the wave-lengths of $H\gamma$, $H\delta$, $H\epsilon$, and $H\zeta$.
Ans. 4340, 4102, 3970, 3889

2. What is the wave-length of the brightest point in the spectrum of a black body whose temperature is 11,000°K? What color would you expect a star of this effective temperature to have (*e.g.*, Vega)?

3. Find the change of wave-length of $H\beta$, due to the orbital motion of the Earth, in the spectrum of a star toward which the Earth is directly moving at the moment of observation (take the Earth's orbital velocity as 30 km./sec. and the velocity of light as 300,000 km./sec.).
Ans. 0.49 Angstrom

4. If a photograph of the spectrum of the star of Example 3 has a length

of 25 millimeters between Hβ and Hγ, what is the linear displacement of Hβ on the plate?

Ans. 0.023 mm.

5. What is the *wave-number* (number of waves per centimeter) of green light, the wave-length of which is 5000 Angstroms? What is the *frequency* (number of waves per second) of such light?

Ans. 20,000; 6×10^{14}

CHAPTER VIII

THE SUN

Importance of the Sun.—From three main points of view the Sun is, to us inhabitants of the Earth, the most important of astronomical bodies. First, it is the ruler of the Solar System, controlling the motions of the Earth and all the other planets, the comets, and the streams of meteoric bodies, and influencing the motions of the satellites. This aspect of the Sun will appear in succeeding chapters.

Second, the Sun is a star; not by any means the largest or brightest or hottest star, but important to us because it is by far the nearest, our next stellar neighbor being some 275,000 times as far away and the great majority of visible stars many times farther still. The Sun is the only star whose features we can study in detail.

Third, the radiation of the Sun is almost the sole source of power, warmth, activity, and life upon the surface of the Earth, the only exceptions worth mentioning being the tides and the activity of volcanoes and geysers which originates in the internal heat of the Earth.

Consider, for example, the manifestations of life on a city street, at night when it might seem that the Sun had no influence. The street is lighted and cars are being propelled upon it by the energy of electric current which is generated, perhaps, at a distant waterfall. The generators are run by turbines which are driven by the weight of falling water; but in order to fall, the water must have been raised to a higher level than that of the sea, and this was done by the radiation of the Sun which warmed the water of the ocean, causing it to evaporate and rise to form clouds, which were wafted over the land by Sun-generated winds to fall as rain. If the Sun's radiation were cut off, the cataract would cease even before the existing supply of water was exhausted, for the temperature would speedily fall so low that the water would all be converted to immobile ice. Or perhaps the current is generated by the burning of coal. The energy then comes from the combination with the oxygen of the air of carbon which was stored in plants ages ago under the mysterious action of the Sun's rays known as **photosynthesis,** and we are thus making use of "canned sunlight." The energy which drives the gasoline engines of automobiles may be traced to a similar ancient source, while that of our own bodies is solar energy, stored not so long ago by photosynthesis in the plants that form our food or the food of animals of whose flesh we have eaten.

General Description of the Sun.—The distance and diameter of the Sun have already been given (page 97). It is impressive to compare the Sun's diameter with that of the Moon's orbit for, if the Earth were placed at the Sun's center, the Moon's orbit would lie only a little more than halfway out toward the surface. The distance is so great that, *to subtend an angle of 1″*—which is pretty small to observe with an ordinary telescope in the poor seeing that usually prevails in the daytime—*a marking on the Sun must be 450 miles in diameter.*

The mass of the Sun (page 232) is about 333,000 times that of the Earth, and its surface gravity about 27.6 times the Earth's (a person who weighs one hundred pounds here would weigh nearly a ton and a half if transported to the Sun). Its form is that of a sphere. Its average density is about 1.4 times that of water. The temperature of its visible surface is about 6,000° C., while that of the interior is certainly much higher, and is estimated at 40,000,000° C.

The Sun rotates in the same direction as the Earth upon an axis which is inclined 83° to the plane of the ecliptic and is directed to a point about halfway between Polaris and Vega, in $\alpha = 18^h\ 44^m$, $\delta = +64°$. The rotation is not uniform all over the Sun, for a point at the equator turns faster than points in higher latitudes, the sidereal rotation period being 25 days at the equator, 27.5 days at latitude ± 45°, and about 33 days at latitude ± 80°. This shows, of course, that the surface of the Sun cannot be solid, for its parts move past one another; and its high temperature and other facts prove it to be gaseous throughout in spite of its high average density.

Upon the intensely brilliant visible surface of the Sun, which is called the **photosphere,** are often seen relatively dark spots called **sun spots,** some of which are many times larger than the Earth. Above the photosphere is the red **chromosphere** from which rise the vast flame-like **prominences** (page 146); while beyond all extends the tenuous **corona** which can be seen only

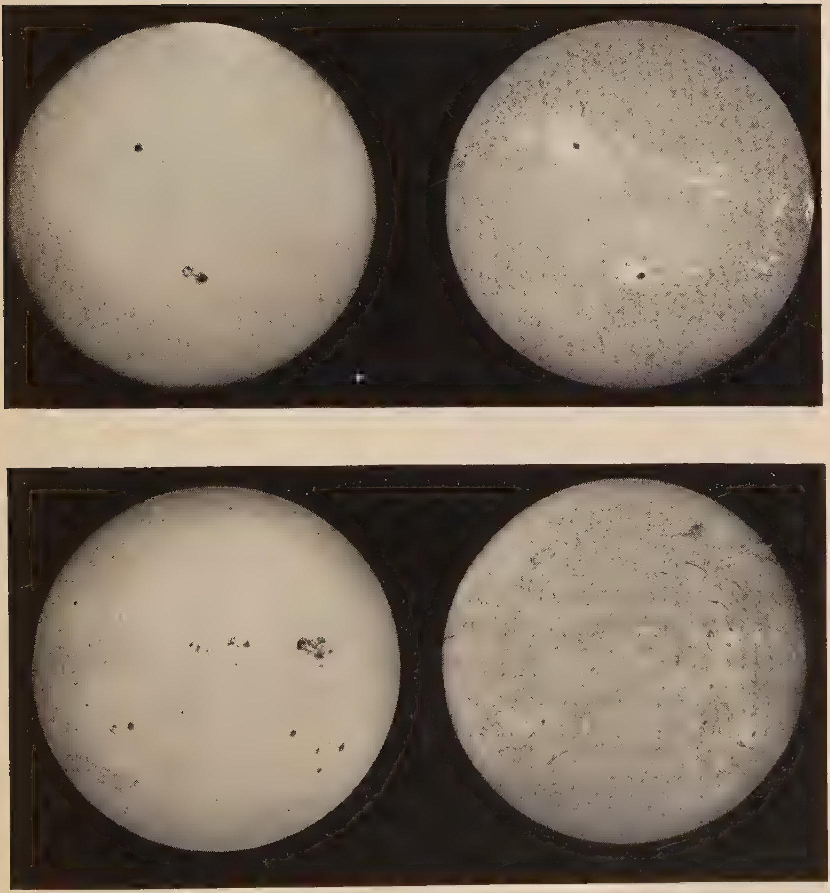
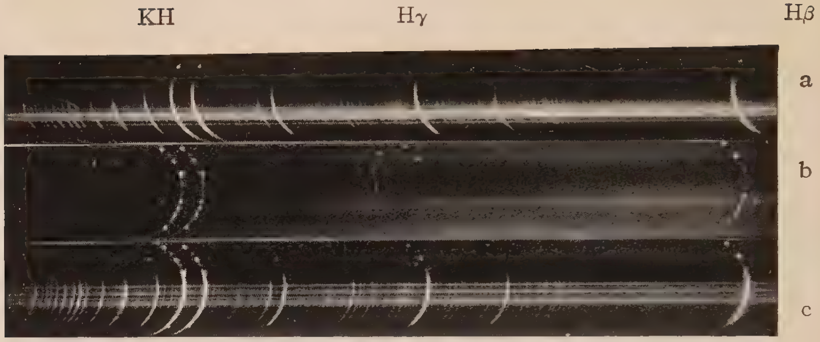


PLATE 14. DIRECT PHOTOGRAPHS OF THE SUN (LEFT) AND SPECTROHELIOGRAMS (RIGHT) MADE AT MOUNT WILSON OBSERVATORY

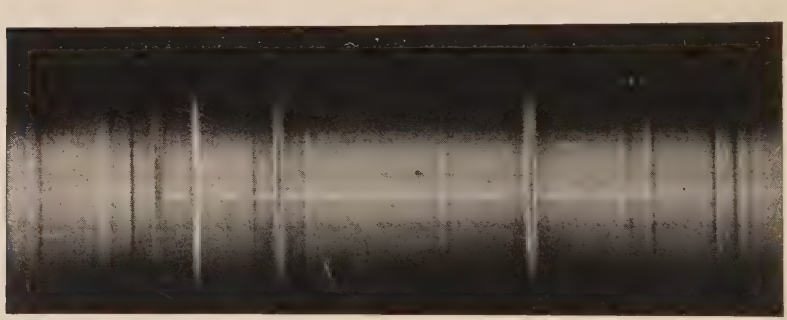
Above—1906 July 30; the Spectroheliogram was made in the light of the H line of Calcium

Below—1917 August 12; Spectroheliogram made in the light of the α line of Hydrogen. Small white dot shows comparative size of the Earth

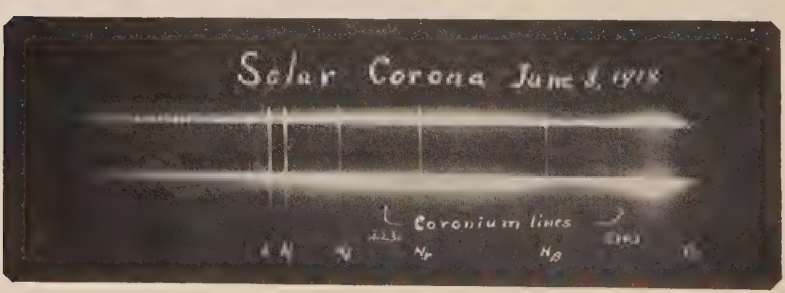
PLATE 15



Spectra photographed with an Objective Prism by Anderson at Middletown, Connecticut, at the Total Solar Eclipse of 1925 January 24. (a) beginning of totality, (b) middle of totality, (c) end of totality.



"Flash Spectrum" photographed without an eclipse with the 75-foot Spectrograph and great tower telescope at Mount Wilson. Region of Fraunhofer's *b* group.



Spectrum of the Eclipsed Sun, photographed by V. M. Slipher with slit spectrograph at Syracuse, Kansas, 1918 June 8. The light of the corona and chromosphere was diffused by a hazy sky so that the bright lines cross the image of the Moon.

at the time of a total solar eclipse. The abundance and size of the spots and prominences, the form of the corona, and the magnetism of the Earth (which is thus shown to be connected with the Sun) all vary, for some unknown reason, in an irregular period which averages 11.1 years.

As shown by the spectrum, the layer of the Sun which produces the Fraunhofer lines contains, in a state of vapor, some sixty of the chemical elements which are known upon the Earth; and there is reason to believe that, except for the effects of high temperature, the chemical constitution of the Sun is not essentially different from that of the Earth.

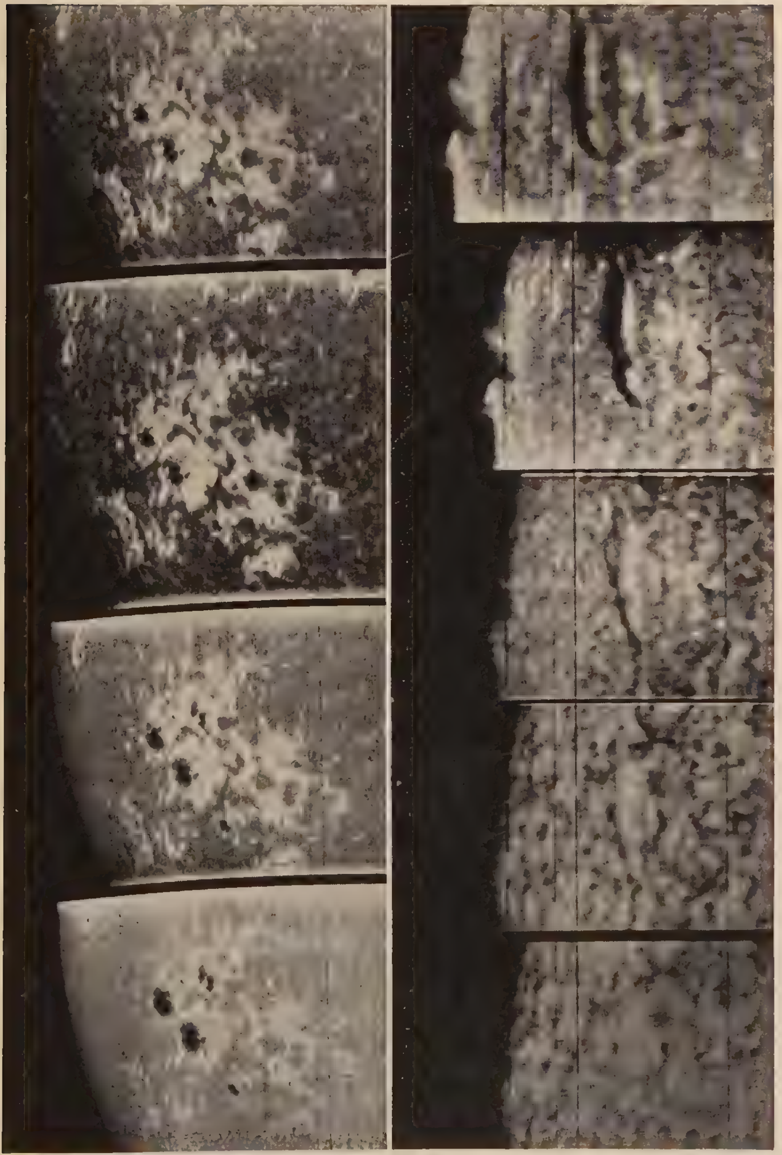
Methods of Observing the Features of the Sun.—To look directly at the Sun through a telescope would be disastrous, for the concentration of rays is so great that a piece of paper or other inflammable object placed in the usual position of the eye at a telescope directed to the Sun is quickly set afire. For visual observations, one may make use of various devices known as helioscopes, which reflect away the greater part of the light after it has entered the telescope. (To reduce the aperture of the objective by a perforated cap would decrease the light but would also diminish the resolving power.) Another method is to project upon a white cardboard screen an enlarged image of the Sun formed by racking the eyepiece outward until the screen and the focal plane of the objective are at the conjugate foci (page 35) of the ocular. The image may then be seen by a number of observers simultaneously.

For photographic observation, which for serious work has now almost entirely superseded the visual, the intensity of the light is an advantage rather than a drawback, for it makes possible the use of slow plates, which are of finer grain than fast plates, and of very short exposures—one one-thousandth of a second or less. The photograph is usually made in the focal plane of a long-focus objective, the telescope often being mounted permanently in a horizontal or a vertical position and "fed" by a **coelostat** which reflects the light in a constant direction.

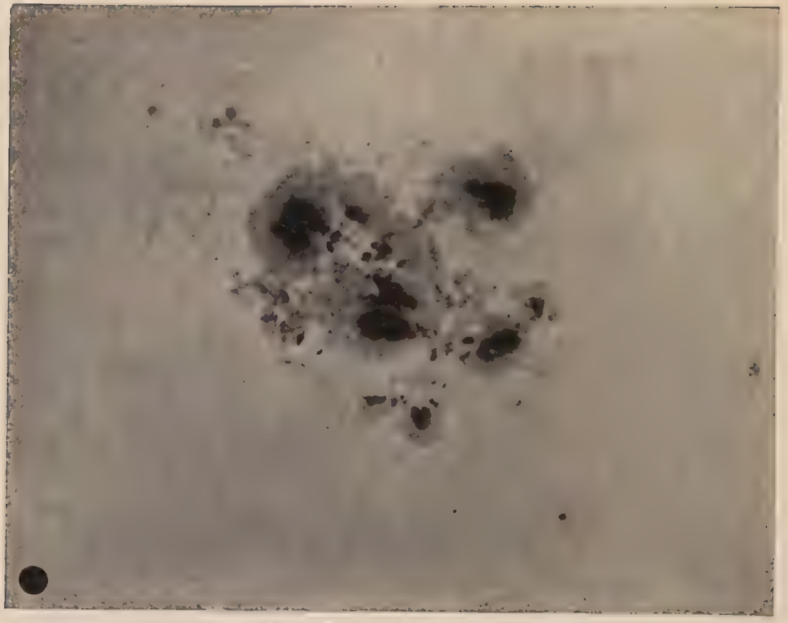
It was discovered in 1868 by Lockyer in England and Janssen in France that the solar prominences, which up until then had been seen only during eclipses, could be observed in full sunlight by the aid of the spectroscope. Prominences cannot be seen directly without an eclipse for the same reason that stars cannot be seen in daylight—the background of sky is too bright. The effect of the spectroscope is to spread the light of the sky (which is reflected sunlight) into a long spectrum and so to reduce its intensity; but the spectrum of the prominences is a bright-line one, and the spectroscope separates the bright lines without widening them and hence without weakening them, so that they may be seen against the weakened background of the sky. To observe the prominences, therefore, a spectroscope of high dispersion—having a fine grating or a train of prisms—is attached to the tube of a telescope with its slit in the focal plane of the objective. The telescope is so directed that the slit is at one point nearly tangent to the image of the Sun, but does not quite touch it; if a prominence is situated at the corresponding point of the Sun's limb, the bright lines of its spectrum may be seen against the continuous sky-spectrum. Since an image of the prominence is formed on the slit-plate by the objective, the prominence-light entering the spectroscope comes from a narrow strip only of the prominence, and the bright lines of the spectrum are themselves images of the telescopic image of this strip, showing interruptions, for example, corresponding to any rifts that exist in that strip of the prominence. By widening the slit, a wider strip may be seen, and it is often possible thus to view the whole prominence if the sky be very clear. Widening the slit, however, admits more sky light without brightening the image of the prominence, and a slight haze, which has the effect of both dimming the prominence and brightening the sky, will make the observation of a whole prominence of any size impossible. The spectral line most frequently used for this kind of observation is the red (C or $H\alpha$) line of hydrogen, because it is the brightest in the prominence spectrum.

About 1890, Hale in America and Deslandres in France invented the **spectroheliograph,** by means of which the entire Sun is photographed in the light of a single spectral line. This instrument consists of a spectrograph of high dispersion which,

PLATE 16. SPECTROHELIOGRAMS TAKEN AT DIFFERENT LEVELS



Left, Calcium H line, Yerkes Observatory, 1919 August 22
Right, Hydrogen α line, Mount Wilson Observatory, 1916 May 29. Highest levels are at the top of the page



Great Spot Group of 1917 August 8.

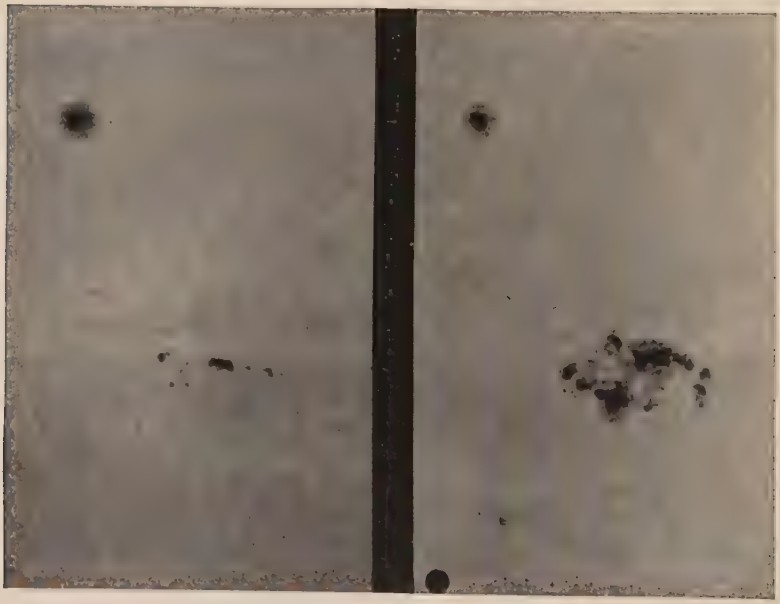


PLATE 17. DIRECT PHOTOGRAPHS OF SUN SPOTS MADE AT MOUNT WILSON OBSERVATORY

Twenty-four-hour development of Sun Spot of 1917 August 18–19. Round black dots show comparative size of Earth.

in addition to the slit through which the light enters (and which we shall here call the first slit) is provided with a second slit placed in the focal plane of the camera objective and adjusted to the width of a line of the spectrum. The spectroheliograph is attached to a large telescope, the objective of which we shall call the main objective. The first slit is placed in the focal plane of the main objective, which is directed to the Sun. A narrow strip of the Sun's image is thus admitted by the first slit and a spectrum formed, each line of which is an image of that narrow strip. The second slit is so placed that the light[1] of a single line passes through it, and behind the second slit is placed the photographic plate. Upon the plate there is thus formed a monochromatic image of the strip of the Sun's surface showing the distribution of the particular element, say hydrogen, which is responsible for the spectral line. A complete picture of the Sun, or rather of that portion of a certain layer which is composed of a single element, is then built up on the plate in one of two ways: either the main telescope is moved slowly so that the Sun's image travels at right angles to the first slit while at the same time the photographic plate is moved across the second slit, causing successive strips of the Sun to fall upon successive strips of the plate; or the plate and the main telescope are both kept stationary while the spectroheliograph with its two slits is moved slowly between them.

In 1925 Hale perfected the **spectrohelioscope,** which makes perceptible to the eye the solar features which before had been detected only by the spectroheliograph. It is arranged precisely like the spectroheliograph except that the photographic plate is replaced by an eyepiece and the two slits are given a vibratory motion so rapid that the eye receives the impression of a persistent image. The range of the vibration is sufficient to admit a field of view covering a considerable portion of the Sun's surface.

The Sun's Appearance.—As observed visually or on direct photographs, the Sun presents a clear-cut, circular disk which is brighter at the center than at the limb, and which has a granular surface, the "granules" being some hundreds of miles in diameter. Photographs made in rapid succession have shown that the granules are short-lived, the majority lasting but about half a minute. They perhaps represent something similar to the crests of waves on a storm-tossed sea. The darkening at the limb, which is due to general absorption of light by the overlying layers of gas, is more pronounced on photographs than in visual observation, the blue and violet light being absorbed more strongly than the light to which the eye is sensitive. Large, irregular, bright areas called **faculæ** may usually be seen in various positions, but especially near the limb, where they appear the more plainly against the less brilliant background. Their spectra show enhanced lines, indicating higher temperature or lower pressure, or both, than in the surrounding photosphere, and it is probable that the faculæ are somewhat elevated above their surroundings. The sun spots, which, when present, are the most prominent feature of all, will be discussed in later sections.

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[1] It should be recalled that the Fraunhofer lines are *dark* only by contrast with the continuous spectrum which lies between them, and are not really without light.

On spectroheliograms, the granules are not seen, but the whole surface of the Sun appears covered with a multitude of light and dark markings to which Hale has given the name of **flocculi.** Many of these directly overlie faculæ and are similar to them in form. Spectroheliograms are made usually in one of the strong lines (H or K) of ionized calcium or in the $H\gamma$ or $H\alpha$ line of hydrogen (the $H\alpha$ line, being in the red, requires the use of specially sensitized plates). The calcium flocculi are bright over extensive areas, especially in the neighborhood of sun spots. The hydrogen flocculi, especially those which appear on $H\alpha$ spectroheliograms, are usually more clearly defined than the calcium flocculi, and the largest ones are dark (Plate 14).

Although the flocculi have somewhat the appearance of clouds, they are not at all like the clouds in the Earth's atmosphere, but are composed of highly heated gases which absorb light of certain wave-lengths only, and are transparent to much the greater part of the Sun's light, so that they are invisible when the Sun is observed directly. The great dark hydrogen flocculi are in many cases prominences projected upon the photosphere, and may be seen as prominences when they have been carried by the Sun's rotation to the limb.

The Flash Spectrum; Layers of Different Height.—Since, according to the third principle of spectral analysis, the Fraunhofer lines are due to the absorption of certain wave-lengths from the light of the photosphere, it is possible, during a few seconds at the beginning or end of a total solar eclipse, to observe these lines as bright or emission lines; for at that time the bright photosphere is hidden by the Moon while the chromosphere, which produces the lines, is still exposed. This phenomenon was looked for and discovered at the eclipse of 1870 by Young of Dartmouth, who named it the **flash spectrum.** It is vividly described in the discoverer's own words[1] as follows:

> . . . At the moment when the advancing Moon has just covered the Sun's disk, the solar atmosphere of course projects somewhat at the point where the last ray of sunlight has disappeared. If the spectroscope be then adjusted with its slit tangent to the Sun's image at the point of contact, a most beautiful phenomenon is seen. As the Moon advances, making narrower and narrower the remaining sickle of the solar disk, the dark lines of the spectrum remain for the most part sensibly unchanged, though becoming somewhat more intense. A few, however, begin to fade out, and some even turn palely bright a minute or two before the totality begins. But the moment the Sun is hidden, through the whole length of the spectrum, in the red, the green, the violet, the bright lines flash out by hundreds and thousands, almost startlingly, as suddenly as stars from a bursting rocket-head.

A slender source of light, such as the crescent-shaped layer of gas here studied, may itself serve as a slit, and the slit and collimator of the spectroscope may be dispensed with. The flash spectrum may be thus seen or photographed by simply using a prism placed over the objective of a telescope. Such a combination is called an **objective prism** or sometimes, if used photographically, a **prismatic camera.** The lines of a spectrum thus produced are of course curved like their sickle-shaped source. The bright lines of the chromospheric spectrum have been photographed by Adams at Mount Wilson without an eclipse by means of the 150-foot tower telescope (page 156), the scale of the image of the Sun being so great as to permit the slit of the great spectrograph to be placed within a very short angular distance of the limb (Plate 15).

The bright lines of the flash spectrum have their sources at different levels of the chromosphere, and so do not all flash out simultaneously; and by a study of their duration or of the length of their arcs on an objective-prism spectrogram, it is possible to acquire information concerning the heights at which they are formed. Mitchell of Virginia finds in this way that ionized calcium (producing H and K) extends 14,000 kilometers above the Sun's limb; hydrogen produces $H\alpha$ at 10,000 km. and the other Balmer lines at about 8,000 km.; helium (which was so named because it was discovered in the chromosphere long before it was known to exist on the Earth) at about 7,500 km.; non-ionized calcium (spectral line at 4227 Angstroms) at 5,000 km.; and other elements, chiefly metallic, at lower levels. The region below 600 km., in which the majority of Fraunhofer lines have their origin, is often referred to as the **reversing layer,** but there is no definite limit between it and the chromosphere above it.

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[1] Young, *The Sun*, p. 82.

Double Reversal.—Young found in 1880 that in the spectrum of the chromosphere observed without an eclipse certain lines, notably those of hydrogen, helium, calcium (H and K), sodium and magnesium, occasionally showed an appearance which he called **double reversal.** The broad dark line of the solar spectrum has superposed upon it a bright line, and this, in turn, a fine dark line through its center; the three parts probably representing successive levels of vapor. At the base of prominences and over bright flocculi, the H and K lines of calcium and the more prominent lines of hydrogen are always thus doubly reversed. Hale has denoted by subscripts the successive parts of the doubly reversed line; thus, K_1 is the broad, dark K line of the Fraunhofer spectrum; K_2 is the double bright part in its center; and K_3 is the fine dark line separating the components of K_2. K_1 is due to the absorption of the Sun's white light by the dense calcium lying at the lowest levels, K_2 to incandescent calcium above this, and K_3 to cooler, rarer calcium vapor at still higher levels. It is K_2 which is most frequently used for photographing the bright calcium flocculi with the spectroheliograph. With the use of spectrographs of higher dispersion, double reversal appears in many other of the Fraunhofer lines. By placing the second slit of the spectroheliograph in different portions of the line, photographs are made which show the distribution of flocculi at different levels (see Plate 16).

Pressures in the Solar Envelopes.—Recent investigations by St. John and by Russell, into which we cannot enter here, lead to the conclusion that the pressure of the gases of the chromosphere, down to about 200 kilometers above the photosphere, is very low, about one ten-millionth that of the Earth's atmosphere at sea level; and that in the reversing layer the pressure rises rapidly and may be as great as one one-hundredth of an "atmosphere" (terrestrial) at the photosphere, at which pressure and at the temperatures

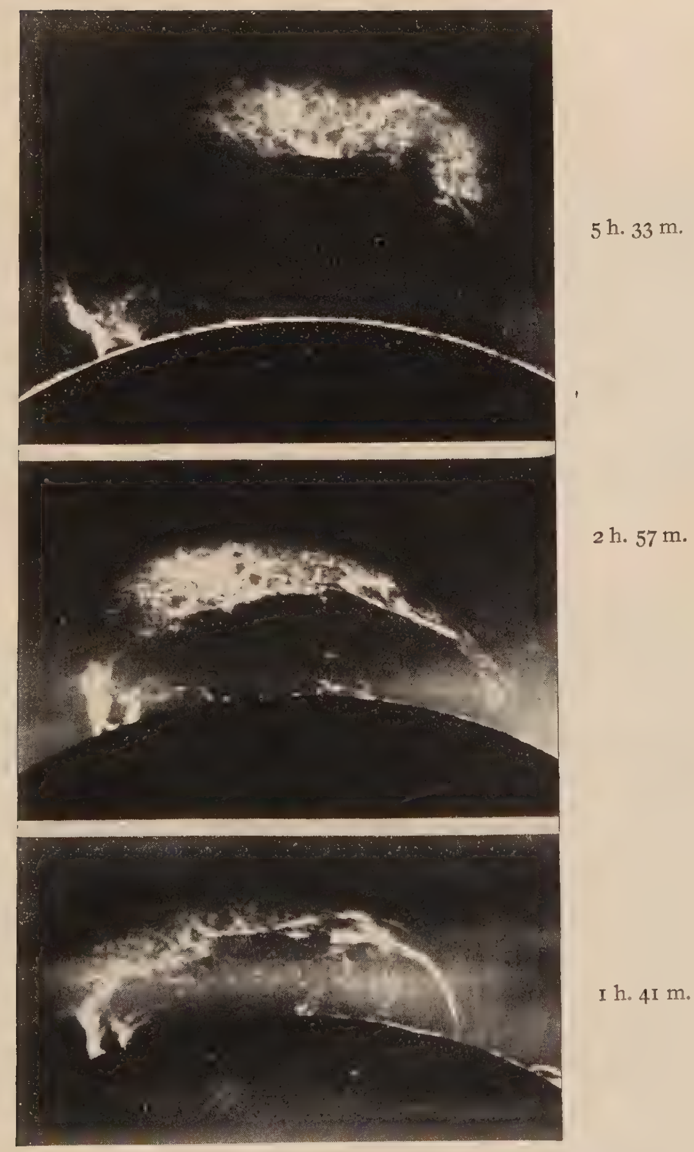
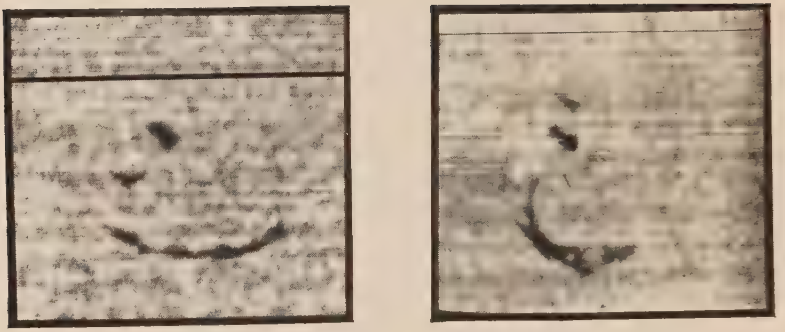
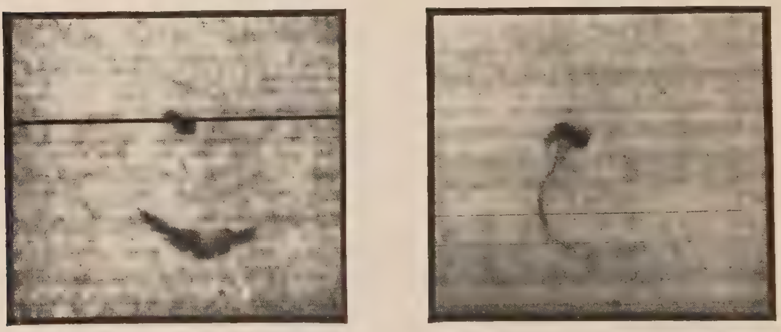

5 h. 33 m.
2 h. 57 m.
1 h. 41 m.

Plate 18. The Great Solar Prominence of 1919 May 29
Photographed by Edison Pettit at Yerkes Observatory



June 2, 6:10 A. M.

June 3, 5:14 P. M.



June 3, 4:58 P. M.

June 3, 5:22 P. M.

PLATE 19. SPECTROHELIOGRAMS MADE IN THE LIGHT OF $H\alpha$ BY HALE AT MOUNT WILSON, 1908 June 2 and 3, showing great dark Hydrogen Flocculus drawn into Sun Spot

there prevailing the gases become sufficiently opaque to give the observed appearance of the Sun's sharply defined limb. The production of a continuous spectrum at this pressure is explained by the presence of a high proportion of free electrons.

The principal forces which act upon a particle of gas at the surface of the Sun are (1) gravity, nearly twenty-eight times as great as on the Earth, acting toward the Sun's center, (2) gaseous expansion, the result of collisions among the molecules, and (3) the pressure of the Sun's radiation (page 163), which acts principally outward. It is believed that the chromosphere is held up against the action of gravity almost entirely by radiation pressure, the gaseous pressure in that region being very low.

Sun Spots.—While there is a great variety in the size and shape of sun spots, the well-developed spot is roughly circular in outline and consists of two parts, an inner, darker part called the **umbra** and an outer, less dark border called the **penumbra.** In contrast to the photosphere, the umbra seems perfectly black, but in reality it gives at least ten per cent as much light per unit area as does the photosphere. The penumbra consists of filaments which converge toward the center of the umbra. The umbra is sometimes as much as 20,000 miles, and the penumbra surrounding a group of spots as much as 100,000 miles, in diameter. Large spots are often seen with no other optical aid than a dark glass, or with the unaided eye when the Sun shines through mists near the horizon, and the Chinese have records of sun spots seen centuries before Galileo applied the telescope to the sky in 1610. Sun spots develop rapidly from small points, persist a few weeks or months, and disappear by breaking up into a number of smaller spots or by contracting in size. (Plate 17.)

The spectrum of a sun spot differs from that of the photosphere in the presence of bands due to chemical compounds, notably titanium oxide, which can exist only at lower temperatures than the 6,000° C. of the photosphere, and also in the relative strengthening of the furnace-lines (lines which are more prominent in the spectrum of the electric furnace than in that of the hotter arc or spark); from which it is evident that the spots are cooler than their surroundings.

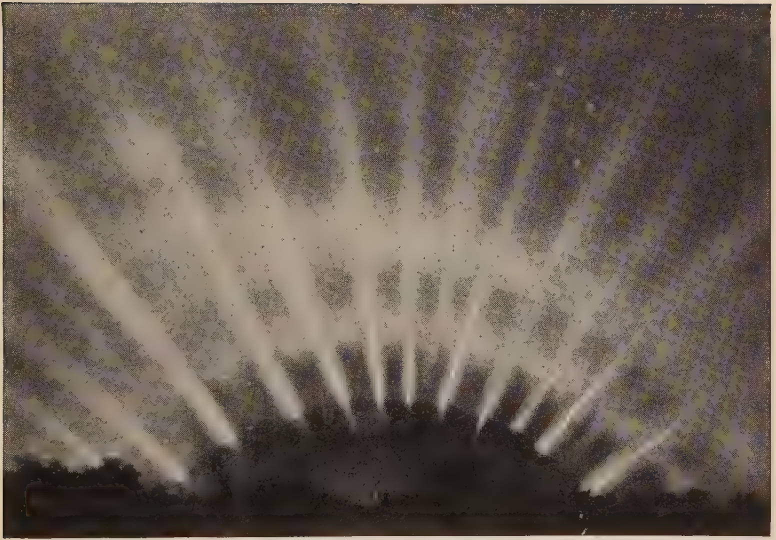
Evershed, in India, by measuring the Doppler-Fizeau displacement (page 167) of lines in the spectra of spots near the limb, showed that the incandescent gases near the photosphere were flowing outward from the umbra, and that the gases of the chromosphere, at higher levels, were flowing inward; a result which was abundantly confirmed by St. John at Mount Wilson.

Sun spots are found in definite **zones** upon the Sun, chiefly between latitudes 10° and 30° on either side of the equator, though occasionally near the equator itself and in latitudes up to 45°; but never near the poles.

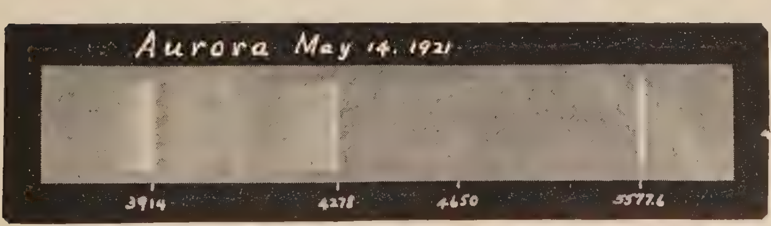
Prominences.—Solar prominences are vast eruptions which rise from the chromosphere to heights sometimes as great as 500,000 miles—greater than the radius of the Sun. They have been divided into two classes: **quiescent** prominences, which appear like great clouds floating in the solar atmosphere and remain unchanged for days at a time; and **eruptive** prominences, which appear more like flames and move with velocities as great as 200 miles a second so that their form may change completely in a few minutes (Plate 18). The spectra of quiescent prominences consist of the bright lines of hydrogen, helium, and ionized calcium; those of eruptive prominences sometimes contain the enhanced lines of metals as well. Prominences of both types are most numerous in the spot zones, but are also seen outside these zones and sometimes even at the poles. Eruptive prominences are often found over active sun spots. When situated over the disk of the Sun instead of at the limb, prominences are often photographed as Hα flocculi.

Vortical Motion in the Gases Surrounding Sun Spots.—Visual observations have sometimes shown a vortical or cyclonic motion in the neighborhood of sun spots, and after his invention of the spectroheliograph Hale hoped to find similar currents in the flocculi. The calcium flocculi gave no indications of such currents, but the hydrogen flocculi, photographed in the light of Hβ, Hγ, and Hδ, showed in many cases a structure consisting of curving lines like those formed by iron filings in the vicinity of a magnet. In 1908, photographic plates which were sensitive to red light became available, and upon applying these (at Mount Wilson) to the photography of the Sun in the light of Hα, Hale found that the vortical structure of the Hα flocculi was very marked. By comparing successive photographs of a series, rapid motion along the curved lines could in some cases be detected; for example, on a series of

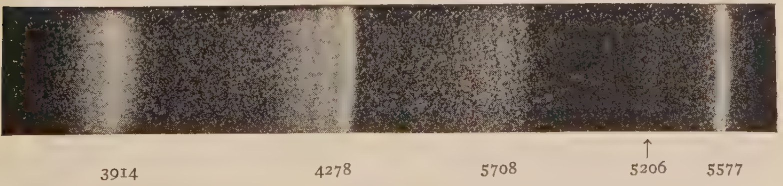
PLATE 20



THE AURORA BOREALIS OF 1878 MARCH 1, from a pastel drawing by Trouvelot

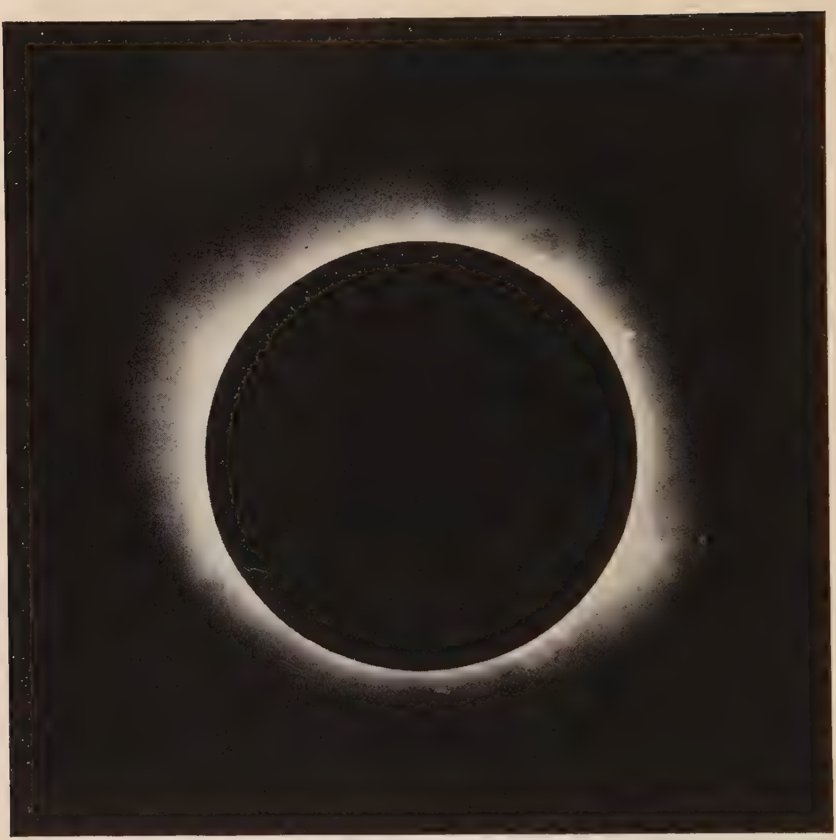


SPECTRUM OF THE AURORA OF 1921 MAY 14, photographed by V. M. Slipher at Lowell Observatory

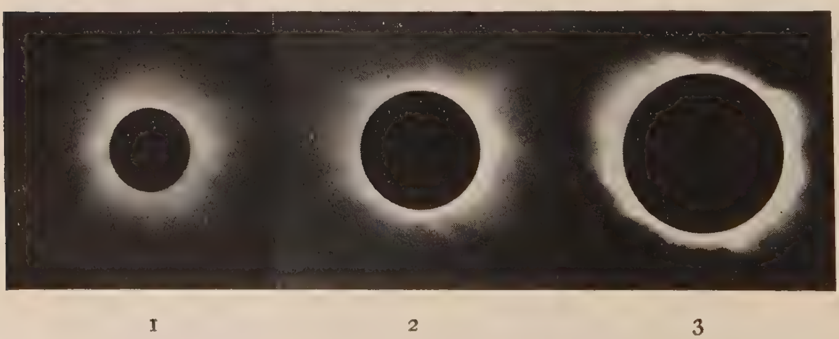


SPECTRUM OF THE AURORA OF 1928 JULY 7, photographed by V. M. Slipher. Shows a previously unobserved faint line at 5206 which is due to atomic nitrogen

PLATE 21. THE SOLAR CORONA



Above—Inner Corona and Prominences, photographed 1926 January 14, with 63-foot camera by Swarthmore College Expedition to Sumatra, Professor John A. Miller in charge



1. Spot Maximum. Lick Expedition to Chile, 1893
2. Intermediate type. Slocum, Middletown, Connecticut, 1925
3. Spot Minimum. Barnard and Ritchey, North Carolina, 1900

plates extending from May 29 to June 4, 1908, a dark hydrogen flocculus, many thousands of miles long, was seen to be sucked into a neighboring spot (Plate 19). Just before its disappearance in the spot its velocity exceeded 100 kilometers a second. Deslandres, at Meudon, by applying a very narrow second slit to the exact center (K_3) of the calcium line, has since shown a structure in the calcium flocculi similar to that of Hα flocculi, and so it appears that the observed vortices exist at high levels, thousands of miles above the spots. In general, the direction of whirl is opposite for two vortices situated in the northern and southern hemispheres of the Sun.

These phenomena led Hale to the conclusion "that a sun spot is a solar storm, resembling a terrestrial tornado, in which the hot vapors, whirling at high velocity, are cooled by expansion, thus accounting for the observed changes of the spectrum lines and the presence of chemical compounds."[1]

Magnetic Properties of Sun Spots.—Immediately after the discovery of solar vortices, Hale found that a sun spot has the properties of a bar magnet, as if a magnetized steel bar were thrust radially into the Sun through the center of the spot.

The reasoning by which he was led to this discovery and the method of making it are of great interest to one acquainted with the principles of physics. In 1876 Rowland had found that an electrically charged disk, when rapidly rotated, produced a magnetic field, showing that, as Maxwell had previously predicted, a moving electric charge was equivalent in its magnetic effects to a current flowing along a wire. For some years previous to Hale's discovery it had been known that hot carbon and hot metals emit negatively charged particles (electrons). It occurred to Hale that there might be a preponderance of positive or negative charges in the gases forming a solar vortex, and that, if so, the rapid whirling of these gases would produce magnetic effects similar to those of a current in a colossal helix, making the vortex into a gigantic electromagnet.

In 1896 Zeeman, in Holland, had found that when a flame or other source of light is placed in a magnetic field, the lines of its spectrum are each doubled, trebled, or even further multiplied, and that, when the source is viewed from a direction at right angles to the lines of magnetic force, the light is plane-polarized, while if viewed along the lines of force—through a hole bored in the pole-piece of the magnet—the light is circularly polarized, the polarization conforming to a simple theory which was given a little later by Lorentz. As early as 1892, Young had noticed that certain lines were doubled in the spectra of sun spots, and Hale suspected that this doubling was a

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[1] Hale, *Ten Years' Work of a Mountain Observatory*, p. 27.

Zeeman effect due to the magnetic field of the vortex. Using the tower telescope and spectrograph in connection with the appropriate Nicol prisms and Fresnel rhombs for studying polarized light, he found that, when the sun spot was near the center of the disk so that the light reaching the spectrograph emerged radially from the Sun, the components of the double lines were circularly polarized, while plane polarization appeared when the spot was near the limb, the light then emerging nearly at right angles to the lines of magnetic force.

A large proportion of the sun spots which have been recorded, from the time of Galileo to the present, are arranged in pairs, or in streams which are nearly parallel to the solar equator. Observations made at Mount Wilson of the Zeeman effect in several thousand spot-groups have shown that in the majority of cases the two spots of a pair, or the clusters at opposite ends of a stream, are of opposite magnetic polarity. Many spots which occur singly are preceded or followed in the solar rotation by groups of faculæ or flocculi, in which magnetic effects conforming to this rule have been detected; to these regions Hale has given the name of "invisible spots." The arrangement of polarities is opposite in the northern and southern solar hemispheres; that is, when the preceding spot of a pair lying north of the Sun's equator exhibits "south polarity"—*i. e.*, like the south-seeking pole of a magnetic needle on the Earth—the preceding spot of a southern pair shows "north polarity," and *vice versa*.

The Eleven-year Cycle.—In 1843 Schwabe, a German amateur astronomer, showed from a record of observations of sun spots which he had kept during the preceding twenty-seven years that the spottedness of the Sun was variable in a period which he placed at ten years. A most laborious search by Wolf of Zürich through all available records made since Galileo's discovery of the spots in 1610 confirmed Schwabe's discovery, and systematic observations made since at Greenwich and elsewhere have placed the fact of the periodicity beyond all doubt. At a time of minimum spottedness the face of the Sun may be wholly unspotted for months at a time, while at maximum it is almost never without spots. The interval between times of minimum spottedness averages 11.2 years instead of 10, but the "regularity is very irregular," the actual observed interval having had a range of at least four years. The time of descent from maximum to minimum is longer than that of the rise from minimum to maximum, the former averaging 6.5 years and the latter 4.6. In this important respect the sun-spot curve resembles the light-curves of variable stars of the Cepheid and long-period types (Chapter XVII).

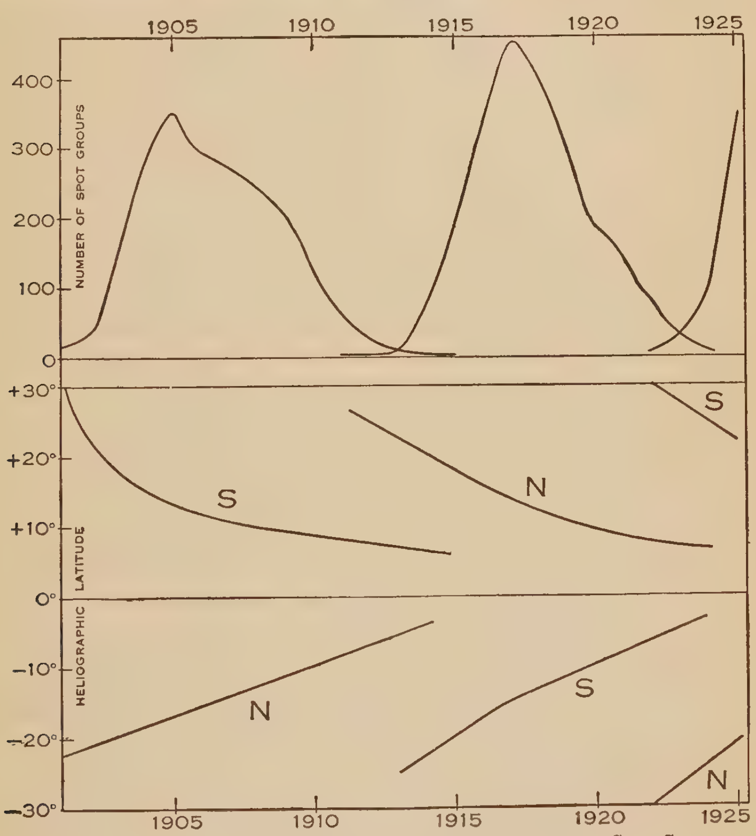

FIG. 106. THE NUMBER, LATITUDE, AND POLARITY OF SUN SPOTS (NICHOLSON)

The upper curves give the number of spot groups observed yearly. The lower curves show that the sun-spots of a new cycle appear in high latitudes during the time of minimum solar activity with opposite magnetic polarities in the northern and southern hemispheres. As the cycle progresses, the mean latitude of the spots in each hemisphere decreases continuously while the distribution of magnetic polarities remains unchanged. In the high-latitude spots of the next cycle, which begin to develop more than a year before the last low-latitude spots of the preceding cycle have ceased to appear, the polarities are reversed. The letters N and S printed on the curve indicate the polarity of the preceding (western) spots of each group.

It has already been remarked (page 180) that the spots are confined to certain zones. A peculiar relation between their latitude and the time of their occurrence was brought out by Carrington and Spoerer about the middle of the nineteenth century and has become as clearly established as the periodicity. The spots belonging to a new eleven-year cycle appear at the outer limits of the spot-zones, and as the cycle progresses they are found farther and farther toward the solar equator. At spot maximum they are seen mostly about latitude ± 14°, while at minimum a few spots of the expiring cycle are seen near the equator and at the same time a few of the beginning cycle appear at high latitudes. It has been found at Mount Wilson that the magnetic polarities of bi-polar spot-groups are reversed at the end of a cycle, and thus the whole interval required for the changes in the spots, including magnetic properties, appears to be 22.2 years instead of 11.1. These relations are illustrated in Fig. 106.

In addition to the number, latitude, and magnetic properties of sun spots, several other phenomena, both solar and terrestrial, are certainly known to fluctuate in the eleven-year period. Such are the numbers of faculæ and of prominences; the form of the solar corona; the intensity of the Sun's radiation as measured by the solar "constant" (page 189); the proportion of ultra-violet in the Sun's light; the number and brilliance of polar auroræ observed on the Earth; the range of variation of the Earth's magnetism; the intensity of radio reception; and the frequency of magnetic storms. No satisfactory explanation of the eleven-year period or of the interdependence of these solar and terrestrial phenomena has yet been found.

The **polar aurora,** often referred to in our hemisphere as the Northern Lights, is one of the most mysteriously beautiful of terrestrial phenomena. As seen in North America and Europe, its most common form is that of an arch of soft light which appears above the northern horizon, usually early in the night, and from which extend needle-like streamers toward the zenith (Plate 20). These streamers are not for a moment still, but pulsate and quiver, at the same time varying in brightness. Sometimes the aurora assumes the appearance of beautifully folded curtains which wave as if in a gentle wind. On rare occasions, the auroral light fills the entire visible sky, in which case the streamers usually converge toward the magnetic zenith (not the true zenith), where they curve spirally to form an auroral **corona,** which may persist for hours while the streamers continue their mystic pulsations. The color of the light is usually apple-green, but rose, lavender, and violet tints are not uncommon. The spectrum (Plate 20) consists of bright lines, the brightest of which, in the green (wave-length 5577 Angstroms) can usually be detected on a clear night whether an aurora is visible or not. Several of the bright lines belong to the spectrum of nitrogen, and the green line has recently been produced at the University of Toronto from a mixture of oxygen with helium and also with neon.

Auroræ are most frequently seen at the time of sun-spot maximum, and the most brilliant auroræ appear on occasions when the largest spots are turned toward the Earth. It is probable that the aurora is due to the impact of streams of electrons arriving from the Sun and impinging upon the upper regions of the air, having been directed in the last few hundred miles of their journey by the magnetic field of the Earth.

It is well known that the needle of the magnetic compass does not, in general, point due north; in the eastern part of the United States, for example, it points several degrees west of north, while in the western part it points east of north. Moreover, its direction continually changes through a range of a few minutes of arc, the most conspicuous change being a **diurnal oscillation.** Soon after the discovery of the periodicity of sun spots, it was found that a similar—in fact, almost identical—periodicity existed in the range of the compass variation and also in the strength of the Earth's magnetic field.

A **magnetic storm** is a sudden violent disturbance of the Earth's magnetism, in which the compass needle oscillates, sometimes through an arc of several degrees within an hour or two, and in which the fluctuations in the strength of the Earth's field are so great as to induce currents which interfere seriously with the operation of telegraph lines. Magnetic storms are most frequent at the time of spot maximum, and usually coincide in time with the presence of very large spots near the center of the visible disk of the Sun.

The Rotation of the Sun.—Immediately after the telescopic discovery of the sun spots by Galileo in 1610, it was noticed that they moved across the disk of the Sun, and they were thought by some to be planets seen in transit (the idea of *spots* on a *celestial* body being obnoxious); but Galileo refuted this opinion by pointing out that they remained behind the Sun the same length of time as in front of it, and were, therefore, on the solar surface. Galileo's German contemporary, Christopher Scheiner, made a fair determination of the rotation period from a record of the position of sun spots obtained by projecting the Sun's image on a screen; but the first accurate determinations were made by Carrington and by Spoerer about 1850. The work of these observers not only gave an accurate description of the position of the Sun's axis, but brought out the remarkable fact that spots near the equator travel around the Sun in a shorter time than spots in higher latitudes. They found that the Sun rotates in the same direction as the Earth about an axis whose north pole is in $\alpha = 18^h\ 44^m$, $\delta = +64°$, or midway between Polaris and Vega; that the plane of the solar equator is therefore inclined 7° to the plane of the ecliptic; that spots near the equator indicate on an average a sidereal rotation in about twenty-five days; and that 30° on either side of the equator the period is twenty-six and one-half days and at 45° about twenty-seven and one-half. They found also that, in general, each individual spot had a motion of its own in both latitude and longitude, the periods mentioned above being averages only.

The path of a sun spot across the disk appears slightly concave northward in summer and autumn, and convex northward in winter and spring. This is because of the inclination of the axis, the north pole leaning 7° toward the point occupied by the Earth on September 7. On June 3 and December 5 the spot-paths are straight, for then the Earth is in the plane of the Sun's equator.

The apparent, or **synodic,** rotation period of the Sun is a little longer than the sidereal, being twenty-seven and a quarter days for a spot at the equator. This is because the Earth advances, during a sidereal rotation of the Sun, nearly a twelfth of the way around its orbit, so that the spot must turn through more than 360° to come back to the same apparent position. The relation between the sidereal and synodic rotation periods is similar to that between the sidereal and synodic months (page 119).

Since sun spots are found only in limited zones of the Sun's surface, they cannot be used to study the rotation of all parts of the solar globe. Faculæ offer no advantage over spots, for their distribution is about the same and they cannot be well observed except near the limb. There are, however, two other methods of approach to the problem: by the motions of the spectroheliographic flocculi, and by Doppler-Fizeau displacements of lines in the spectrum of the limb. The results obtained by Hale and Fox from calcium flocculi agree well with those given by spots; but those derived from hydrogen flocculi, while indicating about the same rate of rotation at the equator as that shown by spots, give no evidence of a retardation at higher latitudes.

When high-dispersion spectra of the east and west limbs of the Sun are confronted, a relative shift of the lines is immediately evident, the lines belonging to the west limb being shifted toward the red (Plate 13). This displacement was first used for studying the solar rotation in 1893 by Duner in Sweden, who observed visually. More accurate observations have been made photographically by a number of observers. Probably the most exhaustive study is that of Adams at Mount Wilson, who finds an average period of 24.6 days at the equator, and 33.3 days at latitude ±80°. Different spectral lines give distinctly different velocities; the lines of lanthanum, titanium, and iron give smaller velocities than the calcium line at 4227, while $H\alpha$ indicates a higher velocity. These differences are probably due to differences of level of the sources of light, the hydrogen that emits $H\alpha$ being higher, and the metals lower, than the non-ionized calcium which emits 4227.

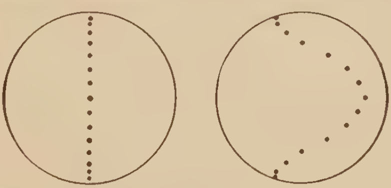

FIG. 107. EQUATORIAL ACCELERATION OF THE SUN

The rate of rotation in different heliographic latitudes may be conveniently represented by a formula. Representing by ξ the daily heliocentric angular motion and by φ the heliographic latitude, the results of Carrington, as reduced by Faye, give

$$\xi = 14^{\circ}.37 - 3^{\circ}.10 \sin^2 \varphi,$$

while Adams' results give

$$\xi = 14^{\circ}.61 - 3^{\circ}.99 \sin^2 \varphi.$$

The effect of the unequal rotation periods in different latitudes is illustrated in Fig. 107, which is constructed according to the formula of Adams. Suppose a row of fifteen spots, one for every tenth degree of latitude from −70° to +70°, arranged upon the same meridian of the Sun as in the left-hand drawing. After one rotation they will have arranged themselves as in the right-hand drawing.

The Solar Corona.—The Sun's corona is a pearly-white atmosphere which extends at least 300,000 miles all around the Sun, and some of its streamers have been known to reach to a distance of 5,000,000 miles. Although its total light, as measured at the 1925 eclipse, is about half that of the full Moon, the light per unit area is so small that all attempts to observe the corona without an eclipse have so far failed. Its extreme tenuity is shown by the fact that the great comet of 1882 passed directly through it without any perceptible change in its speed. Arrhenius estimates its average density as the equivalent of one dust-particle to every fourteen cubic yards.

The form of the corona varies with the progress of the eleven-year cycle (Plate 21). At spot minimum, fine rays arranged like the straws in a sheaf of wheat cluster about the Sun's poles, while from the spot belts and equatorial regions extend broad streamers to enormous distances. At spot maximum, the distribution of the corona around the Sun is more uniform, few pronounced rays are seen, and no portion of the corona extends so far from the limb as at spot minimum. The inner part of the corona is much the brightest, more than half the total light coming from the region within about 3′ of the limb of the Sun.

The spectrum of the corona (Plate 15) consists of three superposed parts. First, there is a continuous spectrum, due possibly to the incandescence of fine solid or liquid particles. Second, the dark lines of the solar spectrum appear, but only faintly, as if partly obliterated by the continuous spectrum first mentioned. These are probably due to reflected sunlight for they are more conspicuous in the light coming from points considerably distant from the Sun, where the light of incandescence would be feebler in proportion to reflected light. Third, there is a spectrum of bright lines, which must be due to luminous gases within the corona.

The brightest line in the visible part of the coronal spectrum was first seen by Young in 1869. It is in the green, at wavelength 5303 Angstroms, and does not belong to the spectrum of any known terrestrial substance; the substance which produces it is accordingly called **coronium.** About thirty other non-terrestrial bright lines have been found in the visible and ultra-violet parts of the coronal spectrum, two of which, in the ultra-violet, have a greater intensity than that of the green line.

J. W. Nicholson, in England, has found that the bright coronal lines form two series such that the cube roots of the wave-lengths in either series form an arithmetic progression, while those of lines in one series bear a constant ratio to those of corresponding lines in the other. These series are not similar to any spectral series so far produced in the laboratory.

For many years after the discovery of its spectrum, it was expected that coronium would eventually be discovered as a terrestrial element, just as helium, first disclosed in the solar spectrum in 1868, was discovered in the laboratory by the chemist Ramsay in 1895; but modern atomic theory shows that only ninety-two elements are possible which have atomic weights no greater than that of uranium (Table 4, page 166), and all but two of these elements have now been discovered. The two undiscovered elements are of so great an atomic weight that they cannot be expected to form a constituent of so light a thing as the corona, and it is therefore necessary to suppose that coronium is an ionized or otherwise modified form of an element already known (compare the case of "nebulium," page 394).

The Rate of the Sun's Outpour of Energy.—The rate at which the Sun radiates energy into space is indicated by the quantity known as the **solar constant of radiation,** which is defined as the number of calories which would be received from the Sun each minute upon a surface one centimeter square, if the surface were exposed perpendicularly to the Sun's rays outside the Earth's atmosphere, at the Earth's mean distance from the Sun. Its value, according to the most generally accepted determinations, averages 1.94, but varies by about five or six per cent.

The word **calorie,** as here used, denotes the amount of energy required to raise the temperature of one gram of pure water from 15° C. to 16° C. It is equal to 4.18×10^7 ergs, and the solar constant is equivalent to 1.35×10^6 ergs per square centimeter per second. The energy of the Sun comes to us in the form of waves of a great range in length, the greater part residing in the infra-red or "heat" waves. This energy has not the form of heat as it passes through space, and assumes that form only when it is absorbed by some non-transparent material. This may be strikingly demonstrated by converging sunlight upon a piece of dark cloth by means of a convex lens made of ice. The Sun's rays heat the ice very little—if it were perfectly transparent to all of them, they would heat it not at all—nor does the ice cool the rays; but they are absorbed by the cloth and so transformed into heat, which is a motion of the molecules, and the cloth is set afire. Dark objects absorb radiation and transform it into heat more readily than do light-colored objects, and this is why, in summer, light-colored clothing is more comfortable than black. Neither a perfectly transparent body nor a perfect reflector, if either existed, would absorb radiation or be warmed by sunlight.

The determination of the solar constant involves two different problems: the measurement of the rate at which energy is received at the surface of the Earth after it has passed through the air, and the determination of the amount absorbed by the air during passage. The first is solved with the instrument known as a **pyrheliometer,** invented by Pouillet about 1838, in which the Sun's radiation is allowed to fall upon a body of known mass and absorbing power, and the rise of temperature in a known interval of time is noted. Corrections must be made for imperfect absorption of the Sun's energy and for radiation of energy from the pyrheliometer. The absorption of the air is more difficult to determine. Information about it is obtained by comparing pyrheliometer readings made at sea level with those made on the tops of mountains, above a part of the atmosphere, and also from readings made at different times of day, with "high Sun" and "low Sun." Since the absorption is different for different wave-lengths, it is necessary also to compare the intensity of the radiation in different parts of the spectrum, particularly in the infra-red; for this, use is made of the **bolometer,** an exquisitely delicate instrument invented by Langley at Allegheny about 1880, the operation of which depends upon the change of electrical resistance produced in a thin strip of platinum when radiation falls upon it.

The value 1.94, given above for the solar constant, is that determined by the Smithsonian Astrophysical Observatory of Washington from a vast quantity of observations made at stations in different parts of the Earth, many in the clear air of desert regions, and at different altitudes. According to Abbot, the Director of that Observatory, the solar constant varies by about five per cent in an irregular interval of a few days, and is about three per cent higher at the time of sun-spot maximum than at spot minimum; but a low value often occurs the day after the passage of a large spot-group across the center of the solar disk.

To put the Sun's radiation in more familiar terms, we may compute the rate at which it could melt ice. The appropriate computation shows that the Sun's radiation would, at the Earth's distance, melt in one minute a sheet of ice 0.0267 cm. thick if placed perpendicular to the Sun's rays and if all the energy were absorbed by the ice. Since the intensity of radiation varies inversely as the square of the distance, to obtain the intensity at the Sun's surface, we must multiply the intensity at the Earth's distance by the square of the ratio of the distance from the Earth to the Sun (149,500,000 km.) to the radius of the Sun (696,000 km.). The square of this ratio is about 46,000. Therefore, at the photosphere, the solar radiation would melt in one minute a shell of ice 46,000 × 0.0267 cm., or about 12 meters (39 feet) thick!

One horse-power is the equivalent of 10,700 calories per minute; hence, at the Earth's distance each square centimeter receives $1.94 \div 10700 = 0.00018$ horse-power, or the radiation falling on a square meter, if utilized in a perfect engine (existing engines are far from perfect) would yield 1.8 horse-power. At the surface of the Sun, each square *centimeter* develops over 8 horse-power continuously.

The above computation involves the assumption that the Sun radiates at the same rate in all directions, just as other luminous bodies do—an assumption against which there is not the slightest evidence. On the same assumption, the Earth receives only about $\frac{1}{2 \times 10^9}$ part of the whole amount of energy produced by the Sun, and only about $1/10^8$ is received by all the bodies of the Solar System combined. A very small amount must be absorbed by the stars, and the remainder of this vast output of energy, to the best of our knowledge, travels forever outward into space.

The Effective Temperature of the Sun's Surface.—The value of the solar constant affords a direct determination of the Sun's effective temperature by the Stefan-Boltzmann law (page 160). The result is 5750° K (K signifying absolute temperature in Centigrade degrees).

Other methods depend upon observations with the bolometer, which give a curve whose ordinates are proportional to the energy of the Sun's radiation and whose abscissæ represent wave-lengths in the solar spectrum. The form of this curve depends somewhat upon the portion of the solar disk which is observed, but when corrected for absorption in the atmosphere of the Earth it resembles the Planck curves shown in Fig. 102. The abscissa of the peak is near λ4700 Angstroms (0.000047 centimeters), so that Wien's law gives for the effective temperature 6150° K. The curve itself conforms closely to the Planck curve for 7000° in the infra-red, but falls below the curve for 6200° in the violet.

We thus have values of the effective temperature of the solar surface ranging from 5750° K to about 7000° K. The disagreement among the results may well be due to the fact that they represent averages, obtained by different methods, of radiations which originate at different levels in the Sun where the actual temperatures vary widely.

The Source of the Sun's Energy.—Calculations based on the laws of physics show conclusively that the Sun's vast production of energy cannot be explained by combustion (if it were composed of pure coal and oxygen its consumption at the present rate of outpour would last only about a thousand years), nor by the cooling of matter previously heated, nor by the fall of meteoric bodies into the Sun.

At the end of the nineteenth century, it was generally conceded that the **contraction theory** of Helmholtz afforded an adequate explanation. A contraction of the Sun is the equivalent of a fall of each particle of its substance toward the center, and in falling the particles must generate heat. From Abbot's value of the solar constant, it may be shown that a contraction of the Sun's radius of 37 meters annually would account for the energy radiated, while the change in the Sun's diameter would not become perceptible from the Earth in several thousand years. Lane proved further that the radiation of energy produced by this contraction would not be as rapid as its generation, and that, therefore, paradoxically, the Sun would actually grow hotter until its contraction proceeded so far that it was no longer gaseous. However, if the Sun has been contracting in the past at the rate required by the Helmholtz hypothesis, it must have been as large as the Earth's orbit as recently as 20,000,000 years ago, and under such circumstances the Earth could not have supported life if indeed it could have existed. Geologists find evidence of life on the Earth as much as 300,000,000 years old, and place the age of the Earth's crust at not less than 1,000,000,000 years; and hence the Helmholtz theory must be regarded as inadequate.

The source of the energy of the Sun and of the other stars is now believed to lie within their atoms. Modern atomic theory shows that the electrons and protons of which atoms consist are themselves particles of energy. According to the theory of relativity, the mass of a body is only the total quantity of energy which it contains *divided by the square of the velocity of light* (9×10^{20}), so that the mass of the Sun, 2×10^{33} grams, is the equivalent of 1.8×10^{54} ergs of energy. From Abbot's value of the solar constant, it may be computed that the Sun is pouring out energy at the rate of 1.2×10^{41} ergs per year, or, if all this energy comes from the annihilation of matter, the Sun's mass is diminishing at the rate of 1.3×10^{20} grams or 1.7×10^{14} tons per year. This is of the order of a fifty-millionth of the Earth's mass and is so small a fraction of the Sun's mass that the radiation could continue at its present rate for 1.5×10^{13} (fifteen million million) years before the sub-atomic supply of energy is exhausted. Surely this period is long enough to satisfy the most exacting demands of geology and the theory of evolution.

Man has not learned to experiment with the vast quantities of energy stored within the atom, and we can only guess as to the way in which the transformation occurs in the interior of a star. Two possible methods have been suggested: first, by the coalescence and mutual destruction of a positive proton and a negative electron; and second, by the transmutation of hydrogen into more complex elements—for example if four atoms of hydrogen, whose atomic weight is 1.008, were transmuted into helium, of atomic weight 4.000, the disappearance of 0.032 of the mass of the hydrogen could be accounted for only by its transformation into radiative energy. The first method seems the more likely because it provides a far greater length of life for the Sun and the other stars.

EXERCISES

1. In what direction do sun spots appear to be carried across the disk by the Sun's rotation?

2. What must be the diameter, in miles, of a group of sun spots which subtends an angle equal to the apparent separation of the two stars composing ϵ Lyræ (about 3′), and might therefore be easily seen with no other optical aid than a dark glass?

Ans. 81,000

3. What is the relative radial velocity of the east and west limbs of the Sun due to its rotation, which produces the Doppler displacement shown in the lower part of Plate 13?

Ans. 4 km./sec.

4. If the Sun's effective temperature were 15,000°K, like that of some stars in the Pleiades, what would be the value of the solar constant?

Ans. Over 75 calories per square centimeter per minute

CHAPTER IX

THE PATHS OF THE PLANETS

The Planets.—A **planet** may be distinguished from a star in three ways: First, the stars twinkle and the planets usually do not; this rule, however, is far from infallible. Second, when magnified by a telescope, the planets show disks of perceptible area, while the stars appear as glittering points. This distinction holds for all the principal planets, but fails for most of the many minor planets, or asteroids. Third, and most important, the stars maintain practically the same relative positions for years while a planet changes its position among them perceptibly from night to night or, seen in a telescope, in the course of a few hours or even minutes.

The word planet is derived from a Greek word meaning *wanderer*, and is so applied because of the third characteristic just mentioned. The ancients recognized seven planets: the Sun, the Moon, Mercury, Venus, Mars, Jupiter, and Saturn. The Sun and Moon are not now so classed, but modern astronomy places the Earth among the planets and has discovered two others, Uranus and Neptune,[1] of a size much greater than the Earth's, and besides these more than a thousand little bodies called minor planets, or asteroids. The word as now applied means an opaque body that shines by reflected sunlight and that moves around the Sun in a nearly circular orbit.

Apparent Motions of the Planets upon the Celestial Sphere.—The apparent motions of the planets are not so simple as those of the Sun or Moon. The Sun seems to move with nearly constant speed and always toward the east in the great circle called the ecliptic, which is practically fixed among the stars. The Moon's apparent motion is also eastward and in a great circle that is slightly inclined to the ecliptic. Although the apparent path of each of the planets (some of the asteroids excepted) lies near the ecliptic, their motion is very different from that of the Sun or Moon, being zigzag or looped—toward the east for a considerable period, then toward the west for a shorter time, and then eastward again. The long, eastward motion is called **direct** and the short, westward motion, **retrograde.** The apparent motion of Mars from 1928 August 1 to 1929 May 1, through the constellations Taurus and Gemini, is shown in Fig. 108. From 1926 December 8, until 1928 November 10, this planet moved directly, but its motion became reversed on the latter date, and remained retrograde until 1929 February 1, when it again became direct. The planet then continued to move directly for about two years, going entirely around the sky and again passing the region shown in Fig. 108, and traversing another loop about 25° farther east in the winter of 1930–1931.

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[1] Announcement of the discovery of an additional major planet beyond Neptune, by the astronomers of the Lowell Observatory, was made 1930 March 12, while the proof of this page was being read. See accounts on pages 209, 237 and 272. The name Pluto was proposed by the discoverers on May 1.

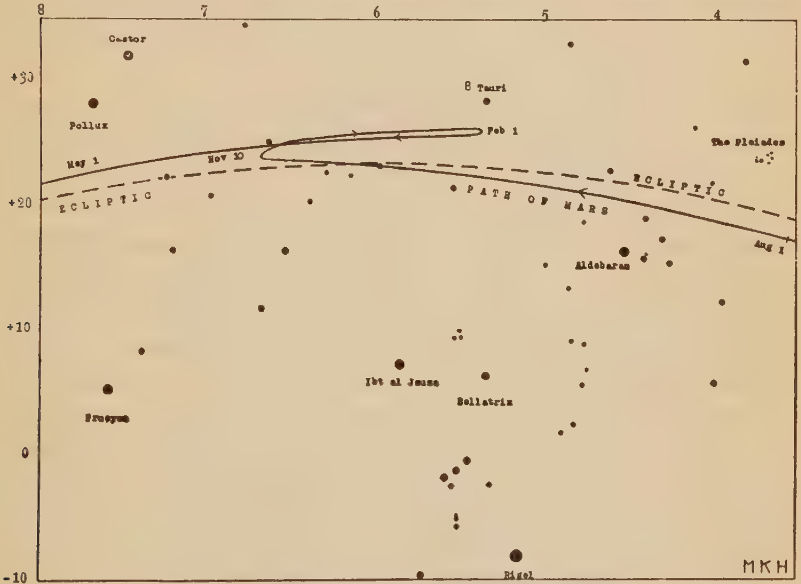

FIG. 108. APPARENT PATH OF MARS IN 1928–1929

Since the Sun also appears to move among the stars, the motions of a planet relative to the Sun and relative to the stars are necessarily different. The direct motion of all but two of the planets is always slower than that of the Sun, so that they drop relatively westward and cross the meridian of any given place earlier each night as counted by solar time. The two exceptions are Mercury and Venus, which drop rapidly west of the Sun during their retrograde motion and then slowly overtake it as they move directly, thus appearing at one time as "morning stars" on the west side of the Sun and then as "evening stars" on the east.

The difference of the longitudes of a planet and the Sun is called the planet's **elongation,** and this may be either east or west elongation. If the elongation is zero, the planet is said to be in **conjunction;** if 90°, it is in **quadrature;** and if 180°, it is in **opposition.** Mercury and Venus never reach quadrature or opposition, the greatest elongation ever reached by the former being 28°, and by the latter 47°. The conjunction reached by either of these planets during its retrograde motion is called **inferior** conjunction, while that reached during direct motion is called **superior.** Opposition is reached by each of the other planets about the middle of its retrograde motion. The time taken by a planet to go from opposition to opposition or from conjunction to conjunction is called its **synodic period.**

Apparent Geocentric Motion in Space.—The motions just described are the apparent motions on the celestial sphere and take no account of the changing distance of the planets from the Earth. In the cases of some of the planets, especially Mars, the brightness changes greatly with the planet's position, being greatest for this planet near opposition and least near conjunction. Mercury and Venus are brightest when a short distance from inferior conjunction and faintest near superior conjunction. This was interpreted centuries ago as being due to change of distance, and the correctness of this interpretation is proved by the telescope, which shows that the apparent diameter is greatest when the planet is near opposition or inferior conjunction.

We have seen (page 99) how the apparent geocentric path of the Sun in space can be pictured by drawing radiating lines from a point to represent the Sun's direction from the Earth

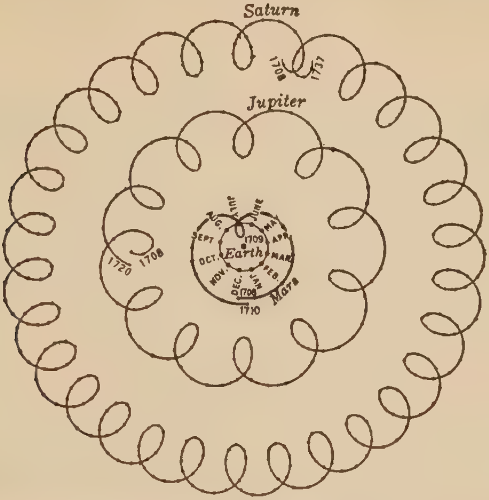
Saturn
1708
1737
Jupiter
1720
1708
JULY
JUNE
AUG.
MAY
SEPT
APR.
1709
Earth
OCT.
MAR.
NOV.
FEB.
DEC.
JAN.
1708
Mars
1710

FIG. 109. GEOCENTRIC PATHS OF MARS, JUPITER, AND SATURN (FROM PROCTOR'S *Old and New Astronomy*)

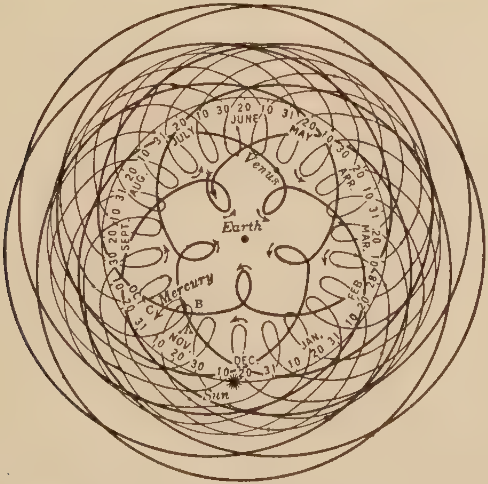
JUNE
JULY
MAY
Venus
AUG.
APR.
SEPT.
Earth
MAR.
FEB.
Mercury
B
C
OCT.
NOV.
DEC.
JAN.
Sun

FIG. 110. GEOCENTRIC PATHS OF MERCURY AND VENUS (FROM PROCTOR'S *Old and New Astronomy*)

at different times, and cutting off the lines to a length inversely proportional to the Sun's apparent diameter; and that the curve so indicated is a nearly circular ellipse with the Earth at one focus. The Moon's geocentric path is also an ellipse and has only a slightly greater eccentricity. If a curve be similarly constructed to represent the geocentric path of a planet, it appears as an intricate series of loops, as shown in Figs. 109 and 110.

The Ptolemaic System of Planetary Motions.—Many philosophers among the ancient Greeks attempted a logical description of the motions of the heavenly bodies. Most of them assumed the Earth to be stationary, but some, among whom was Aristarchus of Samos (310–250 B.C.), taught that both the Earth and the bodies now classed as planets revolved around the Sun. After Aristarchus the principal promoters of theoretical astronomy were the Alexandrians Apollonius (*c.* 230 B.C.), Hipparchus (190–120 B.C.), and Claudius Ptolemy (A.D. 100–170). Probably they did not intend their theories to be accepted unquestioningly as the true physical description of the world, but rather to be used as mathematical tools for computing the positions of the planets at any time.

Ptolemy composed a thorough compendium of the astronomy of his time, which he called the *Mathematical System of Astronomy*, but which came to be known as Μεγίστη Σύνταξις, the Great System. During the centuries of semi-barbarism that stifled Europe after the decline of the great school at Alexandria, this work was honored and preserved by the Arabs, who prefixed their article *al* to the title and corrupted it to **Almagest,** the name the book bears to this day. The description of the planetary system given in Ptolemy's Almagest, known as the Ptolemaic system, exercised a profound influence on literature, science, and religion, which is abundantly evident, for example, in the great epic poems of Dante and Milton.

According to the Ptolemaic system, the Earth is fixed at the center of the universe. (Ptolemy argued that, if the Earth moved, falcons and other birds would be left behind when they flew into the sky.) Around it revolves the Sun in a period of a year in a slightly eccentric circle. Each of the planets revolves, in a small circle called an **epicycle,** around a point which in turn revolves around the Earth in a large circle called a

deferent. The deferents of Mercury and Venus lie within the orbit of the Sun, and the centers of their epicycles lie always on a straight line joining Sun and Earth, thus explaining their apparent oscillations relative to the Sun. The deferents of the other planets lie outside the Sun's orbit. Within all the other orbits revolves the Moon in an epicycle, backward, in a period of one sidereal month, while its epicycle revolves forward in the same period, in a deferent that surrounds the Earth eccentrically. Outside all the orbits Ptolemy places the sphere of the fixed stars, and beyond this is the **Primum Mobile,** which furnishes the motive power that keeps the whole intricate machine turning westward in the diurnal motion while the planetary motions go on inside. The general principles of the system, and the way in which it explained the observed motions, can be readily seen in Fig. 111.

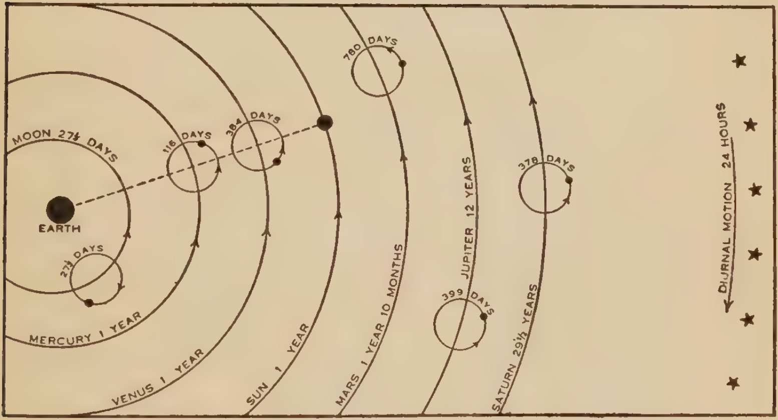
MOON 27⅓ DAYS
EARTH
27⅓ DAYS
MERCURY 1 YEAR
116 DAYS
384 DAYS
VENUS 1 YEAR
SUN 1 YEAR
780 DAYS
MARS 1 YEAR 10 MONTHS
JUPITER 12 YEARS
399 DAYS
378 DAYS
SATURN 29½ YEARS
DIURNAL MOTION 24 HOURS

FIG. 111. THE PTOLEMAIC SYSTEM
(Adapted from *Dante and the Early Astronomers*, by M. A. Orr)

The Copernican System.—The Ptolemaic system was virtually undisputed for thirteen centuries, until the Polish monk Nikolaus Kopernik (1473–1543), or Copernicus (the Latin form of his name), published his book *De Revolutionibus Orbium Cœlestium*, in which he showed that the observed motions of the celestial bodies could be explained more simply and reasonably by placing the Sun at the center of the system and supposing the Earth to be merely one of the planets revolving around it. The looped geocentric orbits of the planets are easily explained on the Copernican theory in this way: In Fig. 112 (left side) let *S* represent the Sun, the small circle the orbit of the Earth, and the large arc a part of the orbit of another planet, say Jupiter. The Earth would occupy the positions 1, 2, 3, etc., at intervals of two months, and Jupiter, whose angular velocity is only a twelfth that of the Earth, would at the corresponding times occupy the points 1′, 2′, 3′, etc. To us who live on the smoothly-moving Earth, it seems that we are stationary while Jupiter is seen in the directions and at the distances represented by the lines 11′, 22′, 33′, etc. Hence, if we construct a drawing like that at the right in Fig. 112, making the Earth stationary at *E* and drawing lines parallel to the lines 11′, etc., and of the same length, the locus of the ends of these lines will represent the apparent geocentric path of Jupiter, which thus turns out to be looped like the observed path.

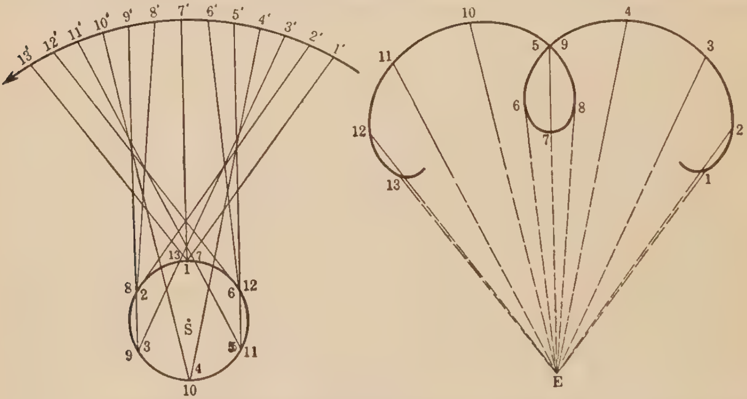

FIG. 112. COPERNICAN EXPLANATION OF THE LOOPED PATH OF A PLANET

It is to be noted that Copernicus's theory explains the observations that had been made up to his time as well as does Ptolemy's, but no better; but it has the advantage of greater simplicity and is free from the difficulty of making the stars, which were by that time known to be very distant, revolve rapidly in enormous orbits.

Tycho Brahe (1546–1601).—Three years after the death of Copernicus there was born a Danish nobleman, Tyghe Brahe, whose name is usually given in the Latin form of Tycho. Although learning of any kind was then considered beneath the dignity of the nobility, Tycho was attracted to astronomy at the age of fourteen by the fulfillment of a prediction of an eclipse, and, overcoming his aristocratic scruples, became so great an astronomer (and astrologer) that the king of Denmark established for him a great observatory on the island of Hven, near Elsinore, which Tycho furnished with the most accurate instruments ever made before the introduction of the telescope. With these instruments and the help of many assistants, Tycho made a long and accurate series of observations of the positions of the stars and planets which formed the greatest contribution to observational astronomy up to his time. Upon his observations of the planets Kepler based his great discovery of the laws of planetary motion, which firmly established the Copernican system and paved the way to Newton's discovery of the law of gravitation.

Tycho took a backward step in theoretical astronomy by rejecting the Copernican system, partly on theological grounds and partly because his most careful observations failed to show any parallactic displacement in the stars. Instead, he substituted a "Tychonic" system in which the Sun revolved around the immovable Earth while the planets revolved around the Sun. This system was not generally accepted, and so did little harm.

Galileo (1564–1642).—The heliocentric theory received support from the brilliant discoveries of the great Italian, Galileo Galilei, when he applied the telescope to the observation of the sky. One of the first discoveries he made was that of the four bright satellites of Jupiter, which in their orbital motion around that planet exemplify almost exactly the Copernican motions of the planets around the Sun.

The news of the discovery soon spread and excited the greatest interest and astonishment. Many, of course, refused to believe it. Some there were who, having been shown them, refused to believe their eyes, and asserted that although the telescope acted well enough for terrestrial objects, it was altogether false and illusory when applied to the heavens. Others took the safer ground of refusing to look through the glass. One of these who would not look at the satellites happened to die soon afterwards. "I hope," says Galileo, "that he saw them on his way to heaven."[1]

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[1] Lodge, *Pioneers of Science*, p. 104.

The most powerful blow to the geocentric theory of Ptolemy was given by Galileo's observation of Venus. According to the Ptolemaic system, as we have seen, both Venus and Mercury were always nearly between the Earth and Sun, and so could never present to the Earth so much as half of their illuminated hemispheres—in other words, could never exhibit the gibbous phase. Copernicus had predicted that, if human sight could ever be sufficiently enhanced, these two planets would show the same phases as the Moon. Galileo observed Venus in the gibbous phase, and, having learned caution, announced the fact in an anagram which, after he had followed the planet until it took the crescent form, he translated by interchanging the letters. The original sentence read "*Haec immatura a me iam frustra leguntur.* o y" (These unripe things are now read by me in vain.) The translation was "*Cynthiæ figuras æmulatur mater amorum*" (The mother of the loves imitates the form of Cynthia); or, Venus goes through the same phases as the Moon.

In his old age Galileo was tried by an ecclesiastic court composed of men who, no doubt, were pious and conscientious, but to whom the idea of the Earth's not being the immovable center of the universe was so repugnant that they convicted Galileo of heresy, and he escaped severe punishment only by publicly abjuring the belief that he knew to be true. Forty years earlier, Giordano Bruno had been burned alive for similar heresies.

Kepler (1571–1630) and the Laws of Planetary Motion.— The great German mathematical astronomer, Johannes Kepler, was a pupil of Tycho Brahe and held friendly correspondence with Galileo. Impressed with the rationality of the Copernican theory, he concerned himself with numerical questions relating to the planets, such as, Why are there just six planets? What law determines their distances from the Sun? Why do the outer planets move more slowly than the inner? and Why does the speed of any given planet vary?

At one time he thought he had solved the first two questions, and it was this solution which attracted the attention of Tycho and brought Kepler into contact with that great man. It is shown in solid geometry that there are just five possible regular solids: those having four, six, eight, twelve, and twenty sides. Kepler found that, if the heliocentric orbit of Saturn were imagined to lie on the surface of a sphere, and a cube were inscribed in this sphere, the sphere inscribed in the cube would nearly fit the orbit of Jupiter; if a regular tetrahedron were inscribed in the sphere of Jupiter it would about contain the sphere of Mars; and, with fair approximation, a dodecahedron might be inserted between the spheres of Mars and the Earth, an icosahedron between those of the Earth and Venus, and an octahedron between those of Venus and Mercury. Fig. 113, copied from the frontispiece to Vol. I of Kepler's *Collected Works*, illustrates this idea.

These relations, however, were not sufficiently exact to satisfy Kepler, and so, following the ancient Pythagorean notion of the **Music of the Spheres,** he imagined that the pitch of the note sung by a planet might depend upon the planet's velocity. He thus made many attempts to discover the celestial harmonies, but found nothing better than the example shown in Fig. 114.

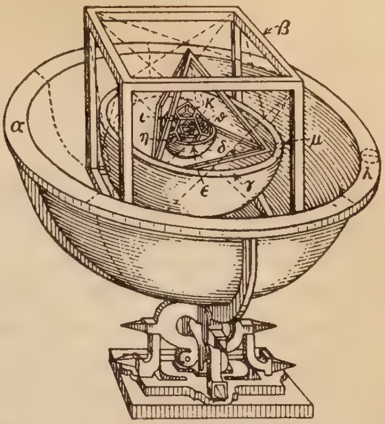

FIG. 113. KEPLER'S THEORY OF THE GEOMETRIC SOLIDS

Finally, after years of incredible labor and many wrong guesses, by using the extensive series of observations of the planets recorded by Tycho, he arrived at the following important conclusions, which are known as Kepler's laws of planetary motion:

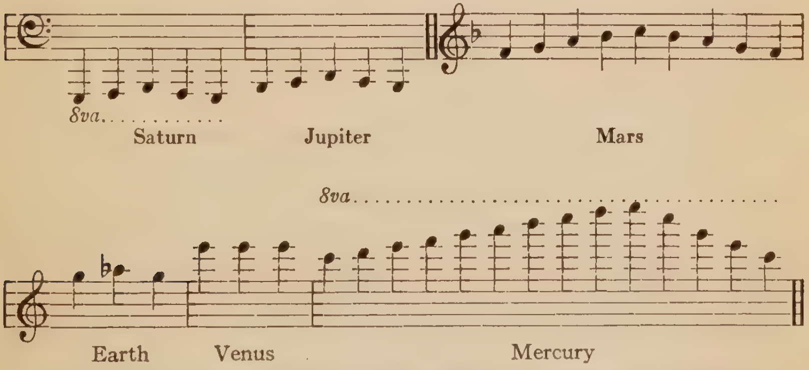
8va. . . . .
Saturn
Jupiter
Mars
8va. . . . .
Earth
Venus
Mercury

FIG. 114. "THE MUSIC OF THE SPHERES" ACCORDING TO KEPLER

I. Each planet moves in an ellipse which has the Sun at one of its foci.

II. The radius vector of each planet passes over equal areas in equal intervals of time. This is known as the law of areas.

III. The cubes of the mean distances of any two planets

from the Sun are to each other as the squares of their periodic times;

$$\text{or, } a_1^3 : a_2^3 :: P_1^2 : P_2^2,$$

where the a's denote the mean distances and the P's the sidereal periods of any two planets.

The third law, which he called the Harmonic Law, especially delighted Kepler, and he wrote of it, "The die is cast, the book is written, to be read either now or by posterity, I care not which; it can await its reader; has not God waited six thousand years for an observer?"

Elements of a Planet's Orbit.—The orbit of a body that revolves around the Sun is most conveniently and accurately described by means of certain numbers known as the **elements** of the orbit. Those most commonly used are the following:

1. The longitude of the ascending node, Ω
2. The inclination to the ecliptic, i
3. The longitude of perihelion, π; or else the "argument of the latitude of perihelion," ω
4. The semi-major axis, a
5. The eccentricity, e
6. The mean heliocentric longitude, L, or the mean anomaly M, of the planet at a specified epoch; or else the time of perihelion passage, T
7. The sidereal period, P, or mean daily motion, μ

The definition of these elements will be assisted by reference to Fig. 115. The plane $EKLI$ represents the plane of the Earth's orbit (or of the ecliptic), and $ORBT$ the plane of the orbit of the planet in question. Their line of intersection NN', which always passes through the center of the Sun, is the **line of nodes.** The planet passes from the south to the north side of the ecliptic at the point n, which is the **ascending node.**

Let the line $S\Upsilon$ be drawn from the Sun toward the position of the vernal equinox on the celestial sphere. The angle between this line and the line of nodes, measured from the vernal equinox toward the ascending node, is Ω, the **longitude of the ascending node.** The angle between the two planes is i, the **inclination.** These two elements define completely the position of the orbit plane.

The orientation of the orbit within its plane may be described by the angle ω, measured in the plane of the orbit and in the direction of the body's motion (eastward in the case of each planet, but often westward in the case of a comet) between SN and SP, P being the **perihelion.** For the principal planets, however, it is customary to substitute for this element the **"longitude of perihelion"** π, which (strictly speaking not a longitude at all) is the sum of the two angles $\Omega + \omega$.

The **semi-major axis,** a, or **mean distance** of the planet from the Sun, defines the size of the orbit. It may be expressed in miles or kilometers, but for most purposes it is better to use the "astronomical unit" of distance, which is the semi-major axis of the Earth's orbit, 92,900,000 miles, or 149,500,000 kilometers.

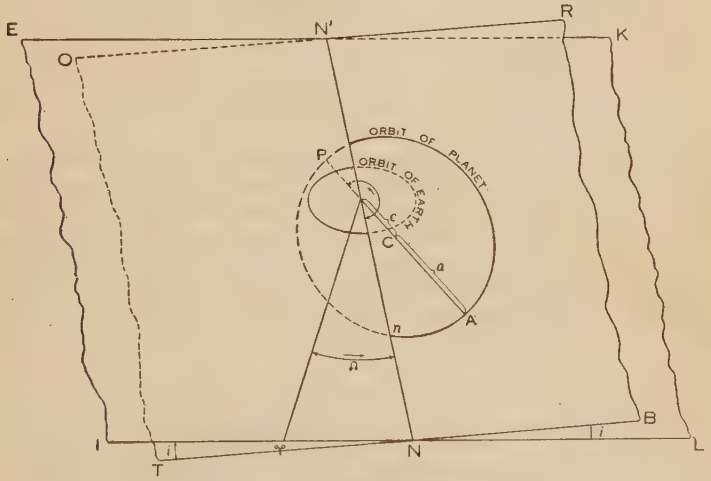

FIG. 115. ELEMENTS OF AN ORBIT

The **eccentricity,** e, defines the shape of the orbit. It is a pure number, defined as the ratio of c, the Sun's distance from the center of the orbit (or half the distance between the foci) to a, the semi-major axis; that is, $e = c/a$.

The above five elements completely describe the orbit itself; but to determine the position of the planet at any time it is necessary to know its position at some specified time, which is given by the sixth element; and also the **time of revolution,**

P, or, what is usually more convenient in computation, the **mean daily motion,** μ, which is simply 360° divided by the number of days in P. In the case of a planet which is so small that its mass may be neglected, this last element is superfluous, since it can be computed from a by Kepler's harmonic law; but where the mass is appreciable as compared to that of the Sun, the harmonic law is not quite exact.

The position of a planet in its orbit is described by two co-ordinates: r, the length of the **radius vector,** or line joining the planet and the Sun; and v, the **true anomaly,** the angle made by the radius vector with the line of apsides, counted from the perihelion in the direction of the planet's motion. In a circular orbit, the calculation of these two co-ordinates is extremely simple, for r is constant and v changes uniformly with the time; but in an elliptic orbit both r and v change in conformity with the law of areas, and the problem of their calculation, known as **Kepler's problem,** is a matter of some difficulty.

Use is made of an imaginary **mean planet** which, coinciding with the true planet at perihelion, moves with a uniform angular velocity equal to the mean angular velocity of the true planet. The planet's **mean anomaly** is the anomaly of this fictitious body, and its **mean longitude** is the "longitude" of the fictitious planet, counted, like the "longitude" of perihelion, in the plane of the ecliptic from the vernal equinox to the ascending node, and in the plane of the orbit from the ascending node to the place of the mean planet.

Kepler's Method of Determining the Form and Size of an Orbit.—Since the discovery of the law of gravitation and the development of modern mathematical methods, all the elements of the orbit of a planet can be very approximately determined, by a computation that in skilled hands requires less than a day, from three observations of the planet's right ascension and declination made on different dates. This method was not available to Kepler, who investigated the planetary orbits and discovered his three great laws by a much more laborious process.

As we have seen, there were available to Kepler the numerous observations of the apparent positions of the planets recorded during many years by Tycho Brahe and his assistants at the great observatory on the island of Hven. The first step was to determine accurately the planet's sidereal period, which could be done in two ways: (1) by observations of the time of conjunction or opposition, and (2) by observations of the time of node passage.

The **sidereal period** is the time of one complete orbital revolution, from a given direction among the stars to the same direction again, as seen from the Sun; the **synodic period** is the time from opposition to opposition or from conjunction to conjunction as seen from the Earth. The synodic periods are variable because of the varying speed of the planets in different parts of their orbits; but if we represent by S the *mean* synodic period of a planet in days, by P its sidereal period, and by E the sidereal period of the Earth (365.25), then

$$1/S = 1/P - 1/E \quad \text{if} \quad E > P, \text{ or}$$
$$1/S = 1/E - 1/P \quad \text{if} \quad E < P;$$

for each side of the equation represents the average daily gain of the Sun upon the planet in their apparent eastward motion, expressed in fractions of a circumference. Oppositions, quadratures, and conjunctions of the planets played an important part in astrology, and hence records of such observations made long before the time of Tycho were available; from these S could be found, and P could then be determined from one of the above equations.

When a planet is at its node, its latitude is zero as seen from either the Earth or the Sun, since it is then exactly in the plane of the ecliptic. One sidereal period later, it will again be at the node and, although its longitude as seen from the Earth will be different because the Earth occupies a different point of its orbit, the planet's latitude will again be zero. Hence, the interval between two successive times when the planet is seen crossing the ecliptic in the same direction is its sidereal period.

Having the sidereal period, Kepler determined the planet's distance from the Sun at different points of its orbit by triangulation. His first thorough investigation was that of the orbit of Mars, the period of which is 687 days. Suppose that Mars is at the point C of its orbit (Fig. 116) when the Earth is at A. After 687 days Mars will be back at C, while the Earth will have made one complete revolution and a large part of another, and be at B. In the triangle ASB the sides SA and SB are radii of the Earth's orbit, and the angle ASB can be found from the interval of time; hence it is possible to compute trigonometrically the side AB and the angles SAB and SBA. Tycho's observations furnished the angles SAC and SBC, the planet's elongations when the Earth was at A and B, respectively; from these, by subtracting the computed angles SAB and SBA, he obtained the angles CAB and ABC; and from these and the side AB he computed the sides AC and BC of the triangle ABC, giving the distances of Mars from the Earth in terms of the Earth's distance from the Sun. Finally, he obtained SC, the radius vector of Mars, from the triangle SAC or CBS, having given two sides and the included angle. From many pairs of observations separated by the interval of 687 days, he was thus enabled to find the distance of Mars from the Sun at many points, and to deduce the size and form of the orbit and the speed with which the planet moved, and so to discover his first two laws. The third law resulted later from a comparison of the distances and periods of different planets.

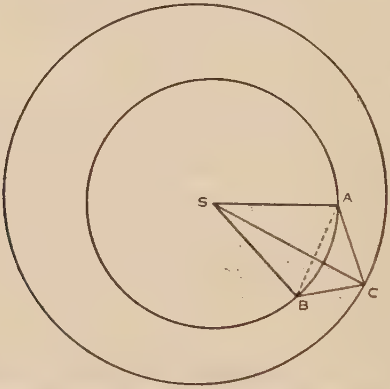

FIG. 116. KEPLER'S METHOD OF DETERMINING THE ORBIT OF A PLANET

Orbital Elements of the Principal Planets.—The elements of the orbits of the principal planets for the year 1925 are given in Table 5. They are subject to slow changes due to perturbations, but the effect of these will be unimportant for many years.

The symbols of the planets given in the second line of the table, which are often encountered in astronomical literature, are mostly of ancient origin

TABLE 5

ORBITAL ELEMENTS OF THE PRINCIPAL PLANETS

Name of Planet	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto[1]
Symbol	☿	♀	⊕	♂	♃	♄	⛢	♆	—
Mean Distance a	0.387	0.723	1.000	1.524	5.203	9.539	19.191	30.071	43
Eccentricity e	0.206	0.007	0.017	0.093	0.048	0.056	0.047	0.009	0.2
Inclination i	7°.0	3°.4		1°.9	1°.3	2°.5	0°.8	1°.8	10°
Long. of Asc. Node ☊	47°.4	76°.0		49°.0	99°.7	113°.0	73°.6	131°.0	?
Long. of Perihelion π	76°.3	130°.5	101°.7	334°.7	13°.1	91°.6	169°.4	44°.0	204°
Mean Hel. Long. of Planet, 1925 Jan. 0.0	105°.7	212°.2	99°.6	39°.0	277°.1	212°.4	350°.8	140°.0	118°
Sidereal Period P	88^{d}	225^{d}	365^{d}	$1^{y}.9$	$11^{y}.9$	$29^{y}.5$	$84^{y}.0$	$164^{y}.8$	282^{y}
Mean Daily Motion μ	4°.09	1°.60	0°.99	0°.52	299″	120″	42″	22″	12″.6

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[1] The ultra-Neptunian planet discovered in 1930. The elements given are those inferred by Lowell from the perturbations of Uranus (page 237) and are very uncertain. Preliminary computations based on observations in 1930 agree fairly well on a present distance of 41 astronomic units, and on values of i and ☊ of 17° and 109° respectively, but are very discordant in regard to the other elements.

and are supposed to be conventionalized pictures of objects associated with the deities for whom the planets are named. The symbol for Mercury represents the Caduceus, a wand with two serpents twined around it, which was carried by the messenger of the gods. Venus, the planet of love and beauty, is symbolized by a hand mirror; Mars, planet of war, by a shield and spear; and Saturn, slowest of the ancient planets, by a sickle, corresponding to the scythe of Father Time. The symbol of Jupiter is perhaps a thunderbolt or the letter *Z*, initial of Zeus. That of the Earth is probably a globe showing the equator and central meridian, and is of more recent origin. The symbol of Uranus is said to represent the heavens (Uranus was god of the sky), and Neptune is represented by his familiar trident. These last two are of course modern.

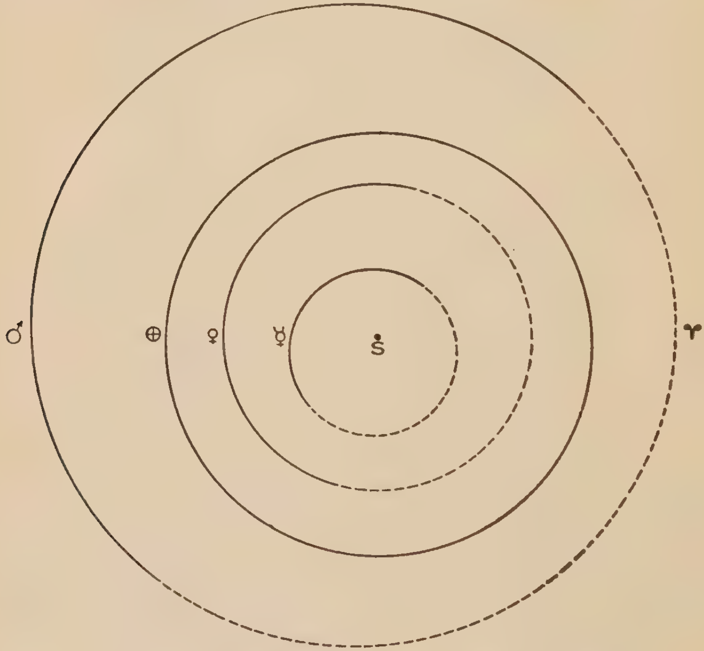

FIG. 117. ORBITS OF THE FOUR INNER PLANETS

Perhaps the most interesting elements are *a* and *P*. The periods range from three months in the case of Mercury to 165 years for Neptune; the age of an earthly octogenarian is less than half of a Neptunian "year." The mean distance of Mercury is some 38,000,000 miles, while that of Neptune is roughly 3,000,000,000. Light, which reaches the Earth from the Sun in 499 seconds, requires more than four hours to make the journey to Neptune. The range of distances is so great that it is impracticable to represent all the planet orbits on a single diagram. The four inner orbits are pictured in Fig. 117, and those of Mars and the four outer planets in Fig. 118.

Bode's Law.—The approximate distances of the planets from the Sun may be conveniently remembered by a relation first pointed out by Titius, but commonly known as Bode's law. If

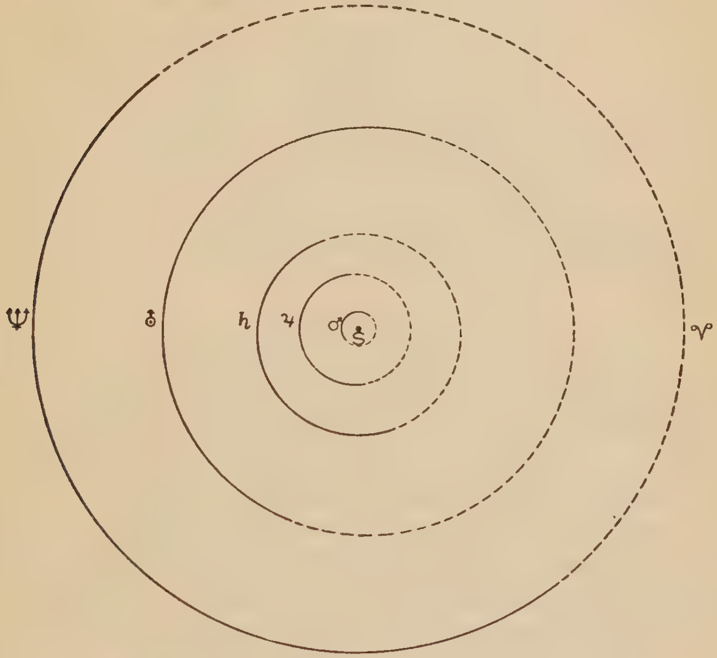

FIG. 118. ORBITS OF THE FIVE OUTER PLANETS

we write a series of 4's and add to them the numbers 0, 3, 3 × 2, 3 × 2 × 2, 3 × 2 × 2 × 2, etc., thus:

4	4	4	4	4	4	4	4	4
0	3	6	12	24	48	96	192	384
4	7	10	16	28	52	100	196	388

we get a series of numbers which are approximately ten times the distances of the planets in astronomic units. Except for the gap between Mars and Jupiter, which is now known to be occupied by most of the asteroids, the relation holds pretty well for all the planets out to Neptune, but there it fails completely. There is no known theoretical reason for the "law," and it may be merely a coincidence.

The Sun as Seen from Different Planets.—As an observer recedes from the Sun, both its apparent diameter and the intensity of its light and heat become less, the apparent diameter being inversely proportional to the first power of the distance and the intensity to the square. As seen from Mercury the Sun's diameter is about two and one-half times as great as it appears from the Earth; as seen from Neptune its diameter is only a little more than a minute of arc, and to the naked eye it would appear as an intensely bright star without perceptible disk.

Taking the intensity of the Sun's light and heat at the Earth's distance as unity, we have the following values of the intensity at the distances of the various planets:

☿	♀	⊕	♂	♃	♄	♅	♆
6.7	1.9	1.00	0.43	0.04	0.01	0.003	0.001

It may well be imagined that there exists a great diversity of climate.

The Earth as Seen from Other Planets.—The Earth, being an opaque body, must shine by reflected sunlight and, to an observer on one of its nearest neighbors, must present much the same appearance as the other planets do to us. The most favorable view would be obtained from Venus. When the Earth and Venus are nearest together, at a distance of some 26,000,000 miles, the latter planet is at inferior conjunction, nearly or (on rare occasions) quite directly between us and the Sun, so that her dark side is turned toward us and at the same time the eye is dazzled by the light of the Sun. For an observer on Venus at the same time, the conditions for viewing the Earth would be the reverse of these: the Earth would be at opposition with its illuminated side turned full upon the observer and would appear as a star about six times as bright as Venus appears to us. The Moon would also be plainly seen as a fainter star passing from side to side of the Earth in the course of a month to a distance of about half a degree.

From Mars the view would be less favorable, since for the Martians the Earth, like Venus and Mercury, would seem to oscillate from side to side of the Sun and would reach a "greatest elongation" of 48°, or about the same as that of Venus as seen from the Earth. When Mars and the Earth are nearest together, the Earth would be at inferior conjunction, and near that time a Martian with a good telescope might see both the Earth and the Moon in the crescent phase.

As seen from Jupiter, the Earth is never more than 12° distant from the Sun, and it is likely that a race of Jovians, if equipped with eyes and telescopes similar to ours, would be unaware of the existence of this little globe. This deplorable state of ignorance is of course still more likely to be true of any possible inhabitants of more distant planets, or their satellites.

EXERCISES

1. Construct a drawing of the complete geocentric path of Jupiter on a scale of one centimeter to the astronomical unit by the method described on pages 196 and 197, assuming for Jupiter a distance from the Sun of five astronomical units and a period of twelve years.

2. What would be the period of a planet having a mean distance from the Sun one-fourth that of the Earth?

Ans. ⅛ of a year or 45.7 days

3. What must be the mean distance of a planet in order that it will revolve around the Sun in 216 years?

Ans. 36 astronomical units

4. If the Earth's mean distance from the Sun were doubled, what would be the value of the constant of aberration?

Ans. 14″.6

5. Calculate the mean synodic periods of Venus and Jupiter from their sidereal periods given in Table 5, assuming 1 year = 365.25 days.

Ans. Venus, 586 days; Jupiter, 399 days

6. What is the phase of Venus at greatest elongation?

Ans. The quarter

7. What is the elongation of Mars as seen from the Earth when the Earth as seen from Mars is in the quarter phase?

Ans. Quadrature

CHAPTER X

THE LAW OF GRAVITATION

Isaac Newton (1643–1727).—Kepler's laws of planetary motion were simply descriptive statements of the behavior of the planets. Neither Kepler nor any one else had given an explanation of the force that causes the planets to move in just this way and no other. It was generally supposed that, to keep a planet moving, some "projectile force," acting along a tangent to the orbit, was required, and Kepler seems to have been somewhat inclined to attribute this force to the will of a supernatural being—perhaps an angel that had the charge of each planet—or to invisible spokes that radiated from the Sun and pushed the planets along. It remained for the great English mathematical philosopher Sir Isaac Newton to show that the planetary motions were but manifestations of a universal principle and to derive the mighty generalization known as the Law of Gravitation.

The contributions of Newton to astronomy, mathematics, and physics are numerous and exceedingly important. Before reaching the age of twenty-four he had discovered the binomial theorem, formed his theory of colors, founded the branch of mathematics which he called "fluxions" and which has grown into the modern calculus, and laid the basis of the law of gravitation. It was not, however, until 1687 that he published his immortal work *Philosophiæ Naturalis Principia Mathematica*, commonly known as Newton's *Principia*, the appearance of which probably marks the greatest forward step ever made in physical science.

Newton's Laws of Motion.—The science of mechanics consists of theorems built upon certain laws or principles just as geometry is built upon its familiar axioms. The motions of bodies as ordinarily observed, whether they be the planets or familiar bodies at the surface of the Earth, may be calculated precisely by the system of mechanics built up by Newton. He used as a basis the axioms of Euclidian geometry together with three principles that are known as Newton's laws of motion because they were first definitely stated by Newton, although they were at least partly understood by Galileo and, before him, by the great artist-inventor Leonardo da Vinci. These laws of motion are:

I. Every body continues in a state of rest, or of uniform motion in a straight line, unless it is compelled to change that state by a force impressed upon it.

II. The rate of change of motion is proportional to the impressed force, and the change takes place in the direction of the straight line in which the force acts.

III. To every action there is an equal and opposite reaction.

Comment on the First Law.—If a block of wood resting on a level floor be given a sufficient impulse, it will slide along the floor, but if then left to itself it will come quickly to a stop because of the friction between it and the floor. If the floor be made very smooth, the block will slide farther before coming to a stop because the friction is less. If the block be placed on smooth ice, a given impulse will carry it farther still. We cannot remove from a body all forces such as friction, gravity, etc., and so cannot completely verify the law experimentally; but experiment shows that, the more nearly we eliminate the forces acting on a body, the more nearly does its motion conform to the law; and Newton assumed that a body moving in free space, unacted upon by any force whatever, would travel forever uniformly in a straight line. Simply to keep moving, then, a planet or any other body needs no force such as the "projectile force" of Aristotle. That which requires an explanation is the curvature of the planet's path and the variation of its speed.

Comment on the Second Law.—By "the rate of change of motion" Newton meant what many later writers call the rate of change of **momentum**, momentum being the product of mass times velocity. The rate of change of motion of a body whose mass remains constant is thus the mass times the rate of change of velocity, or mass times acceleration.

Up to the time of Galileo it had generally been supposed, in agreement with Aristotle, that a ten-pound weight would fall ten times as fast as a one-pound weight. Galileo, by a famous experiment at the Leaning Tower of Pisa, showed that spheres of different masses fell from the tower to the ground in the same time. The reason is that, while the weight, or force of gravity, acting on a ten-pound ball is ten times as great as that acting on a one-pound ball, and it therefore produces in the former ten times the "quantity of motion," the **mass** of the ten-pound ball is also ten times as great and so the acceleration, or rate of change of velocity, is the same.

The fact that all bodies, such, for example, as a bullet and a feather, do not fall a given distance in air in the same time is due to the difference of the resistance of the air caused by their different shape or density. If dropped in a tube from which the air has been exhausted, the feather and the bullet fall in the same time.

In the statement of the second law, nothing is said about the condition of rest or motion of the body at the time the force is applied, nor of other forces that may be acting at the same time. It is therefore implied that, whether the body is at rest or in motion, and whether other forces be acting or not, a given force produces the same change of momentum that it would produce if it alone acted on the body at rest. Thus, if a body at *A* (Fig. 119) be acted upon simultaneously by a force which would cause it to move in one second to *B* and by another which would send it in one second to *C*, it must, at the end of one second, arrive at a point *D* which is at the distance *AB* from *A* as measured in the direction of the first force and at the distance *AC* from *A* as measured in the direction of the second force; that is, *D* is at the opposite corner of a parallelogram of which the lines *AB* and *AC* are adjacent sides. The important principle of the **parallelogram of forces** is thus a corollary to the second law of motion.

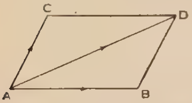

FIG. 119. THE PARALLELOGRAM OF FORCES

Comment on the Third Law.—Suppose a man on a raft that floats freely in still water pulls on a rope attached to a similar raft that is heavily loaded, say with scrap iron. Both rafts will move, and the more lightly loaded raft will move the faster. If there were no friction or other forces except the pull on the rope, the product of the mass of each raft (plus its load) by its rate of change of velocity would be the same—the "action" of the man is met by an equal and opposite "reaction" on the part of the inanimate scrap iron. A little reflection will disclose a similar balance wherever forces are applied.

The first two laws are sufficient for a discussion of the results of applying various forces to one body; the third is needed for studying the motions of a system of bodies.

The Theorem of Areas.—As an example of Newton's methods and the important results to which they led we may prove the following theorem:

To Prove.—If a body move subject to no forces except one that is directed always toward the same fixed point O, the line joining the body to O must pass over equal areas in equal intervals of time.

Proof.—Suppose, first, that the body is subject to no forces whatever, so that, by the first law of motion, it will move uniformly in a straight line. Let the line of its motion be *AC* (Fig. 120) and let it move the distance *AB* in one second. In the next second it will travel an equal distance BC. Let O be any point not in the line AC and draw OA, OB and OC. The triangles OAB and OBC, having equal bases AB and BC and the common altitude OK, are equal in area.

Suppose, however, that upon arriving at B at the end of the first second, the body is acted on by an instantaneous force—like a blow from a hammer, say—acting along the line BO, and that this force is of such magnitude that, had the body been at rest, it would have been sent to D (Fig. 121) in one second. According to Newton's second law, this instantaneous force will produce a change of motion parallel to the line BO, causing the body to be displaced in one second a distance BD in the direction of that line; but it will not cause it to lose its original motion which, acting alone, would carry the body in one second from B to C. The actual path is therefore BE, the diagonal of the parallelogram of which BD and BC are adjacent sides. Draw OC and OE. The triangles OBC and OBE have the common base OB and equal altitudes CL and EM, the distance between the parallel lines BD and CE. Therefore triangle OBE is equal in area to triangle OBC, which we have already shown to be equal to OAB.

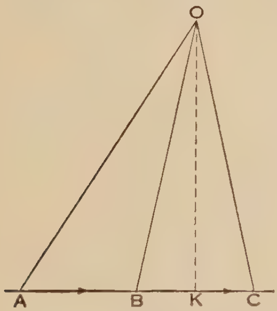

FIG. 120

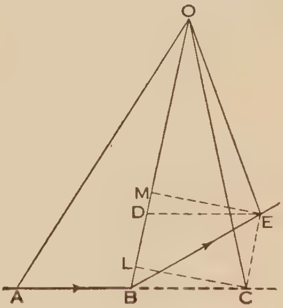

FIG. 121

In the third second, if the body were left to itself, it would according to the first law of motion, move from E to G where $EG = BE$ (Fig. 122); but let it again be acted upon, when at E, by an instantaneous force directed toward O and of such magnitude as to carry it to F had it been at rest at E. It will travel to H, the opposite corner of the parallelogram whose sides are EG and EF. The triangles OEH and OEG are equal in area since they have the common base OE and equal altitudes GP and HQ.

We now have (Fig. 123)

$$\Delta OAB = \Delta OBE = \Delta OEH;$$

that is, if the body be subject to no forces except one which acts instantaneously toward O at the end of each second, the lines joining O to the positions occupied by the body at the instants of action of the force form triangles of equal area. Our proof will hold as well if we suppose the force to act at intervals of half a second, or a millionth of a second, or at intervals that are smaller than any assignable quantity, however small. In the limit, the intermittent, instantaneous forces are replaced by a continuously acting force (still supposed to be always acting along the line joining the body to O), and the path of the body becomes a smooth curve of a form depending on the way in which the force varies. Yet, however small the interval at which the force be supposed to act, the area described in any unit of time, as one second, will be the same as that described in any equal unit since it is the sum of the same number of equal areas; and hence, however the force may vary in intensity, so long as it acts always along the line joining the body to O (it may be directed away from O as well as toward O), that line will pass over equal areas in equal intervals of time. Q. E. D.

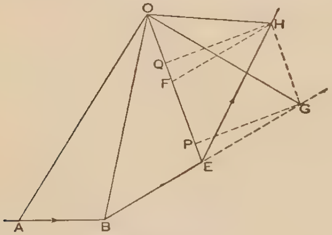

FIG. 122

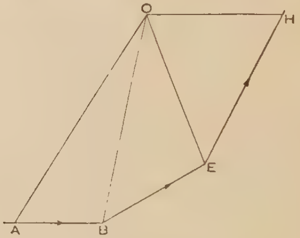

FIG. 123

We have placed no limitations upon the force except that it shall always act along the line joining the body to O. It may be of constant or variable intensity, or it may be intermittent, or alternately attractive and repulsive; it may vary in intensity directly as the distance, or inversely as the square of the distance, or according to any other law, or even capriciously; and the law of areas proved above must still hold.

This theorem illustrates one application—the only one that we shall attempt to give in detail—that Newton made of his laws of motion to the study of mechanics. He proved also that the converse is true: If a body be found moving in such a way that the line joining it to any point passes over equal areas in equal intervals of time, then the only force (or the resultant of all the forces) acting on the body must be directed toward (or away from) the point. Kepler had, as we have seen, already shown that each planet moves in that way in regard to the Sun—*i. e.*, that its radius vector describes equal areas in equal times. The conclusion was therefore obvious that *the only force required to explain the motion of any one of the planets is a force directed toward the Sun.*

Newton's Inferences from Kepler's Laws.—By mathematical processes based upon the laws of motion as in the above example, Newton made an important inference from each of the laws of Kepler. These inferences, including the one just stated, are:

1. From Kepler's law of areas: *The force that controls the motion of each planet is directed toward the Sun.*

2. From Kepler's first law: *The Sun's attraction for a given planet varies inversely as the square of the planet's distance from the Sun.*

3. From the harmonic law: *The Sun's attraction varies from one planet to another inversely as the square of the distance.*

The second statement means that, as a planet approaches perihelion, the intensity of the attraction rapidly increases; if the distance be halved, the attraction is increased fourfold. It depends both on the fact that the orbit is an ellipse and on the fact that the Sun occupies its *focus*; if, for instance, Kepler had found that the Sun was in the center of the planetary ellipses, Newton must have concluded that the attraction varied directly as the first power of the distance instead of inversely as the square.

The third statement means that the attraction for a planet of given mass depends only on the distance and not on any other properties of the planets such as temperature, chemical constitution, etc. If Mars were placed at the Earth's distance from the Sun, the Sun's attraction for it, except for the difference of mass, would be the same that it is in the case of the Earth. Newton showed, however, that for planets of appreciable mass, the harmonic law is only approximately true; the precise relation is

$$a_1^3 : a_2^3 : : P_1^2\,(M + m_1) : P_2^2\,(M + m_2)$$

where M is the mass of the Sun and the subscripts refer to the different planets. In every case in the Solar System, M is so much greater than m (the mass of the Sun is about a thousand times that of even Jupiter, the greatest planet) that Kepler's statement of the law, in which the masses are omitted, is very nearly correct.

The Law of Gravitation.—It is related that Newton was sitting one day in his garden, reflecting upon the force that bends the planets from straight paths and upon the force that holds the Moon in its orbit around the Earth, when his attention was diverted by an apple falling from a near-by tree. It occurred to him that the fall of the apple and the divergence of the Moon and planets from straight lines might be manifestations of the same force. Becoming convinced that this was true, he was eventually led to the Law of Gravitation, which is:

Every particle of matter in the universe attracts every other particle with a force that varies inversely as the square of the distance between them and directly as the product of their masses.

This great law may be expressed very simply in symbols if we let m_1 and m_2 be the masses of any two particles, d the distance between them, F the force of their attraction, and G a constant; thus:

$$F = G\,\frac{m_1 m_2}{d^2}.$$

Newton did not state the law of gravitation in just these words in the *Principia*, nor does it appear that he explicitly extended it beyond the Solar System; but the whole of celestial mechanics, including Newton's own contribution, is in harmony with the law as above stated, and the elliptical motion of double stars shows that the law holds between them as between the Sun and planets.

Test of the Law of Gravitation by the Motion of the Moon.—By means of his principles of "fluxions" Newton was able to prove the difficult proposition that, if the law of gravitation be true, a homogeneous sphere of any size attracts an exterior particle (or sphere) inversely as the square of the distance from its center; that is, as if the whole mass of the sphere were concentrated at its center. He was then in a position to test the law of gravitation by comparing the distance which an apple falls in the first second with the distance that the Moon "falls" toward the Earth, or in other words deviates from a straight line, in the same time.

At a given distance from the Earth the Moon and the apple would, according to the second law of motion, fall at the same speed just as did Galileo's unequal weights at the Leaning Tower of Pisa; but, as Newton knew, the Moon is about sixty times as far from the center of the Earth as is the apple, and hence, if they are both controlled by an attraction that varies inversely as the square of their distances from the center of the Earth, the fall of the apple in the first second should be 60^2, or 3,600 times the Moon's departure from a straight path in an equal time. The apple falls in the first second 16.1 feet, or 193 inches; hence the Moon should depart in one second from a straight line 1/3600 of this distance, or 0.0535 inch.

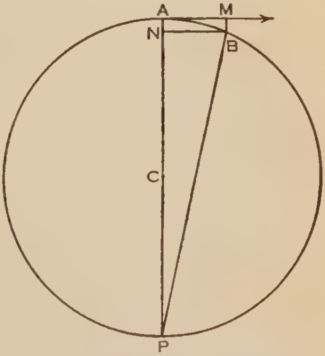

FIG. 124. VERIFICATION OF THE LAW OF GRAVITATION

The actual deviation may be easily determined from a knowledge of the Moon's distance from the Earth and the time of its revolution. In Fig. 124 let the circle, of radius R, represent the orbit of the Moon, and let the Moon move from A to B in one second. Draw the diameter AP and the chord BP, and draw BN at right angles to AP. The fall of the Moon toward the Earth in one second is the distance BM or its equivalent AN. The arc AB is in reality so short that it may be taken as a straight line, being one side of a right triangle whose hypotenuse is AP, and therefore a mean proportional between AN and AP; that is,

$$AN = \frac{\overline{AB}^2}{AP}.$$

But AB, the distance traveled along the orbit in one second, is $2\pi R/T$, where T is the number of seconds in a month; and, as $AP = 2R$,

$$AN = \frac{2\pi^2 R}{T^2}.$$

To compare the fall of the Moon with that of the apple, we must have T in seconds and R in inches. Although Newton knew that R was sixty times the Earth's radius, he at first used an incorrect value of the Earth's radius in inches, and so the computation failed to verify the law. He then reluctantly laid aside that part of his work until six years later, when a new determination of the size of the Earth was made by Picard (page 85). Newton then applied this value and found the Moon's deviation from the tangent to coincide with the fall of the apple within one ten-thousandth of an inch.

In Newton's day modern methods of mathematical analysis were unknown and he developed his theorems by the comparatively cumbrous processes of elementary geometry, aided by his principles of "fluxions." Celestial mechanics was brought to a high state of perfection in the latter half of the eighteenth century by the French mathematicians Lagrange and Laplace by applying to Newton's laws the methods of the calculus.

Detection of Gravitation between Small Bodies.—A number of experiments have been performed whereby the attraction between two bodies of much smaller mass than that of the Moon or planets has been detected and measured. In 1740 Bouguer detected the deviation of the plumb-line caused by the attraction of Mount Chimborazo in South America, and in 1774 Maskelyne made a similar observation of greater accuracy at Mount Schiehallien in Scotland, from which he deduced the mass of the Earth by comparing its attraction with that of the mountain. A more accurate experiment fulfilling the same purpose is that of the **torsion balance**; it was first performed by Lord Cavendish in 1798 and is known as the Cavendish experiment.

Cavendish's torsion balance consisted of a light rod about six feet long which was supported in a horizontal position by a slender wire, and which carried at each end a lead ball about two inches in diameter. Suspended at the same level with these balls and on opposite sides of the rod were two 12-inch balls, also of lead. Any rotation of the rod around the supporting wire could be observed by small telescopes directed to small mirrors attached to the ends of the rod. The whole apparatus was inclosed in a case to prevent disturbance by air currents.

Fig. 125 is a diagram of the apparatus as seen from above. Suppose the normal position of the rod was xx. With the large balls in the positions W_1W_1, their attraction, which was resisted only by the twist of the slender wire, brought the rod into position x_1x_1. The large balls were then turned to W_2W_2, and their attraction brought the rod into the line x_2x_2. The total angle moved through by the rod from position 1 to position 2, which is observed by the telescope, is four times the angle through which it would be moved by the attraction between one small ball and one large one. This angle is proportional to the force of attraction and therefore to the resistance due to the torsion of the wire, and the factor of proportionality can be determined by observing the time of swing when the balance is disturbed slightly from its position of equilibrium and then released, the large balls W being for this purpose removed. The stiffer the wire the more rapid are the vibrations of the balance.

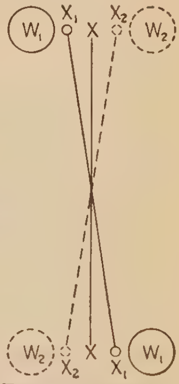

FIG. 125. METHOD OF USING THE TORSION BALANCE.

Cavendish was able with this apparatus to detect and measure the attraction between the balls, but the accuracy of the measurements was impaired by the effect of air currents which could not be entirely eliminated by the inclosing case. The experiment has been repeated a number of times. In 1895 Boys used gold balls 5 mm. in diameter, attracted by lead balls 10 cm. in diameter. The rod of his torsion balance was a mirror 24 mm. long, which was suspended by an exceedingly fine fibre of spun quartz. The gold balls were suspended from the mirror at different levels so that the lead balls might not interfere with one another, and the balance was inclosed in an air-tight case from which the air was partially exhausted. Boys's results are probably the most accurate yet obtained.

Determination of the Constant of Gravitation.—If we let f represent the attraction between the large and the small ball, d the distance between their centers, and B and b their respective masses, we have by the law of gravitation

$$f = G\frac{Bb}{d^2}$$

or, solving for G,

$$G = \frac{fd^2}{Bb}.$$

The masses B and b can be determined by weighing the balls, d can be measured, and f is determined by the experiment; hence the equation gives the numerical value of G, the constant of gravitation, which is believed to have the same value throughout the universe. The value determined by Boys is $6658 \cdot 10^{-11}$ C.G.S. units, which means that two spheres, each having a mass of one gram, if placed with their centers one centimeter apart, attract each other with a force of 6658/100,000,000,000 of a dyne.

The **dyne** is the C.G.S. unit of force, and is defined as the force required to give a mass of one gram an acceleration of one centimeter per second per second. The *weight* of a body at the Earth's surface, which is the force of attraction between the Earth and the body, gives it an acceleration, commonly denoted by g, of 981 cm. (about 32 feet), per second per second; hence, the weight of a body whose mass is one gram is 981 dynes.

The force of gravitation acting between bodies of ordinary size is exceedingly small when compared, for instance, with the attraction between two magnets. An illustration involving larger masses will perhaps make this clearer. Suppose two spheres, of mass 10^{11} grams each (iron balls 100 feet in diameter would have about this mass), placed with their centers one kilometer (10^5 cm., 0.62 mile) apart. No matter what buildings or other obstacles intervene, the two spheres will attract each other, but the attraction will be only 66,580 dynes, a force equal to the weight of a body whose mass is 68 grams, or less than three ounces. Even if all friction and other forces that might affect their motion were eliminated, either ball could be kept from moving by a slight pressure of the finger. If perfectly free from other forces, the balls will begin to approach each other, but so slowly that a microscope will be required to detect the motion. At the end of an hour, each ball will have moved only about 4.4 centimeters, and will then be moving at the rate of a fortieth of a millimeter per second. This speed will be accelerated at a slowly increasing rate, and at the end of eighty-four hours the two balls will come together at a relative velocity of 2.1 cm./sec.

Determination of the Mass of the Earth.—Let E be the mass of the Earth, and w the weight of the small ball used in the Cavendish experiment. The centers of the Earth and the ball are a distance r apart, where r is the radius of the Earth. The weight w is simply the force of the attraction between the ball and the Earth, and is found by multiplying the mass b of the ball by the quantity g; also, it is given by the law of gravitation,

$$w = G\frac{Eb}{r^2}.$$

Solving for E,

$$E = wr^2/Gb,$$

or, substituting the value of G found in the preceding section,

$$E = wr^2B/fd^2.$$

From this equation, expressing, the forces w and f in dynes, the mass B in grams, and the distances r and d in centimeters, it is possible to compute E in grams. The result is $6 \cdot 10^{27}$ grams, or 6×10^{21} metric tons.

The Earth's mass and that of the vastly larger Sun are so great that, despite the smallness of G, the force of their mutual attraction is sufficiently strong to pull asunder a solid steel rod nearly 3,000 miles in diameter; and yet, because of the Earth's great inertia, this force causes the Earth to deviate from a straight line only about one-ninth of an inch while traveling eighteen miles along its orbit.

Calculation of the Superficial Gravity of a Heavenly Body.—Since a sphere attracts an external body as if its mass were all situated at its center, if m represent the mass of the Earth, r its radius, and g its attraction for unit mass at its surface, the law of gravitation gives

$$g = Km/r^2$$

where K is a factor of proportionality. Similarly, if m', r', and g' represent the corresponding quantities for any other sphere,

$$g' = Km'/r'^2.$$

Hence, dividing the second equation by the first,

$$g'/g = m'r^2/mr'^2.$$

The quantity on the left has already been defined as the **superficial gravity** of the sphere (page 122). It may readily be computed by the last equation when the body's mass and radius are known in terms of the Earth's.

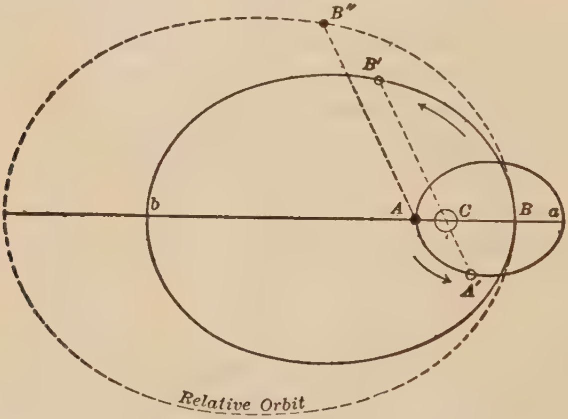

FIG. 126. ORBITS OF SIRIUS AND ITS COMPANION

The Problem of Two Bodies.—The following problem has been completely solved, first by Newton and later by other mathematicians who used modern and more powerful methods.

Given the masses of two particles or spheres which are subject to their mutual gravitational attraction and to no other force; given also their positions and velocities at any moment; to determine the orbits which they will follow and their positions at any other time.

Strictly speaking, the conditions of this problem are never fulfilled in nature, since every particle is attracted by *all* others instead of by a single one; but they are fulfilled approximately in the case of each planet and the Sun, since the attraction of the latter vastly preponderates over that of the other planets and the stars; and probably still more nearly in the cases of some double stars. The details of the solution of the problem cannot be given here, but some of the important results may be mentioned as follows:

1. The position of the center of mass (page 122) of the two bodies is unaffected by their attraction, and it remains at rest or in uniform rectilinear motion.

2. The two bodies will follow orbits around their common center of mass which are similar in form but of size inversely proportional to their masses, the more massive body moving in the smaller orbit. The orbit of each body relative to the other will be a curve similar to its orbit relative to the center of mass, but of a size proportional to the *sum* of the masses of the two bodies.

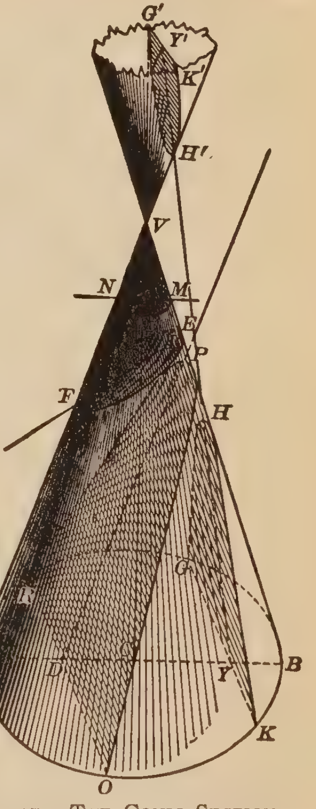

FIG. 127. THE CONIC SECTIONS (FROM YOUNG'S *Manual of Astronomy*)

Observations of the right ascension and declination of Sirius made with meridian circles before 1844 showed that this brightest of stars was moving slowly in a little orbit upon the celestial sphere, and the motion was attributed by Bessel to the attraction of an unseen companion. In 1862, during a test of the new eighteen-inch objective of the Dearborn telescope, the companion was seen by Clark, and it has since been observed in different parts of its orbit which it requires fifty-two years to circumvolve. Fig. 126 shows the orbits of the two stars, the fainter of which is exceeded about four times in mass by the brighter. *C* is the center of mass, and the continuous curves are the absolute orbits of the two bodies, the smaller ellipse belonging to the larger star, *A*. The orbit of the small star relative to the bright one, as determined by micrometer measures of distance and position angle, is shown by the dotted ellipse.

3. The orbits of the two bodies will not necessarily be ellipses, but must in every case be one of the species of curves known as **conic sections,** of which the ellipse is a member; and the center of mass will be at the focus of the conic.

The conic sections are so called because each may be defined as the intersection of a plane with a right circular cone, the different forms resulting from different positions of the cutting plane. A right circular **cone** (Fig. 127) is defined as the surface generated by a straight line which passes always through a fixed point *V* called the **vertex,** while one point of the line travels around a circle whose center *C* is at the point of intersection of a perpendicular drawn from *V* to the plane of the circle. This perpendicular, *VC*, is called the **axis** of the cone. If this surface is cut by a plane passing at right angles to the axis, as at *NM*, the curve of intersection is a **circle** of a size depending on the angle of the cone and the distance of the cutting plane from the vertex. If the plane is gradually inclined from this position the curve of intersection becomes an **ellipse,** as at *FE*; and the eccentricity of the ellipse increases until the plane makes an angle with the axis equal to half the vertical angle of the cone as at *DP*, when the ellipse becomes a **parabola.** If the cutting plane is still further inclined, as at *YHH′Y′*, it cuts the cone above the vertex as well as below, and the curve becomes a **hyperbola.** The circle and parabola are thus seen to be limiting cases of the ellipse. Parabolas as well as circles are all of the same shape, though of different sizes; ellipses and hyperbolas have various shapes as well as sizes. If the cutting plane passes through the vertex of the cone, the circle and ellipse each degenerate to a point, while the parabola narrows to a line and the hyperbola becomes a pair of crossed lines.

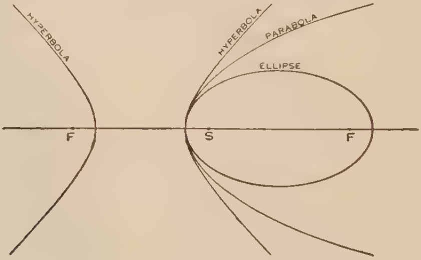

Fig. 128. Confocal Conics

The properties of conics were worked out from the above point of view by Apollonius of Perga about 230 B.C. In modern analytic geometry the curves are treated by means of their equations in rectangular or polar coordinates. The equation of a conic is always a quadratic.

The ellipse is a closed curve, returning into itself as does the circle. The parabola does not return into itself, but extends to infinity. A body moving around the Sun on a parabola, as many comets appear to do, makes but one visit to the Sun in all eternity. The parabola may be regarded as an ellipse with its second focus infinitely removed from the first; the distance between its foci is therefore infinite and so is its major axis, and its eccentricity is exactly 1. The hyperbola consists of two infinite branches. The second focus is behind the first, and the eccentricity is greater than 1. In the case of an attractive force varying inversely as the square of the distance, the body will move on the branch of the hyperbola that is concave to the center of the attraction; if the force were repulsive, it would move on the convex branch. The ellipse is the locus of points the *sum* of whose distances from the foci is constant; the hyperbola is the locus of points the *difference* of whose distances from the foci is constant. In Fig. 128 are shown an ellipse, a parabola, and a hyperbola which have a common focus at S.

Speed in the Relative Orbit.—In the solution of the two-body problem it is shown that the speed of one of the bodies in its orbit around the other is always the same at a given distance r, irrespective of the direction of one body from the other, and is given by the formula

$$V^2 = k^2 (m_1 + m_2) \left(\frac{2}{r} - \frac{1}{a}\right)$$

where a is the semi-major axis of the orbit and k a constant. For a given pair of bodies, m_1 and m_2 are also constant, and we may write

$$V^2 = \mu \left(\frac{2}{r} - \frac{1}{a}\right).$$

If the orbit be circular, $a = r$ and $V^2 = \mu/r$; if it be parabolic, $a = \infty$ and $V^2 = 2\mu/r$. The velocity of a body in a parabola at a given distance from the Sun is therefore $\sqrt{2}$ times that of a body moving in a circle at the same distance. Meteors are observed to enter the Earth's atmosphere at an average speed of about twenty-six miles a second, which is $\sqrt{2}$ times eighteen miles per second, the orbital speed of the Earth. It is therefore inferred that these bodies travel on orbits that are either parabolas or very long ellipses.

Representing the velocity $\sqrt{2\mu/r}$, which is called the **parabolic velocity,** by U, the equation for the velocity in an orbit of semi-major axis a becomes

$$V^2 = U^2 - \mu/a,$$

from which

$$a = \frac{\mu}{U^2 - V^2}.$$

Therefore, if a planet were launched in an orbit at a given distance r from the Sun, the length of the major axis of the orbit would be the greater the greater the speed of projection, for an increase in V decreases the denominator and hence increases a. If $V = U$, the orbit is of course parabolic; if $V < U$, it is elliptic, and if $V > U$ it is hyperbolic. Fig. 129 shows a number of orbits described by planets projected from the same point in the same direction, but with different speeds. If projected in different directions with the same speed, the bodies would move in orbits of the same length but of different eccentricities and lines of apsides, as shown in Fig. 130.

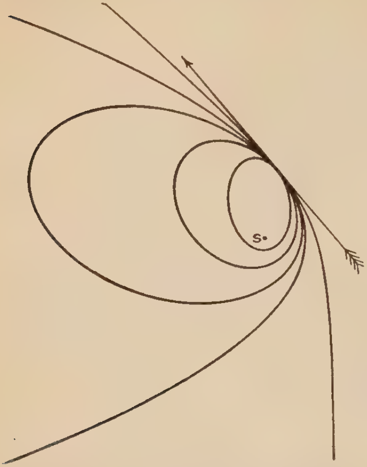

FIG. 129. ORBITS OF BODIES PROJECTED IN THE SAME DIRECTION WITH DIFFERENT SPEEDS

Projectiles Near the Earth; Escape of Atmospheres.—When a ball is tossed into the air, it forms with the Earth a two-body system, and, if there were no friction, the ball would move along an ellipse having the center of the Earth as its distant focus and the highest point to which it is thrown as its apogee. Since the ball is stopped by the Earth's surface after describing only a short arc, the orbit is sensibly a parabola. If the speed of projection is increased, the major axis of the orbit is also increased, and the ball rises higher. The greatest speed yet attained by a projectile from a gun (the German "Big Bertha") is less than a mile a second, and the greatest

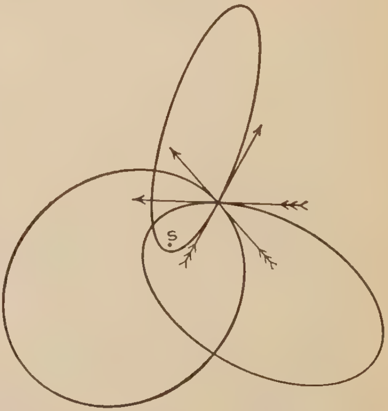

FIG. 130. ORBITS OF BODIES PROJECTED WITH THE SAME SPEED IN DIFFERENT DIRECTIONS

height of projection about twenty-four miles—very small in comparison with the dimensions of the Earth. If the speed were increased until it equaled the parabolic velocity due to the Earth's attraction, the projectile would move on a parabola and depart an infinite distance from the Earth, never to return. The parabolic velocity at the surface of a planet is therefore sometimes called the **velocity of escape.** It is equal to $\sqrt{2\mu/\rho}$ where ρ is the radius of the planet and μ is to be computed from its mass. The velocities of escape for various bodies of the Solar System are given in Table 6.

TABLE 6

(From Moulton's *Celestial Mechanics*, p. 48)

Body	Velocity of Escape						
Earth	11,180	meters,	or	6.95	miles,	per	sec.
Moon	2,396	"	"	1.49	"	"	"
Sun	618,200	"	"	384.30	"	"	"
Mercury	3,196	"	"	1.99	"	"	"
Venus	10,475	"	"	6.51	"	"	"
Mars	5,180	"	"	3.22	"	"	"
Jupiter	61,120	"	"	38.04	"	"	"
Saturn	37,850	"	"	23.53	"	"	"
Uranus	23,160	"	"	14.40	"	"	"
Neptune	20,830	"	"	12.95	"	"	"

We are assured by physicists that any body of gas, such as the atmosphere of a planet, consists of molecules which are in rapid motion, each molecule moving in a straight line until it collides with another molecule or with the wall of the containing vessel. The mean velocity of the molecules depends on the temperature and pressure of the gas, but that of any individual may be indefinitely increased or lessened by collisions with others. A molecule near the upper limit of an atmosphere may thus attain a velocity exceeding the velocity of escape, and it is possible that the absence of an atmosphere on the Moon, and on other bodies for which the velocity of escape is small, may be explained on this basis.

Determination of the Mass of a Heavenly Body.—In the mathematical treatment of the problem of two bodies, it is shown that

$$m_1 + m_2 = a^3/P^2,$$

where the sum of the masses $m_1 + m_2$ is expressed in terms of the Sun's mass, the mean distance a in astronomic units, and the period P in years. This formula gives at once the combined masses of a planet and its satellite, or of the components of a double star, when the period and mean distance have been determined by observation. As the planets Mercury and Venus have no known satellites, their masses can be determined only by their attractions for other planets or for occasional comets that pass near them, and are not very accurately known.

The mass of the Sun is found in terms of that of the Earth-Moon system by a simple application of Kepler's third law as modified by Newton; for, if M be the sum of the masses of the Earth and Sun, m of those of the Earth and Moon, A and a the semi-major axes of the orbits of the Earth and Moon, and P and p the lengths of the sidereal year and the sidereal month, then

$$M : m : : \frac{A^3}{P^2} : \frac{a^3}{p^2}.$$

The Problem of Three Bodies.—The problem of the motion of three mutually attracting bodies is just as determinate as that of two—that is, a given set of initial conditions leads as certainly to a definite result; but the problem is of such difficulty and complexity that only in special cases is even the most powerful of modern analysis capable of producing formulae by which this result can be computed. The simplest two of these special cases were solved by Lagrange; in one, the three bodies remain always in a rotating straight line; and in the other they remain at the vertices of an equilateral triangle.

If an infinitesimal body were launched with the proper velocity at a point about a million miles from the Earth on the side directly opposite the Sun, and then not disturbed by any outside force, it would remain in that relative position and, like the Earth, revolve around the Sun in a period of one year. The motion, however, would be *unstable*, and if the body were disturbed ever so slightly—and it certainly would be disturbed by the attraction of the other planets and the Moon—it would depart from the position indefinitely. The dim light known as the Gegenschein (page 262) may, as suggested by Moulton, be due to sunlight reflected from tiny bodies which, moving through interplanetary space, are caught temporarily at the antisolar point by a sort of "dynamic whirlpool."

The equilateral-triangle solution is *stable*, and examples are found in the motion of certain of the asteroids known as the Trojan group (page 262). Each of these asteroids revolves at the mean distance of Jupiter from the Sun and at the same time oscillates slowly about a point equally distant from the Sun and Jupiter.

Perturbations.—The practical problem of calculating the positions of bodies in the Solar System is the problem of n bodies, and would be entirely unmanageable were it not for the fortunate circumstance that one of the bodies, the Sun, so far exceeds the others in mass that, in a first approximation to the orbit of any one of the planets, the attraction of the others may be neglected. This first approximation, which, as we have seen, is an ellipse, may then be corrected by computing the effects of the attractions of the other planets at as many points of the orbit as desired. The deviations of the planets from exact elliptic motion are called **perturbations.**

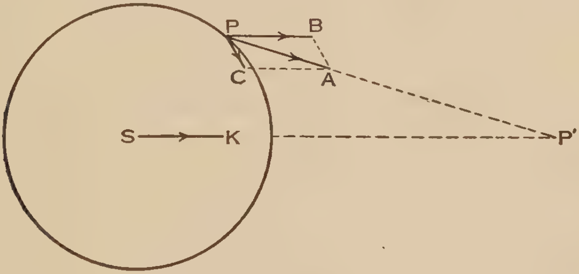

FIG. 131. PERTURBATIONS

The perturbing effect of a third body upon the relative motion of two depends upon the *difference* of the acceleration produced in the two bodies by its attraction—difference in direction as well as in amount; for, if the third body moved both the others in the same direction and at the same rate, it could produce no change in their *relative* position. To make this clear, let S, Fig. 131, represent the Sun and P a planet revolving around it, and let a third body, say another planet, be at P'. P' will attract both S and P, producing in them accelerations inversely proportional to the squares of their distances. Let the lines PA and SK represent, in magnitude and direction, the acceleration produced by P' in P and S, respectively. Both will be directed toward P', and PA will be the greater. The acceleration PA is, by the second law of motion, equivalent to two accelerations that may be represented by the sides of a parallelogram of which PA is the diagonal, and we are at liberty to choose the length and direction of one of these sides. Let it be PB, equal and parallel to SK. If PB alone acted on P, it would change the relative position of P and S not at all, for the two bodies would then be pulled in the same direction and by the same amount; hence, the *perturbative* action of P' upon P is represented by PC, the other side of the parallelogram; and this component is what is meant by the *difference* of the accelerations produced by P' in P and S.

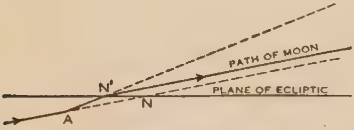

FIG. 132. THE REGRESSION OF THE MOON'S NODES

As may be inferred from the above discussion, it is not a difficult problem to compute the perturbations produced by a third body in a given relative position of the three bodies; but, as the bodies move in their orbits, the perturbations continually change in both direction and amount, and to determine accurately the path of P they must be found for *every* position. What is worse, P' is perturbed by the attraction of P so that it also departs from an elliptic orbit, and these perturbations of P' produce perturbations of the second order in P, and so on *ad infinitum*. Since, however, the perturbations of the planets are small because of the small masses and great distances of the perturbing bodies, their positions may, over long periods of time, be computed as accurately as they can be observed.

The Regression of the Moon's Nodes: Precession.—The regression of the Moon's nodes (page 118) is a perturbation of the Moon's motion produced by the Sun. It has just been shown that the perturbative action of P' on the motion of P is directed toward a point of the line SP', and it may be shown that this is the case whatever the position of P on its orbit around S (cf. page 240 and Fig. 133). In the problem we are now considering, S is the Earth, P the Moon, and P' the Sun, and the perturbative action of the Sun is always directed to the plane of the ecliptic. Consider the Moon's motion as it approaches the node, as in Fig. 132 at *A*. If unperturbed, it would cross the plane of the ecliptic at *N*, which would thus be the ascending node; but the Sun's perturbative action causes it to move more nearly at right angles to the ecliptic and so to cross at *N′*, farther west, and the node is thus moved backward. After node passage, the action is again toward the ecliptic and the inclination is restored after undergoing a slight temporary disturbance.

Precession may similarly be explained by the perturbative action of the Moon (and Sun) on the protuberant matter at the Earth's equator. If the Earth were a homogeneous sphere there would be no precession. Suppose it were such a sphere except for a single mountain at its equator. The perturbative action of the Moon upon this mountain would be directed toward the plane of the Moon's orbit, and as the Earth's rotation brought the mountain across this plane the crossing point (the equinox) would be moved backward as in the regression of the Moon's nodes. Instead of a single mountain, the spherical core of the Earth is surrounded by a ring of matter constituting the equatorial bulge, and every particle of this ring is affected in the way just described. Since the particles are fastened together to form the rigid Earth, this means a precessional motion such as was described in Chapter IV.

Perturbations of the Earth.—The perturbations of a planet are conveniently classed as **periodic** and **secular.** The former set the planet forward or back or sidewise by slight amounts and run through their changes in a century or less. The secular perturbations are more conveniently regarded as changes in the orbit itself, and operate in one direction for thousands or millions of years, or indefinitely. The periodic perturbations of the Earth never exceed about 1′ as seen from the Sun—about 30,000 miles. Its principal secular perturbations are as follows:

1. The line of apsides is revolving eastward at a rate that would, if continued, carry it entirely around in about 108,000 years; but it will not continue always at the same rate.

2. The eccentricity of the orbit, which is now 0.016, is diminishing and will continue to do so for some 24,000 years, when it will be about 0.003; then it will increase for some 40,000 years, but will never exceed about 0.07.

3. The plane of the orbit is slowly changing its position, resulting in a change of the obliquity of the ecliptic. The value of the obliquity is now 23° 27′, and is diminishing at the rate of 0″.5 a year. This decrease will continue about 15,000 years, after which the obliquity will increase. It oscillates in this manner about 1°.5 on either side of the mean.

Perturbations of the Moon and Planets.—The perturbations of the Moon, produced principally by the powerful attraction of the Sun, are much greater than those of the Earth; and the **lunar theory,** or mathematical treatment of the Moon's motion, forms an important branch of astronomy in itself. The complexity of the subject is illustrated by an equation which alone covers no less than 250 quarto pages in Delaunay's *Théorie du Mouvement de la Lune.* This work, published in 1860–1867, is now superseded by the monumental *Tables of the Motion of the Moon*, published in 1919 by Brown of Yale and used by the national almanacs in the computation of the Moon's hourly ephemeris.

We have already noted two of the most important of these lunar perturbations, the regression of the nodes and the advance of the line of apsides. Another famous one is the **evection,** which was known in the time of Hipparchus and is so large that it may set the Moon forward or back about a degree and a quarter. This is a periodic perturbation with a period of one and one-eighth years, the time required for the Earth to revolve from the direction of the Moon's perigee to the same direction again. Other important periodic perturbations are the **variation,** the **annual equation,** and the **parallactic inequality.**

The diminution of the eccentricity of the Earth's orbit, noted above, results in a slight increase of the orbital speed of the Moon known as the **secular acceleration of the Moon's mean motion.** The theoretical amount of this perturbation, according to Brown, is about 6″ a century, but Fotheringham, from a comparison of ancient and modern eclipses, finds a value of about 10″ a century. The discrepancy is believed to be due, not to a real change of the Moon's motion, but to a gradual slowing of our fundamental timepiece, the Earth, produced largely by the friction of the tides. There are still other minute discordances between theory and observation, which Brown attributes to an alternate but irregular expansion and contraction of the Earth, amounting to only a few feet and quite imperceptible by direct observation, but sufficient to produce a slight irregularity in the Earth's rotation.

The most famous of planetary perturbations is perhaps the "long inequality" of Jupiter and Saturn. Five of Jupiter's periods are nearly equal to two of Saturn's, so that these planets are repeatedly brought to their nearest approach in nearly the same part of their orbits and their mutual perturbations repeated. Since, however, the commensurability of their periods is not exact, the perturbation is periodic, having a period of about 900 years.

The Discovery of Neptune.—One of the most dramatic events in the history of astronomy was the discovery of the planet Neptune. Uranus was discovered in 1781 and its orbit was computed from observations made during the next few years; but it refused to follow the orbit computed for it, even when the perturbations of all the known planets were taken into account. By 1845 the difference between computation and observation amounted to the "intolerable quantity" of nearly two minutes of arc—almost enough to perceive without a telescope—and it had by that time long been suspected that Uranus was perturbed by an unknown planet. Two young mathematicians, Adams in England and Leverrier in France, undertook the difficult and laborious task of locating the disturber by solving the problem of perturbations inversely. Each assumed a distance from the Sun of about 38 astronomic units, in accordance with Bode's law, and this turned out to be incorrect; but the orbits calculated by them were sufficiently in accordance with the true path of the new planet to determine its direction from the Earth, and that was all that was needed to find it in the sky. Adams secured his result first, but was unable to awaken much interest among English astronomers, and Neptune was first recognized in 1846 by Galle, a German astronomer to whom Leverrier had communicated his result, and who found the planet within half an hour, less than a degree from the point that Leverrier had indicated.

Discovery of a Trans-Neptunian Planet in 1930.—The success of Adams and Leverrier naturally led to efforts to extend the boundaries of the known Solar System still farther. Even after the perturbations of Neptune were taken into account, observations of the position of Uranus did not agree perfectly with theory, although the discrepancies have never exceeded 4″.5, a very small quantity compared with the "intolerable" 2′ of 1845. Neptune itself could not well be used as a guide-post to a farther planet, because even today it has traversed scarcely half its orbit since its discovery, and the available observations do not go far enough back. The problem was attacked by a number of investigators, of whom the most determined and thorough was Percival Lowell. Before 1905 he had made estimates of the probable position of the trans-Neptunian planet, and in that year he inaugurated a photographic search by his assistants at the Lowell Observatory. In 1915, a year before his death, he published the results of mathematical work on the subject which, with the help of a corps of computers, he had been conducting for several years. His investigation disclosed two possible solutions which placed the planet in either of two opposite regions of the sky, with considerable uncertainty as to its exact location in either region. As Lowell said, "Analytics thought to promise the precision of a rifle and finds it must rely upon the promiscuity of a shot gun after all." His results "indicated for the unknown a mass between Neptune's and the Earth's; a visibility of 12–13 magnitude; and a disk of more than 1″ in diameter."

It soon became evident that no distant planet as bright as this was in either of the regions indicated by Lowell's analysis, but the search was continued with improved photographic means by members of the Lowell Observatory staff, and on January 21, 1930, in one of the two predicted regions, an object was found by C. W. Tombaugh which in apparent path and rate of motion conformed to Lowell's predictions of the trans-Neptunian body. No visible disk has been reported, and the new planet is unexpectedly faint (fifteenth magnitude); facts which are sufficient to account for the difficulty of its detection. At the present time of writing the elements of the orbit have not been computed, nor indeed can the orbit of so distant a planet be determined with any degree of accuracy until after it has been observed for several months or even for years.

The Theory of Relativity.—It is the great triumph of the law of gravitation that it explains not only the regularities of the motions of the planets, as described by Kepler's laws, but also their minute and irregular departures from elliptic motion, so that calculation is abundantly confirmed by observation. In only one case is an exception to this known, and that is the case of the line of apsides of Mercury, which advances at the rate of 574″ per century, 43″ faster than it should according to the law of gravitation and the positions of known planets. On this account it was at one time proposed to amend the Newtonian law by substituting the exponent 2.00000016 for 2; but this would introduce perturbations in the other planets for which we have no observational evidence, notably in the motion of the line of apsides of Venus.

What many regard as the greatest advance in physical theory since Newton is the **Theory of Relativity,** developed principally by the German physicist Einstein in papers published in 1905 and 1915. Many unsuccessful attempts had been made to detect the Earth's motion relative to the luminiferous ether. For example, it had been reasonably expected that the velocity of light would appear to be slightly different when measured along and across the line of the Earth's orbital motion; but many experiments known to be capable of detecting the theoretical difference, notably a famous experiment first performed in 1887 by Michelson and Morley, had all failed. The only possible explanation seemed to be that the measuring apparatus became shorter when placed in the line of the Earth's motion than when placed at right angles to this position—an effect known as the FitzGerald contraction. The principle of relativity goes much farther than this, and assumes that mass, length, and time are all relative, their value depending on the speed with which the observer is moving, but changing very little for velocities that are small compared with the velocity of light—which even planetary velocities are. Newton's law is expressed in terms of mass, distance, and time (time being included in the conception of force), and hence, in the light of the principle of relativity, it is ambiguous. Einstein derived a new law of gravitation (which cannot be stated in simple terms, but which, for the bodies of the Solar System, differs but minutely from the Newtonian law), and showed that, if it be true, the orbit of a planet at Mercury's distance must revolve 43″ per century even if unperturbed.

Most surprising results flow from the theory of relativity. It appears that the universe is four-dimensional, its dimensions being length, breadth, thickness, and *time*; and this "space-time" is curved in a fifth dimension, the curvature being greatest in the neighborhood of bodies of greatest mass. Although the human mind cannot picture such a state of affairs, it can treat it adequately by mathematical methods, and the theory has, as far as it can be tested, received striking confirmation. Einstein showed that, according to the principle of relativity, a ray of light which passes near a body of the mass and dimensions of the Sun should be deviated by 1″.74. At the total eclipse that was visible in Africa and Brazil in 1919, an unusually fine opportunity existed for testing this prediction, for the Sun at the time was among the bright stars of the Hyades, so that their light had to pass near the Sun to reach the Earth. These stars were photographed during the eclipse by Eddington and Davidson, who later photographed the same star field with the same instruments when the Sun was in another part of the sky. A comparison of the plates revealed the fact that, during the eclipse, the stars were apparently shifted in a direction and by an amount to fulfill very nearly Einstein's prediction. This result was abundantly confirmed at the Australian eclipse of 1922 by Campbell.

The relativity theory is further confirmed by the fine structure of the lines in the spectra of the elements, which under very high dispersion are found to consist of groups of lines. This is explained by relativists by a revolution of the orbits of the electrons within the atom somewhat analogous to the motion of Mercury's line of apsides. Another predicted effect is a slight redward shift in the spectral lines of a massive, luminous body like the Sun. Many of the Fraunhofer lines certainly are so shifted, but the Einstein effect is so complicated with effects of pressure, radial velocity, etc., that observers are not yet agreed as to its detection. (See also pages 192, 361, and 419.)

Elementary Theory of the Tides.—The cause of the periodic rise and fall of the water of the ocean, known as the **tides,** may be regarded as a case of perturbations due to the action of the Moon and the Sun. The connection of the tides with the Moon is evident from the fact that the average interval between successive high waters is $12^h\ 25^m$, which is exactly half the average interval between successive upper culminations of the Moon. Their connection with the Sun is shown by the occurrence of spring tides, the highest tides of the month, near the time when the Moon is in syzygy—that is, in the line joining the Earth and the Sun.

In Fig. 133, let the center of the Earth be at C and that of the Moon at M, and consider the perturbative action of the Moon on drops of water situated at A and B. The tide problem can be considered in the same way as the problem of the perturbations of a planet, which was treated in Fig. 132. Since the lithosphere of the Earth is rigid,[1] it moves as a single body and the Moon's effect on it is the same as if its mass were concentrated at C; but each drop of the ocean being free to move independently, the drops at A and B may be moved relatively to the lithosphere and to the other drops. Let CK represent the acceleration produced by the Moon's attraction in the solid Earth, and let AS and BT be the accelerations produced in A and B. Since A is nearer the Moon than is C, while B is farther away, AS will be longer than CK and BT will be shorter. Resolve AS and BT as was done with the planet's attraction in Fig. 132, taking in each case one side of the parallelogram equal and parallel to CK. The other side, AR or BQ, will represent the Moon's perturbing acceleration on the drop of water, or what is called the **tide-raising accelera-**

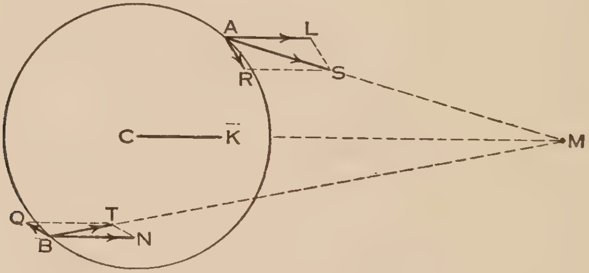

FIG. 133. TIDE-RAISING FORCES AT OPPOSITE POINTS

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[1] It is not absolutely rigid, and the Moon creates in it minute "Earth-tides" which have been detected and measured by Michelson.

tion. It is important to note that, since the diagonal of the lower parallelogram is shorter than its horizontal side, the other side is here directed toward the left—that is, the tide-raising force is directed away from the Moon. The tide-raising acceleration in different parts of the Earth's circumference is shown by the arrows in Fig. 134, by which it is seen that the tidal forces of the Moon tend to heap the water up in two places on the Earth's surface situated at opposite ends of a diameter—one directly under the Moon and the other on the opposite side of the Earth. In fact, the tendency of the Moon's attraction is to deform the spherical Earth into a prolate spheroid with its longest axis directed toward the Moon.

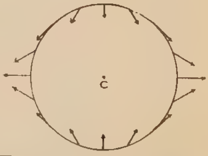


Fig. 134. The Two Tidal Bulges

The maximum tide-raising acceleration, which is effective just under the Moon, is the difference of the accelerations produced by the Moon at the distances, respectively, of the center and the surface of the Earth, and varies very nearly as the inverse *cube* of the Moon's distance. To prove this, let the Moon's distance from the Earth's center be D and the radius of the Earth r. Then the two accelerations just mentioned are proportional to $1/D^2$ and $\frac{1}{(D-r)^2}$ and therefore their difference is proportional to $\frac{1}{(D-r)^2} - \frac{1}{D^2}$ or to $\frac{D^2-(D-r)^2}{D^2\,(D-r)^2}$. This last expression is equal to $\frac{2Dr-r^2}{D^2\,(D-r)^2}$, or, since r is small compared with D, nearly to $\frac{2Dr}{D^4}$, or to $\frac{\text{constant}}{D^3}$. Since the Moon is some 400 times nearer to us than is the Sun, its tide-raising force is 400^3, or 64,000,000, times as great as the tide-raising force of an equal mass of the Sun; and, as the Sun is only about 27,000,000 times as massive as the Moon, the latter has about 64/27, or more than twice, the tide-raising effect of the former. The Moon's tide-raising acceleration at the point where the Moon is in the zenith is about 1/8,500,000 of g, the acceleration due to gravity.

When the Moon is in syzygy, its tidal bulges are added to those of the Sun, and we have **spring** tides; when it is at quadrature, the solar tides occur between the lunar ones, resulting in a smaller difference between high and low water (**neap** tide).

Suppose that the Earth consisted of a spherical, rigid lithosphere entirely covered by a frictionless fluid, each particle of which yielded immediately to the tide-raising force of the Moon. This covering would take the form of a prolate spheroid, the longest axis of which would pass always through the Moon's center as suggested above. Let the lithosphere rotate every twenty-four hours about an axis as in Fig. 135. A point *O* on its surface will be brought successively under the two tidal bulges and so will experience high water twice in each interval between upper culminations of the Moon—that is, one in every $12^h\ 25^m$. In general, the heights of the two daily tides will be unequal, but twice a month, when the Moon is in the plane of the equator, this **diurnal inequality** will vanish.

In the actual case, the water of the ocean is not frictionless, and so the tidal bulge is partly carried forward by the Earth's rotation. Moreover, when a protuberance is once formed it has a tendency to travel as a wave with a velocity which depends upon the depth, and the progress of the tidal wave around the Earth is vastly complicated by the varying depth of the sea, the presence of continents and islands, and the irregularity of coast lines. While in most localities the two daily tides and their diurnal inequality are recognizable, as are also the spring and neap tides, the actual prediction of the time and height of high water cannot be made from astronomic data alone, but depends also upon local observations of the tides.

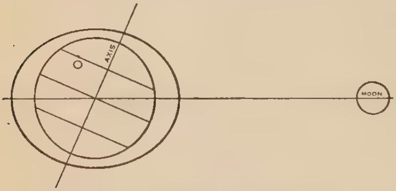


FIG. 135. THE SEMI-DIURNAL TIDE

Effect of the Tides on the Rotation of the Earth and of the Moon.—The friction of the tides, slight though it is in comparison with the momentum of the Earth, must act as a brake upon the Earth's rotation, and, continuing through millions of years, must inevitably tend to lengthen the day.

This tendency is reinforced by the slow deposit of meteoric matter upon the Earth, and opposed by the shrinkage of the Earth due to the radiation of its internal heat; but both these effects are very slight indeed. A comparison of the recorded times of ancient eclipses with their times computed from modern observations of the motions of the Earth and Moon indicate that the day actually is lengthening at a rate of about one second in 7,000,000 years. It is believed that most of this change is due to the friction of the tides in shallow seas such as Bering Sea, the friction in mid-ocean being practically negligible.

Unless the Moon is perfectly rigid, the Earth must create tidal bulges upon it, and the tide-raising effect of the Earth must be greater than that of the Moon because of the Earth's greater mass. If in the past the Moon was in a liquid condition, and rotated more rapidly than it revolved, the friction of the Earth's tides upon it must have tended to lengthen its rotation period, and this tendency must have persisted until the Moon presented always the same face to the Earth as it does now. It is, in fact, believed that the Moon's slight departure from perfect sphericity, which results in the minute physical libration (page 127), is due to the tidal protuberances having become solid and permanent.

Effect of the Tides on the Revolution of the Moon.—The tides raised upon the Earth must affect not only the rotation of the Earth, but also the revolution of the Moon. Consider Fig. 136, in which the lithosphere of the Earth is represented as a sphere rotating within a hydrosphere which completely covers it, its equator and also the center of the Moon being in the plane of the paper. The tidal friction of the Moon creates bulges in the hydrosphere which are carried forward by the Earth's rotation. As soon as their deepest points are no longer in the line of centers of the Earth and Moon, a component of the Moon's attraction will pull back upon them toward that line, as may be shown by the reasoning that was applied to Figs. 132 and 133. This force will resist the tendency of the Earth's rotation to carry the bulges forward, and they will therefore be carried only to certain points *A* and *B*, where they will be in equilibrium between the frictional forward pull of the Earth and the gravitational backward pull of the Moon; and they will remain just so far in advance of the Moon while the body of the Earth turns under them and, as pointed out in the last section, their friction acts as a brake on the Earth's rotation. (This does not mean that the separate particles of water would remain stationary with respect to the Moon; each drop would be carried along by the Earth's rotation, but

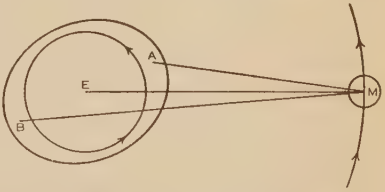


Fig. 136. Effect of the Tides on the Revolution of the Moon

would rise gently to the crest of a wave at *A* and at *B* and would sink to a trough at the points midway between.)

The protuberance whose summit is at *A* will attract the Moon approximately along the line *MA* and more strongly, mass for mass, than does the lithosphere which is centered at *E*; while the other bulge will attract the Moon along *MB*, more feebly than does the lithosphere. But the attraction at *A* pulls forward on the Moon, and makes a greater angle with the line of centers than does the weaker attraction of *B*, which pulls backward; hence, the net effect is to hasten the Moon in its eastward orbital motion. Paradoxical as it may seem, this does not shorten the month, but lengthens it; for it increases the centrifugal force due to the Moon's orbital motion, causing the Moon to move farther from the Earth and actually to move more slowly in a larger orbit.

Evolution of the Earth-Moon System.—At present, the change in the Moon's revolution, like that in the Earth's rotation, must proceed with extreme slowness; but in the remote past, when the Moon was nearer the Earth, the tidal action must have been much greater than now, since the tide-raising force varies inversely as the cube of the distance; and the tides must have been especially potent when both bodies were plastic throughout, as they are believed once to have been. Upon this line of thought is based the famous theory of **tidal evolution,** which was first elaborated by the English mathematician Sir George Darwin late in the nineteenth century.

According to this theory, the Earth and Moon evolved from a single liquid body which rotated on its axis in a little less than five hours. Owing to its rapid rotation, this body was very oblate, and the motion of its particles was continually disturbed by tides raised upon it by the Sun, which of course had half this period. Darwin showed that, granted certain reasonable assumptions, the free period of vibration of this body must have been the same as that of the tides, in which case a vibration of very large amplitude must have been set up, and, according to the theory, this vibration caused a portion of the body to be separated from the remainder.

Probably the smaller portion was not originally all in one piece, but for some unknown reason it became consolidated into a single body; we know this body as the Moon, while the remainder of the original spheroid is the Earth. The two bodies then had the form of prolate spheroids with their long axes in the same line, and this line was rotating in a period of five hours. But this condition was unstable and the disturbance of the solar tide caused the Moon to fall back a little, whereupon the friction of their enormous mutual tides came into play, and their periods, both of rotation and of revolution, were lengthened, the latter the faster. This process, at first rapid, has continued with diminishing speed until the present, when the Earth's rotation period has reached twenty-four hours, while both the rotation and revolution of the Moon are twenty-seven times as slow.

According to Darwin, the lengthening of the month was more rapid than that of the day until the former attained a length of about twenty-nine days, when the day came to be lengthened faster than the month; and the process will continue (unless the oceans freeze and the whole Earth becomes rigid) until both the day and the month have a length of about fifty-five of our present days. The Earth will then turn always the same face to the Moon as the Moon now does to the Earth.

EXERCISES

1. Two balls, of masses 10 kilograms and 100 kilograms respectively, are placed with their centers 1 meter apart. With what force do they attract each other?

Ans. 0.006658 dyne

2. The balls are moved so that their centers are only 50 centimeters apart. What is now their mutual attraction?

Ans. 0.026632 dyne

3. What is the weight, in dynes, of the smaller ball?

Ans. 9,810,000 dynes

4. Calculate the superficial gravity on the planet Jupiter, whose mass is 318 and radius 11 times that of the Earth.

Ans. 2.6

5. Prove that the values of superficial gravity on spherical planets of equal density are proportional to the radii of the planets.

6. If a body weighs 100 pounds at the surface of the Earth, what would be its weight at the surface of an asteroid having the Earth's mean density and a radius 1/20 that of the Earth?

Ans. 5 pounds

CHAPTER XI

THE PLANETS

Mercury and Venus.—The principal numerical facts concerning the two planets whose orbits lie within that of the Earth are given in Table 7:

TABLE 7

	Mercury	Venus
Mean distance from Sun	0.39 astron. unit	0.72 astron. unit
Intensity of solar radiation received.........	6.7	1.9
Orbital velocity........	23 to 36 mi./sec.	22 mi./sec.
Eccentricity of orbit....	0.20	0.007
Inclination of orbit.....	7°.0	3°.4
Sidereal period.........	88 days	225 days
Mean synodic period....	116 days	584 days
Diameter..............	5″ to 13″ = 3,000 miles	11″ to 67″ = 7,575 miles
Greatest elongation.....	18° to 28°	47°
Distance from Earth in miles...............	57,000,000 to 129,000,000	26,000,000 to 160,000,000
Mass (Earth's mass = 1)	0.05	0.82
Density (water = 1)....	4.5	4.8
Surface gravity........	0.31*g*	0.85*g*
Albedo................	0.06	0.60

Mercury, of all the planets, is nearest the Sun, and so it has the swiftest motion and receives the greatest intensity of radiation. Of non-trans-Neptunian planets (except asteroids) it has the most highly inclined and most eccentric orbit and the least diameter and mass. Venus has the most nearly circular orbit and a diameter most like that of the Earth, only about 400 miles smaller.

At his brightest, Mercury appears to the unaided eye as a star nearly as bright as Sirius, but so near the Sun that (as seen from northern latitudes) he is never in the unilluminated part of the sky and so is never conspicuous.

The preceding sentence, which in the first edition of this book was printed without the parenthesis, brought forth protest from Australia, where Mercury at times really is visible in the unilluminated part of the sky. The advantage possessed in this respect by southern observers is due to the form and situation of Mercury's orbit. The *greatest* "greatest elongation" (28°) occurs when the planet is at aphelion, and a study of Fig. 117 will show that such eastern elongations (evening star) occur in late summer or early autumn and such western elongations (morning star) in early spring. In either case the Sun is near the equator while Mercury is south of both equator and ecliptic, and so the interval between its setting or rising and the setting or rising of the Sun is longer for an observer in the southern hemisphere than for one in the northern.

Venus is very much brighter than Mercury and can be easily seen at midday if the sky is very clear; and, besides, attains so great an elongation that she is at times visible, from points in the United States, more than three hours after sunset or before sunrise. Of all the planets (one or two asteroids excepted) Venus approaches nearest to the Earth, and with the exception of the Sun and Moon and very rare comets she appears the brightest of all the heavenly bodies.

Telescopic observation of these planets is best carried on in the daytime because at night, when visible at all, they are at too low an altitude for good seeing. To the eye they reveal very little surface detail; each presents a nearly uniformly illuminated disk, and the few markings that have been reported are vague and exceedingly difficult of detection. The surface of Venus is intensely white, contrasting beautifully with the blue of the sky. That of Mercury is, as would be inferred from its low albedo, much duller.

Ordinary photographs show as little as do visual observations; but photographs of Venus in ultra-violet light, made by Ross with the Mount Wilson reflectors in 1927 (Plate 23), show bright clouds near the cusps and, in other parts of the disk, dark belts which change from day to day and which suggest those of Jupiter (page 263).

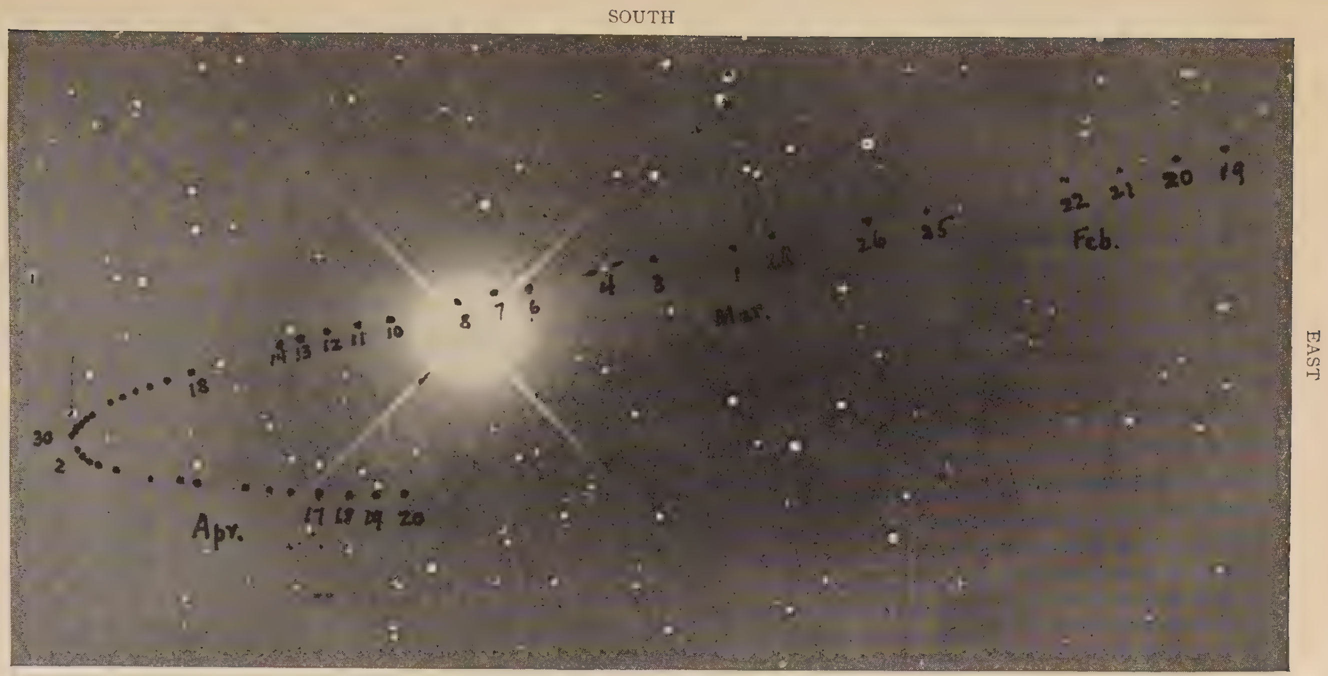
The most striking feature of the telescopic appearance of either Venus or Mercury is its change of phase and of apparent diameter as the planet changes position with respect to the

Earth and the Sun. The change of apparent diameter is especially noticeable in Venus (Plate 23), her maximum diameter being about six times the minimum. Venus's greatest brilliancy occurs at thirty-six days before and after inferior conjunction, when about one-fourth of her earthward hemisphere is illuminated.

The Atmosphere of Venus.—That Venus has a fairly dense atmosphere is certain, for it is plainly visible when the planet is near inferior conjunction, between the Earth and the Sun. The illuminated disk then appears as a very thin crescent which is extended beyond the ends of a diameter by thin bright arcs. When the apparent distance of the planet from the Sun is about two degrees or less, these arcs are prolonged until they meet and form a complete ring of light; an appearance due to sunlight illuminating the planet's atmosphere from behind.

It would be reasonable to suppose that the light of Venus, being reflected sunlight, must have passed twice through her atmosphere before reaching the Earth. Since the oxygen and water-vapor of the Earth's atmosphere produce absorption bands in the spectra of celestial bodies (page 155), it is to be expected that the atmosphere of Venus would have a similar effect and that these bands would be more intense in the spectrum of Venus than in that, for example, of the airless Moon. No such effect has been observed, however, although looked for assiduously at the Lowell and Mount Wilson observatories, and we must infer either that Venus's atmosphere is lacking in water-vapor and oxygen or else that these gases lie wholly in the lower layers to which the sunlight does not directly penetrate.

It is probable that the visible surface of Venus consists of a dense layer of white clouds which prevent our seeing the solid surface, and which also, probably, prevent direct sunlight from reaching that surface. It is not impossible to imagine that Venus is inhabited by beings who, because of this impenetrable mantle of clouds, are ignorant of the existence of the Sun and of all other bodies exterior to their own planet.


SOUTH
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Apr.
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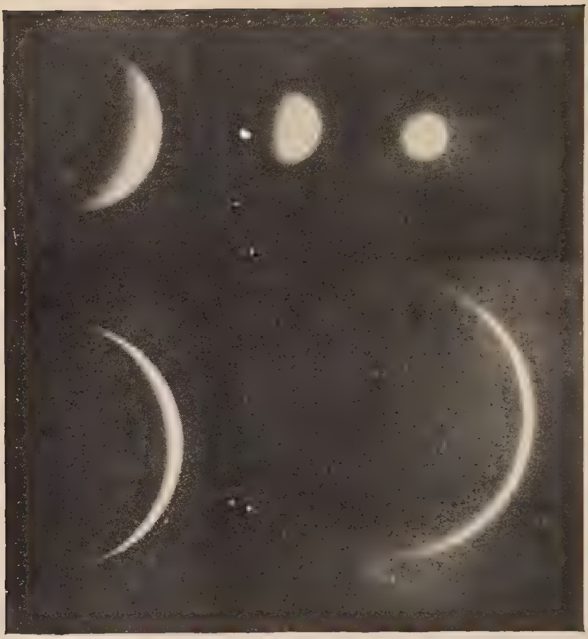
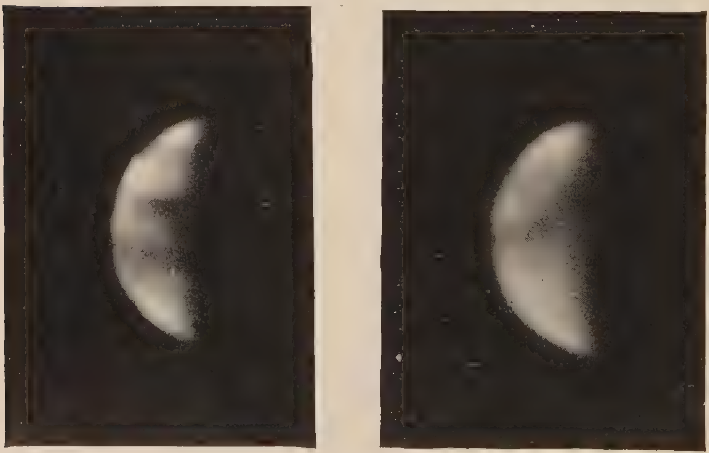
PLATE 22

APPARENT PATH OF THE TRANS-NEPTUNIAN PLANET FROM FEBRUARY 19 TO APRIL 20, 1930, as shown on photographs made by C. O. Lampland with the 42-inch reflector of the Lowell Observatory. The positions are plotted upon the photograph of March 4. The photographic image of the planet is the white dot marked by two black dashes, just above the figure 4. The great white image is that of the star δ Geminorum; the four rays extending from it are caused by diffraction of light around the steel supports of the secondary mirror of the telescope. Scale of photograph: 1 mm = 7″.

PLATE 23



PHOTOGRAPHS OF VENUS IN DIFFERENT PHASES, showing relative sizes of the disk. E. C. Slipher, Lowell Observatory



PHOTOGRAPHS OF VENUS made by Ross in 1927 with the 60-inch Mount Wilson reflector, using a screen which transmitted ultra-violet light only. Shows dark bands suggesting those of Jupiter

No evidence of any atmosphere on Mercury has ever been found, and it is believed that this planet, like the Moon, is without air.

The Temperatures of Mercury and Venus.—The surface temperatures of the planets have been computed by means of the laws of radiation (page 160) from measures of their radiation made at the Lowell and Mount Wilson observatories with exceedingly delicate radiometers (page 320).

Though all the visible light of a planet is reflected sunlight, much of its invisible, long-wave radiation is due to its own heat. The proportion of reflected to directly emitted radiation can be determined by transmitting the light through screens which are opaque to certain parts of the spectrum—for example, a cell containing pure water, which is transparent to visible light but opaque to the invisible, long-wave, planetary radiation.

From these observations it appears that the temperature of the sunlit side of Mercury is about 350° C.—above the melting point of lead—while that of the dark side is too low to be measured and is probably not much above the absolute zero (–273° C.). The condition of this airless planet is thus similar to that of the Moon (page 132), except that the temperature of its sunward side is much higher, due to its proximity to the Sun.

The temperature of the dark side of Venus is found to be about –25° C., while that of the bright side seems to be only a few degrees higher. Here the measures refer to the upper atmosphere, or upper surface of the bright envelope of cloud. A similar condition is known to exist in the "stratosphere," or upper atmosphere of the Earth (page 58), which has an apparently uniform temperature of about –55° C.

Evidence on the Rotation of Venus and Mercury.—Until 1880 the impression prevailed very generally among astronomers that both Venus and Mercury rotated in about twenty-four hours. This impression was based upon the apparent motion of markings which various observers had detected or thought they had detected, mostly with very small telescopes in unfavorable climates and when the planets were at low altitudes. In 1880 Schiaparelli, observing in the daytime under favorable conditions in Italy, found for each of these planets a rotation period identical with its revolution period, indicating that, except for the effect of librations (page 125), it keeps always the same face toward the Sun just as the Moon keeps the same face toward the Earth. Schiaparelli's results were confirmed by a number of observers, especially Lowell, but are disputed by others. The markings on both planets are too indefinite and those of Venus too changeable to permit a reliable value of the period to be obtained by direct observation.

When the image of a rapidly rotating planet is formed so that its equator falls on the slit of a spectrograph, one end of the slit is illuminated by that part of the planet which is approaching the Earth and the other end by the part which is receding. Therefore, according to the Doppler-Fizeau principle (page 167), one end of each line of the spectrum is displaced toward the violet, and the other end toward the red, so that the lines are inclined. This effect is very marked in the case of Jupiter (Plate 26) and is noticeable in the case of Mars, whose period is slightly more than twenty-four hours. Careful observations of the spectrum of Venus by V. M. Slipher at the Lowell Observatory, which have been amply confirmed by St. John and Nicholson at Mount Wilson, disclose no inclination of the lines whatever, and make it certain that the rotation period is at least several days. So long a period as Schiaparelli's 225 days could not be detected by this method. Spectrographic evidence on the rotation of Mercury has not been obtained, and the small disk of this planet and its proximity to the Sun would make the securing of such evidence very difficult.

The fact that the dark side of Venus, though cold, is far above the absolute zero of temperature is regarded by many as evidence that the rotation and revolution periods are not identical—if the same side were always turned away from the Sun, it should have no heat at all. Others think that the observed slight warmth of the dark side may be accounted for by convection currents in the atmosphere producing an interchange of warm air from the sunlit side and cold air from the dark. Ross regards the changing parallel bands which he has photographed (page 247) as an indication of a rather rapid rotation, and, reviewing all the evidence, suggests a compromise of about thirty days as the best estimate of the rotation period of Venus.

In the case of Mercury, the best evidence seems to point to identity of rotation and revolution periods.

Transits of Venus and Mercury.—On rare occasions, one of these planets passes directly between the Earth and the Sun and appears as a black dot upon the photosphere. Such an occurrence is called a **transit.** The conditions necessary for a transit are analogous to those for an eclipse of the Sun: the planet must be at inferior conjunction (corresponding to new Moon) and must be near the node; that is, both the planet and the Earth must be nearly on the planet's line of nodes and on the same side of the Sun.

Transits of Mercury can occur only in May and November, when the Earth crosses Mercury's line of nodes; those of Venus occur only in June and December. The relations of the year and the synodic period of Mercury are such that transits at a given node occur usually at intervals of thirteen or sometimes of seven years, with transits at the other node between. The November transits are much more frequent than the May transits because Mercury's perihelion is nearly in the direction of the November node and its orbit is so eccentric that its perihelion distance is only about two-thirds its aphelion distance.

Transits of Venus at a given node occur either in pairs eight years apart or singly; the interval between single transits or the midway dates of pairs is 243 years; and transits occur at the other node about halfway between.

The first authentically observed transit was one of Venus, which occurred in 1639. It was predicted by Jeremiah Horrocks, a young English clergyman, and as it occurred on a Sunday his ministerial duties prevented him from seeing the beginning of the transit; but his friend Crabtree, whom he had informed of the impending phenomenon, witnessed the entrance of the planet on the solar disk, and both young amateurs saw it before the end. It was seen by no other observers. Four later transits of Venus have been observed, the last one in 1882; but the next will not occur until the year 2004.

The dates of transits of Mercury during the second quarter of the twentieth century are November 8, 1927, May 10, 1937, and November 12, 1940.

From observations of the times of beginning and ending of a transit of Venus, made at two widely separated stations on the Earth, it is possible to determine the parallax of the Sun, and the transits of the nineteenth century were very thoroughly observed for this purpose; but other methods of determining the solar parallax have proved more accurate. Transits of either planet are of value for determining the planet's position on its orbit at a known time and thus for improving the accuracy of the orbital elements.

Facts Concerning Mars.—

TABLE 8

Mean distance from Sun	1.52 astronomic units
Intensity of solar radiation received	0.43
Orbital velocity	15 miles per second
Eccentricity of orbit	0.09
Inclination of orbit to ecliptic	$1^\circ.85$
Sidereal period	1.88 years
Mean synodic period	2.14 years
Distance from Earth in miles	35,000,000 to 247,000,000
Mean albedo	0.15
Mass, Earth = 1	0.106
Diameter	4,215 miles = $3''.6$ to $24''.5$
Density, water = 1	3.92
Surface gravity	0.38g
Rotation period	$24^h\ 37^m\ 22^s.58$
Inclination of plane of equator to plane of orbit	$23^\circ.5$

Mars is notable among the planets and stars for its red color. It is inconspicuous during the greater part of its synodic period and invisible for several months near conjunction, when it is beyond the Sun; but at opposition, when it is relatively near the Earth and is above the horizon all night, rising at sunset

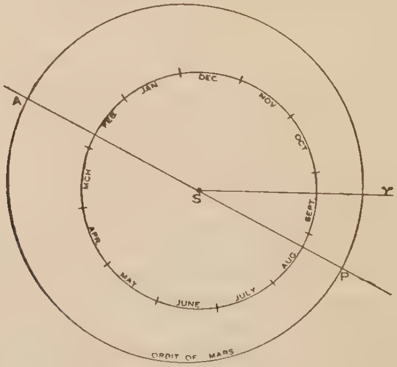


FIG. 137. THE ORBITS OF MARS AND THE EARTH

and reaching its greatest altitude at midnight, it is very bright and conspicuous. The relation of its orbit to that of the Earth is shown in Fig. 137. Owing to its rather high eccentricity, its distance from the Sun varies by about 26,000,000 miles from perihelion to aphelion. The longitude of its perihelion is 333°, which is the heliocentric longitude of the Earth about August 25; hence, when opposition occurs on this date the two planets are at their least possible distance, about 35,000,000 miles. At the next opposition, which occurs in October or early November two and one-seventh years later, the distance is about 43,000,000 miles; and opposition then continues to occur at greater distances for about seven and one-half years, when it takes place in February with Mars at aphelion, more than 61,000,000 miles from the Earth. After this the opposition distance decreases for seven and one-half years more, when it is again a minimum. The last August opposition occurred in 1924.

Rotation and Seasons of Mars.—The first known telescopic drawing of Mars, which was made by Huyghens in 1659, shows the dark marking which is now called the **Syrtis Major** (see map, Plate 24). It soon became evident that this and many other markings were of a permanent nature and that they were being carried around by the planet's axial rotation in the direction of the rotation of the Earth and of the Sun. In 1666 Cassini determined the rotation period as forty minutes longer than that of the Earth. The long interval over which observations of the markings have now been made, and especially the definite and permanent character of the markings themselves, have made it possible to determine Mars's rotation period more exactly than that of any other planet. The period given in the table above is that determined by Lowell.

The north pole of rotation of Mars is in $\alpha = 21^h\ 10^m$, $\delta = +54\overset{\circ}{.}5$, close to the bright star Deneb (α Cygni) which is therefore the Martian Polaris. The inclination of the planes of Mars's equator and orbit is almost exactly the same as the obliquity of the ecliptic, which is the corresponding angle in the case of the Earth; and hence the Martian climatic zones are similar to those of the Earth, but the Martian seasons are nearly twice as long as the terrestrial and must be less pronounced because of the diminished intensity of the Sun's rays. Moreover, the greater eccentricity of Mars's orbit must result in a greater difference between the seasons of the two hemispheres; those of the southern hemisphere are the more extreme since Mars passes perihelion in the southern summer.

Telescopic Appearance of Mars.—Since its orbit lies outside that of the Earth, Mars cannot present to us the crescent phase, but at quadrature it is distinctly gibbous and at conjunction and opposition it is full. Because it is full at opposition, when nearest the Earth, it is a more favorable object for study than Venus, whose dark side is turned toward us at inferior conjunction, the occasion of her nearest approach. Although Mars certainly possesses an atmosphere, it is probably of low density as compared with our air and is never filled with clouds; and so, when the planet is favorably placed, its surface is plainly visible and is seen to be covered with fine detail.

About three-fifths of the surface is of a reddish-ocher color and upon this are darker regions of permanent form which, seen under the best conditions of instrument and atmosphere, appear dark green. Around the poles are areas of pure white which are known as the **polar caps.** Usually only one is visible, the other pole being turned away from the Earth. The reddish areas were called "continents" by the early observers of Mars, and the dark areas "seas" or "lakes"; but it is now certain that the "seas" are not water and that there is in fact but little water on the planet.

In 1877 Schiaparelli of Milan discovered in the "continents" a network of exceedingly fine lines which he called *canali*. This word was translated "canals," and as the **Canals of Mars** they have ever since been known. Their reality was at first universally doubted, as they were so difficult that they were not seen by other observers. In 1888, however, they were detected by Perrotin at Nice and later by many others, although a few good observers, as Barnard and Antoniadi, have been able to see them only as broad, indefinite shadings or as interrupted lines. In 1892 W. H. Pickering at Arequipa, and in 1894

Lowell and Douglass at Flagstaff, found that the canals were present in the "seas" as well as the continents, thus showing that the former are not bodies of water.

The most complete study of Mars and its canals is probably that carried on at the Lowell Observatory, which was founded in 1894 by Percival Lowell of Boston for the express purpose of studying the planets and their satellites, particularly Mars. It is situated at Flagstaff, Arizona, in an exceptionally fine climate for astronomical observing, and at an altitude of 7,000 feet. Lowell observed Mars assiduously at every opposition from the time of the founding of his observatory until his death in 1916, and his work has been supplemented by that of other members of his staff, past and present, particularly Douglass, W. H. Pickering, and E. C. Slipher. The canals of Mars were first photographed in 1905 by Lampland, using the twenty-four-inch Lowell refractor. But photographs have failed to show all the fine details of the best drawings and are useful chiefly in confirmation of the latter. In recent years, observations of Mars have been carried on systematically by W. H. Pickering at Mandeville, Jamaica, using an eleven-inch refractor. Since 1907, M. Jarry Desloges has maintained observatories in France and Algeria which have been devoted entirely to the study of planetary surfaces.

Although, if perfect seeing could be secured, the power of a telescope for showing fine detail on a planet would be proportional to its aperture, such seeing is never actually known; and it is found that, in such conditions as really exist, a moderate-sized telescope is often better for the purpose than a large one, since the larger light-beam must traverse a greater area of disturbance in the terrestrial atmosphere. It is the practice of most observers who use large telescopes for observing the planets to stop down the apertures except when the seeing is at its best.

The nomenclature most commonly used for the features of Mars is that founded by Schiaparelli, in which the names are taken from ancient or mythological geography. The surface of the planet is laid off by meridians and parallels, the origin of longitudes being the **Fastigium Aryn** between the branches of the forked **Sabaeus Sinus.** The most prominent markings are perhaps the **Solis Lacus,** the dark "eye" of Mars in longitude 90°, latitude 30° south; and the **Syrtis Major** and **Margaritifer Sinus,** dark horn-shaped markings which extend from the southern hemisphere to middle latitudes in the northern. The most easily detected canal is usually the **Nilosyrtis,** which forms a sort of tail to the Syrtis Major. Maps of Mars are ordinarily printed with south at the top, since the planet is most frequently observed with inverting telescopes (Plate 24).

Changes in the Features of Mars.—Each of the white polar caps changes very markedly, increasing in size when turned away from the Sun and shrinking as it is turned sunward—a behavior first noted by Sir William Herschel, who attributed the caps to snow which formed in winter and melted in summer. The southern cap, which is turned sunward when the planet is near perihelion, disappears completely, but the northern one has not been observed to do so.

Careful, systematic observation discloses seasonal change also in other parts of the planet. From the great quantity of observations that have been made during the last half-century it has become apparent that, in general, the change is as follows: Just after the Martian solstice, when the sunward polar cap has grown small, the hue of the dark areas in its vicinity begins to deepen. The deepening spreads slowly toward the equator, pervading both "seas" and canals, and finally crosses the equator and may be detected in the low latitudes of the opposite hemisphere. Then, as the planet moves around its orbit and approaches the other solstice, the dark areas of the hemisphere which is turning away from the Sun begin to fade, first near the pole and later in lower latitudes. These changes are very suggestive of the vernal quickening and autumnal fading of vegetation.

On occasions at almost every opposition, bright spots appear at various points of the planet's surface, move across it with speeds large enough to be detected in a few hours' observation, and disappear after a few days. Some of these are white and resemble clouds, while others are yellowish and may be dust storms. Sometimes when Mars is in the gibbous phase one of these clouds is seen in projection beyond the terminator, like a high peak on the Moon (Fig. 88), showing that it lies high above the general surface.

Frequently, large white areas appear at the eastern or sunrise limb, dwindle rapidly, and disappear before reaching the middle, or noon point, of the disk. Their behavior is very suggestive of hoar-frost which, having formed during the Martian night, melts when carried by the planet's rotation into the morning sunlight.

Climatic Conditions on Mars.—The existence of an atmosphere on Mars is proved beyond a doubt by the occasional presence of floating clouds and the melting and re-forming of the polar caps. That it is much rarer than the Earth's air is almost equally certain because of the low surface gravity of Mars and the clearness with which the solid surface is seen. The appearance and behavior of the polar caps suggest strongly that they are frozen water, but the absence of oceans indicates that the total quantity of water on the planet is small.

The presence of water-vapor and of oxygen in the planet's atmosphere has been detected spectroscopically, but the observation is rendered difficult by the fact that these substances exist in our own air and produce bands which tend to mask those produced by the planet. The inquiry has been conducted in two different ways: (1) By comparing spectrograms of Mars and the Moon, taken at equal altitudes so that their light passes through equal depths of the Earth's atmosphere; since the Moon is airless, any water or oxygen on Mars must produce a relative strengthening of the bands which may be perceptible. (2) By taking advantage of the Doppler-Fizeau effect (page 167), which, when the radial velocity of Mars is considerable, should displace the bands of the planet's spectrum, separating them from the telluric bands or at least producing a widening of the composite band.

Spectrograms of Mars and the Moon (Plate 25), secured by V. M. Slipher at Flagstaff in 1908 on nights when the air above his telescope was very dry, show a distinct intensification of the "a" band in the spectrum of Mars, thus indicating the presence of water-vapor in the planet's atmosphere. The oxygen bands are likewise intensified, and Very, from photometric measures of Slipher's spectrograms, estimated the oxygen in the atmosphere of Mars to be about 50 per cent as dense as that in the atmosphere of the Earth. Adams and St. John, applying the Doppler-Fizeau principle in 1925, found that the quantity of water-vapor in Mars's atmosphere, area for area, was 6 per cent, and the quantity of oxygen 16 per cent, of that over Mount Wilson. This indicates, for the date of their observation, extreme desert conditions on Mars and an atmosphere less dense than that at the tops of the highest terrestrial mountains.

As the intensity of solar radiation at the distance of Mars is only four-ninths that received by the Earth, it is to be expected that the temperature of the planet would be low, and this has been considered a serious drawback to the belief that the polar caps and other white areas which melt are frozen water. However, ice and snow evaporate at temperatures far below zero when the air above them is very dry, as the Martian atmosphere appears to be.

Direct evidence in regard to the temperature of the planet's surface was obtained in 1924 when measures of its radiation were made with the thermocouple (page 320) at Flagstaff by Lampland and Coblentz, and at Mount Wilson by Pettit and Nicholson. At both observatories it was found that the temperature rose several degrees above the Centigrade zero near the center of the illuminated disk, but was usually low near the limb. At Flagstaff it was found that the temperature of the south polar cap was −100° C. about two months before the the southern summer solstice, and that it gradually increased to about 0° C. a few days after the solstice; and that the temperature of the east limb (where the surface had just emerged from darkness into morning sunlight) was as low as −85° C., giving evidence of an enormous diurnal fluctuation. It appeared also that the bright areas were at a lower temperature than the dark areas.

Mars as the Abode of Life.—Life as we know it on the Earth, both animal and vegetable, depends on a number of special conditions, among which are a favorable temperature and a supply of water and, for animals, of oxygen. Oxygen and water appear to be very scarce on Mars, and the temperature far from salubrious; yet the seasonal changes of the dark areas are best interpreted as due to vegetation, and where vegetation flourishes, at least on the Earth, animal life is likely to exist also.

The great protagonist of the theory of the habitability of Mars was Lowell. He believed the reddish-ocher areas which compose most of the planet's surface to be deserts and the "seas" to be wooded or grass-covered plains. He pointed out that the canals intersect one another at large angles, often three or more crossing at the same point, quite differently from rivers; that to be seen at all they must be at least several miles wide; and, in particular, that they are of too straight and geometric a character (though their straightness is disputed by some observers) to be natural lines such as rivers or cracks. He therefore interpreted them as strips of vegetation along the sides of artificial lines of irrigation which have been built by intelligent beings in order to make the best use of the limited supply of water. He explained the wave of deepening color which sweeps from pole to equator in the Martian summer by the quickening of vegetation in the dark areas and along the canals by water from the melting polar cap.

Lowell's theory accounts for all the observed facts and encounters no really insurmountable obstacles, but there are yet few astronomers who regard it as proved; while there are many who do not agree with Lowell in his statement, "That Mars is inhabited by beings of some sort or other we may consider as certain as it is uncertain what those beings may be."[1]

Deimos and Phobos.—Mars has two tiny satellites, each probably less than twenty miles in diameter, which were discovered by Asaph Hall at Washington in 1877. They revolve in circular orbits in the plane of the planet's equator. The nearer one, **Phobos,** is only 5,800 miles from the center of the planet and therefore less than 4,000 miles from its surface. Although, like the Moon, it revolves in the direction of its planet's rotation, its period is only $7^h\ 40^m$, about a quarter of Mars's day, and so it moves faster than the surface and, to a Martian observer, must rise in the west and set in the east. The other satellite, **Deimos,** revolves at a distance of 14,600 miles in a period of $30^h\ 18^m$, which is so little greater than the Martian day that the satellite remains above the horizon more than sixty hours or two of its months, so that it must go twice through all its changes of phase between rising and setting.

The Asteroids.—Between the orbits of Mars and Jupiter revolve more than a thousand bodies known as **asteroids, planetoids,** or **minor planets.** Most of them are tiny telescopic specks which can be distinguished from faint stars by their motion only. The four which were first discovered, and which are among the largest and brightest, show perceptible disks when nearest the Earth, and their diameters were measured by Barnard at the Yerkes Observatory. His results and the albedos computed from them by Russell are contained in Table 9.

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[1] Percival Lowell, *Mars and Its Canals*, p. 376.

TABLE 9. THE LARGEST ASTEROIDS

Name	Diameter	Albedo
Ceres......................	488 miles	0.06
Pallas.....................	304 "	0.07
Vesta......................	248 "	0.26
Juno.......................	118 "	0.12

Vesta, although not the largest, is the brightest asteroid, and when nearest the Earth is sometimes visible to the naked eye. Most of the others are probably less than fifty miles in diameter. According to Crommelin, the combined volume of all the asteroids known is less than one-twentieth that of the Moon.

Discovery of the Asteroids.—About the end of the eighteenth century, after the discovery of Uranus (page 270), whose distance fitted nicely into Bode's law (page 211), it was suspected that an undiscovered planet revolved in the space corresponding to the fifth term of Bode's series and a number of astronomers planned to search for it. On the first of January, 1801, Piazzi, who was observing on the island of Sicily and was not one of the organized searchers, found a little planet which he named Ceres. After observing it a short time, Piazzi became ill, and before he recovered or the news of his discovery had reached other astronomers the Earth had moved so far in its orbit as to leave Ceres in a position unfavorable for rediscovery. The elements of the orbit of the little planet were unknown and there was great danger that it could never be identified among the multitude of stars; but the German mathematician Gauss, then twenty-four years old, discovered a method of determining the elements of the orbit of a planet from three observations, calculated the orbit of Ceres, and predicted its apparent position in which it was rediscovered on the last day of the same year.

Ceres was so small that it was thought by the German astronomer Olbers that other planets like it might exist, and the search was continued. Pallas was found by Olbers in 1802, Juno by Harding in 1804, and Vesta by Olbers in 1807. No more asteroids were then discovered until 1845, when Hencke found Astræa; but beginning with 1847 at least one has been found every year. Since 1891 most of the discoveries have been made by photography, a field in which Wolf of Heidelberg and Palisa of Vienna have led.

Orbits of the Asteroids.—The mean distances of the asteroids range from 1.46, which is that of Eros, to more than 5 astronomic units; their periods from two to more than twelve years; the inclinations of their orbit planes from 0 up to 48°; and their eccentricities from 0 to 0.65; but most of the orbits are fairly round, lie rather near the plane of the ecliptic, and have semi-major axes averaging about 2.8, the number in Bode's series between Mars and Jupiter.

It was noticed by Kirkwood of Indiana in 1866 (when only eighty-eight asteroids were known), and has been abundantly verified since their numbers have so greatly increased, that no asteroids have periods which are simple fractions of that of Jupiter. When the mean distances of the asteroids are plotted as in Fig. 138, conspicuous gaps are found at the distances at which, if an asteroid did revolve, its period would be ½, ⅓, ⅔, etc., of Jupiter's period. Although the fact has not been demonstrated mathematically, it is probable that the perturbations of Jupiter have cleared the asteroids from **Kirkwood's gaps** or else prevented their ever entering them.

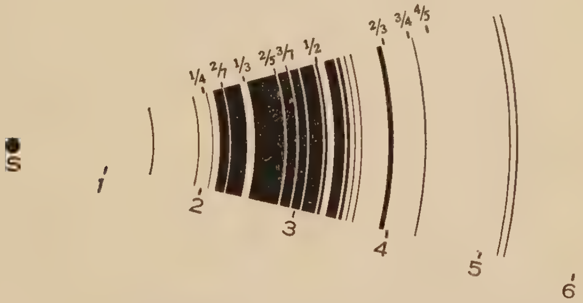


FIG. 138. KIRKWOOD'S GAPS IN THE ASTEROID BELT

At first, the names applied to the asteroids were chosen from Greek mythology; but as they became more numerous other mythologies were invoked, the works of Wagner, Shakespeare, and lesser writers were pressed into service, and many strange new names were invented. All the names are supposedly feminine except those of a few asteroids which have extraordinary orbits. The masculine ones include Eros, Albert, and Ganymede, whose perihelia lie within the orbit of Mars and which therefore at times approach closely to the Earth (Alinda, though bearing a feminine name, belongs to this group also); Hidalgo, whose perihelion lies near Mars's orbit and aphelion near Saturn's; and the "Trojan group" Hektor, Nestor, Achilles, Patroclus, Priamus, and Agamemnon, each of which, with Jupiter and the Sun, fulfills the equilateral triangular solution of the problem of three bodies (page 233).

Eros is of particular interest for its value in determining the distance of the Sun and the scale of the Solar System. It approaches at times within about 13,000,000 miles of the Earth, which is closer than any other planet. Its geocentric parallax is then correspondingly large and for this reason, and also because the little asteroid presents a point-image, its parallax and distance in miles can be determined with considerable accuracy. From the elements of its orbit its distance in astronomic units can be accurately computed, and from this and the distance in miles the number of miles in the astronomic unit is found.

The Zodiacal Light and the Gegenschein.—The region of the zodiac on either side of the Sun is illuminated by a faint, soft light of triangular or lenticular shape which somewhat resembles the Milky Way, but lacks the sparkle of the latter's millions of distant stars. This **zodiacal light** is most easily seen in our latitudes on spring evenings or autumn mornings when the portion of the ecliptic above the horizon is most nearly vertical. Its spectrum, as observed by Fath at the Lick Observatory in 1908, seems identical with that of sunlight, and there is little doubt that the illumination is caused by the reflection of sunlight by a belt of small bodies revolving in orbits which lie mostly within that of Venus, but some of which may extend beyond the Earth. The size of the bodies is unknown, but they are certainly smaller than the asteroids and are very probably mere dust-particles.

Far more difficult to see, and quite imperceptible when there is the least haze or artificial light, is the **Gegenschein,** a dim, oval spot of light which lies exactly opposite the Sun, sharing its annual motion as does the zodiacal light. Moulton's explanation of the Gegenschein is mentioned on page 233.

The Giant Planets.—The four planets next beyond the asteroids are so much larger than those nearer the Sun that they may well be called the **giant planets.** The principal numerical facts regarding them are contained in Table 10.

TABLE 10. THE GIANT PLANETS

	Jupiter	Saturn	Uranus	Neptune
Mean distance from Sun, astronomic units	5.20	9.54	19.19	30.07
Sidereal period, years	11.86	29.46	84.02	164.79
Mean synodic period, days	399	378	369	367.5
Eccentricity of orbit	0.048	0.055	0.047	0.009
Inclination of orbit plane	1°.3	2°.5	0°.8	1°.8
Apparent diameter	32″ to 50″	14″ to 20″	3″.8	2″.5
Equatorial diameter in miles	88,700	75,060	30,900	32,900
Oblateness	0.06	0.11	0.09	?
Mass, Earth = 1	318	95	15	17
Mean density, water = 1	1.32	0.72	1.22	1.11
Surface gravity, $g = 1$	2.65	1.18	0.90	0.89
Mean albedo	0.44	0.42	0.45	0.52
Mean rotation period	$9^h\ 55^m$	$10^h\ 14^m$	$10^h\ 45^m$ ±	16^h ±
Inclination of equator to orbit	3°.1	26°.8	82°?	?

The spectra of all four giant planets, as discovered by V. M. Slipher, contain broad dark bands of greater wave-length than Hβ, the intensity of which is greatest for Neptune, less for Uranus, and least for Jupiter. These bands prove the existence of dense atmospheres, but give no interpretable evidence as to their constitution, since the bands have no known counterpart in any other spectra.

No distinct phases are perceptible on any of these outer planets because the angle *Sun-planet-Earth* is always small; but when Jupiter is at quadrature his eastern and western limbs are perceptibly of unequal brightness.

On each giant planet, the telescope reveals light and dark **belts** arranged parallel to the equator, which are very conspicuous on Jupiter (Plate 26), faint on Saturn, and almost imperceptible on Uranus and Neptune. Great changes are observed in the details of the markings on Jupiter and Saturn, and the rotation periods determined from spots in different latitudes are different, as in the case of the Sun (page 186), so that it is evident that their surfaces are in a state of great disturbance. Until recently it was supposed that these planets were at a high temperature, but in 1923 Jeffreys, in England, showed on theoretical grounds that the contrary is more probably true, and radiometer measures made in 1924 by Lampland and Coblentz resulted in a mean temperature for the surface of Jupiter of −130° C., for Saturn of −150° C., and for Uranus of −170° C.

Jupiter.—To the unaided eye Jupiter appears as a star five or six times as bright as Sirius. It is the largest of the planets, larger than all the others combined, with a diameter 11 times, a volume 1,300 times, and a mass over 300 times the Earth's. Its rotation is so rapid that it may be noticed in less than an hour by observation at the telescope under good conditions, and has produced a very noticeable equatorial bulge. With even a very small telescope the belts are easily seen, and with a large instrument and good seeing the markings show a wealth of detail. The colors are delicate and beautiful, consisting mainly of red, yellow, tan, and brown tints, but excellent conditions are required to show them, since a little atmospheric disturbance blurs the details.

The chief markings are of a durable, but not permanent, character (Plate 26). The most famous one is the **great red spot** which has been observed in the Jovian southern hemisphere since 1857 and was "startlingly conspicuous" from 1878 to 1881. In its prime it was about 30,000 miles long and 7,000 wide. It has disappeared and reappeared at times, but in its absence its place has been marked by the **great red spot hollow.** It cannot be permanently attached to a solid nucleus for its rotational speed, which is usually slower than that of its surroundings, is perceptibly but temporarily accelerated when the spot is overtaken by the **south tropical disturbance,** another salient feature which has been known since 1901. Other markings sometimes change their appearance much more rapidly. The visible surface appears to be composed of clouds, but they must be clouds, not of water, but of some substance which vaporizes at a much lower temperature.

The Satellites of Jupiter.—Jupiter's system of nine satellites is described in Table 11. The four large satellites, which were discovered by Galileo, are numbered in the order of their distance from Jupiter; the others in the order of discovery. The

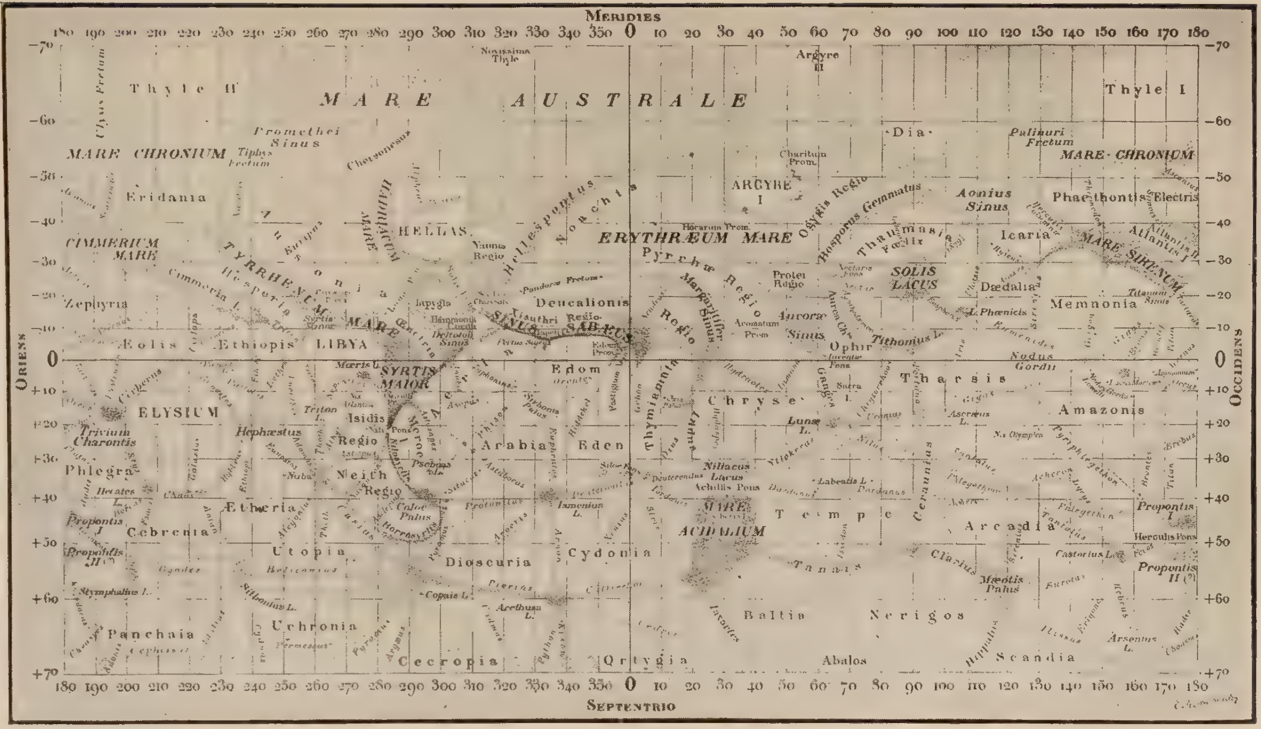
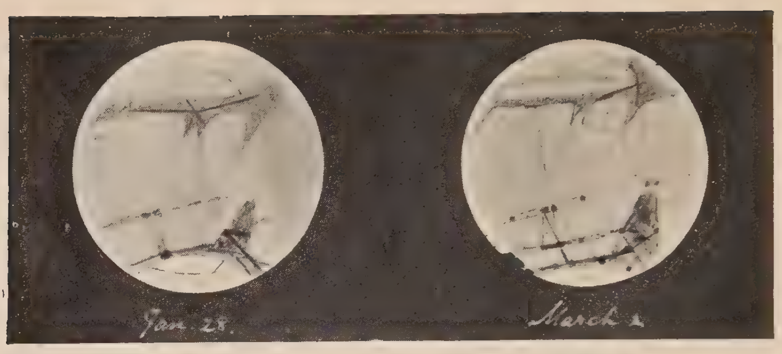
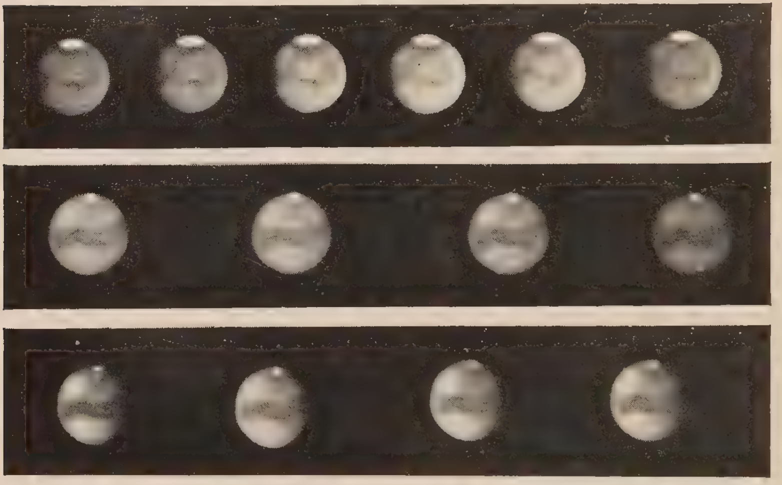

MERIDIES
SEPTENTRIO
ORIENS
OCCIDENS
MARE AUSTRALE
Thyle II
Thyle I
Novissima Thyle
Argyre II
MARE CHRONIUM
Promethei Sinus
Tiphys Fretum
Chersonesus
Eridania
CIMMERIUM MARE
HADRIACUM MARE
HELLAS
Yaonis Regio
Hellespontus
Noachis
ERYTHRÆUM MARE
ARGYRE I
Charitum Prom.
Dia
Palinuri Fretum
Aonius Sinus
Phaethontis
Electris
Icaria
MARE SIRENUM
Atlantis
Bosporus Gemmatus
Ogygis Regio
Thaumasia Fœlix
SOLIS LACUS
Dædalia
Memnonia
Zephyria
TYRRHENUM MARE
Hesperia
Cimmeria
Deucalionis Regio
Pyrrhæ Regio
Margaritifer Sinus
Protei Regio
Auroræ Sinus
Ophir
Tithonius L.
L. Phœnicis
Iapygia
Deltoton Sinus
SINUS SABÆUS
Æolis
Ethiopis
LIBYA
Nodus Gordii
SYRTIS MAJOR
ELYSIUM
Trivium Charontis
Hephæstus
Isidis Regio
Triton
Meroe
Edom
Arabia
Eden
Thymiamata
Chryse
Tharsis
Amazonis
Luna L.
Phlegra
Neith Regio
Nilosyrtis
Nuba
Nilíacus Lacus
Achillis Pons
MARE ACIDALIUM
Tempe
Ceraunius
Arcadia
Ætheria
Cebrenia
Propontis I
Propontis II
Utopia
Coloe Palus
Ismenius L.
Dioscuria
Cydonia
Tanais
Castorius L.
Herculis Pons
Mæotis Palus
Copais L.
Arethusa L.
Panchaia
Uchronia
Cecropia
Ortygia
Baltia
Nerigos
Abalos
Scandia
Arsenius L.
Stymphalius L.
180
190
200
210
220
230
240
250
260
270
280
290
300
310
320
330
340
350
0
10
20
30
40
50
60
70
80
90
100
110
120
130
140
150
160
170
180
−70
−60
−50
−40
−30
−20
−10
0
+10
+20
+30
+40
+50
+60
+70

PLATE 24. MAP OF MARS, by Flammarion and Antoniadi

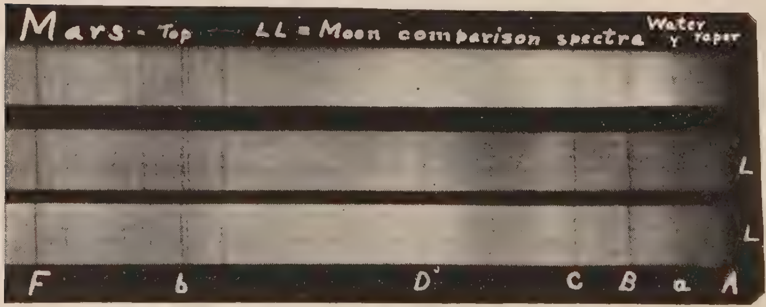
PLATE 25. MARS



Drawings by Percival Lowell, 1916



Photographs by E. C. Slipher, showing seasonal change in south polar cap and southern dark areas from Martian date May 10 to July 19



Spectrograms of Mars and the Moon by V. M. Slipher, 1908 January 15

TABLE 11. THE SATELLITES OF JUPITER

No.	Name	Discovery	Distance, miles	Period	Diameter, miles	Density water = 1	Albedo
5	....	Barnard, 1892	112,500	$11^h\ 57^m$	small	....	......
1	Io	Galileo, 1610	261,000	$1^d\ 18^h$	2,450	2.7	0.69
2	Europa	Galileo, 1610	415,000	$3^d\ 13^h$	2,050	2.6	0.76
3	Ganymede	Galileo, 1610	664,000	$7^d\ 4^h$	3,560	1.5	0.45
4	Callisto	Galileo, 1610	1,167,000	$16^d\ 17^h$	3,350	1.0	0.16
6	....	Perrine, 1905	7,300,000 ±	266^d ±	v. small	....	......
7	....	Perrine, 1905	7,500,000 ±	277^d ±	v. small	....	......
8	....	Melotte, 1908	14,600,000 ±	740^d ±	v. small	....	......
9	....	Nicholson, '14	14,900,000 ±	758^d ±	v. small	....	......

names of the Galilean satellites were given them by Simon Marius, who claimed priority of discovery; as his claim was believed fraudulent, the names have never been extensively used, and the fainter satellites have never been named.

The five small satellites are exceedingly faint and all but the inmost were discovered by photography. The eighth was discovered at Greenwich; the four others at the Lick Observatory. The four outer satellites are so far from Jupiter that they are subject to enormous perturbations by the Sun, and for some time after their discovery there was doubt as to whether they were really satellites to Jupiter or asteroids. The planes of their orbits are highly inclined to the plane of Jupiter's equator, whereas those of the inner five sensibly coincide with it. Numbers 8 and 9 revolve in the retrograde direction.

Phenomena of the Galilean Satellites.—The four great satellites, which are so bright as to be visible in the smallest telescope—or even to the naked eye were it not for their proximity to the planet—present most interesting phenomena. The plane of their orbits (and that of Jupiter's equator) so nearly coincides with that of the planet's orbit, and also with that of the ecliptic, that their orbits are presented almost edgewise to both the Sun and the Earth. The satellites, therefore, appear always in a nearly straight line passing through the planet, and all but Callisto are eclipsed at every revolution. Callisto escapes eclipse when Jupiter is far from his equinoxes.

When Jupiter is not at opposition, four distinct phenomena present themselves, which may be understood from a consideration of the orbit of Ganymede in Fig. 139. When the satellite arrives at *A*, it is **occulted** behind the planet; it emerges from occultation at *B* to disappear in **eclipse** at *C*, reappearing at *D*. On arriving at *H* it passes in front of Jupiter and a **transit** of the satellite occurs, during which it may be seen against the planet as a dot of nearly the same brilliancy as the planet itself. From *M* to *N* the satellite's shadow falls upon the planet and may be easily seen as a black dot (**transit of the shadow**). At opposition, eclipses take place simultaneously with occultations, and the shadows are concealed behind the satellites which cast them (Fig. 140).

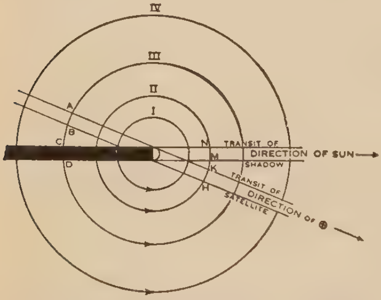


FIG. 139. PHENOMENA OF JUPITER'S SATELLITES; JUPITER AT EAST QUADRATURE

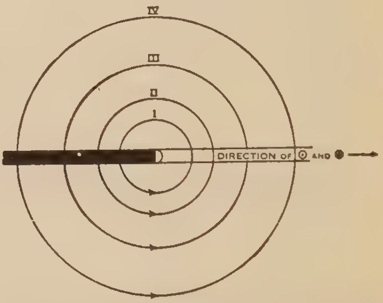


FIG. 140. PHENOMENA OF JUPITER'S SATELLITES; JUPITER AT OPPOSITION

Callisto has a much lower albedo than the other large satellites, one of which, Europa, is about as white as snow. When in transit across Jupiter, Callisto appears dark. All four, especially Callisto, are variable in brightness in periods coinciding with their revolution periods, probably because one side is brighter than the other and because the satellites behave like the Moon and keep always the same face toward Jupiter.

Saturn and its Rings.—To the naked eye, Saturn appears as a star of the first magnitude, but not eminent in any way above several other stars. By a good telescope, it is revealed as a delicately banded globe poised within a shining **ring**—a spectacle which, once seen, can never be forgotten. So far as is known, no other body in the universe is like it (Plate 27).

Galileo's telescope was not quite good enough to show the details of this planetary spectacle, but he perceived that Saturn was not like an ordinary star, and sent a mysterious anagram to Kepler, later transposing it into the sentence *Ultimum planetam tergeminum observavi* ("I have observed the farthest planet triple"). Subsequent observers drew Saturn as triple or as having a pair of ansæ, or "ears," of various types; and it was not until 1655 that Huyghens perceived the planet's true form which he announced, first in an anagram and later in its translation, *Annulo cingitur, tenui, plano, nusquam cohaerente, ad eclipticam inclinato* ("It is surrounded by a ring, thin, flat, nowhere touching, inclined to the ecliptic"). In 1675 D. Cassini found that the ring was divided in two by the space since known as **Cassini's division.** In 1850 G. P. Bond of Harvard discovered, within the two bright rings, a faint, semi-transparent **crêpe ring.**

The rings lie in the plane of the planet's equator, which is inclined about 27° to the plane of its orbit. Twice in the sidereal period of nearly thirty years this plane passes through the Sun just as does the plane of the Earth's equator in March and September (Fig. 67); the Sun then shines only on the thin edge of the ring, and at about the same time the ring is presented edgewise to the Earth. At such times it disappears in small telescopes, while in large ones it seems a fine needle thrust through the ball. About seven and one-half years later, the Saturnian solstice occurs, and the Earth and Sun have an elevation of about 27° above the plane of the ring, which then appears as an ellipse with a minor axis nearly half the major axis. When not presented edgewise to the Sun, the ring casts a shadow on Saturn which may usually be seen as a dark band outlining the edge of the ring. At the same time the shadow of the ball extends across the ring and may be seen, except when directly behind the planet, as a black band outlining one limb of the globe against the ring. The ring reflects so much sunlight when most widely displayed that, to the unaided eye, the planet then appears more than three times as bright as when the ring is placed edgewise.

Saturn's oblateness is greater than that of any other known planet, and its equatorial bulge is conspicuous; but its rotation period is slightly longer, and its diameter considerably less, than Jupiter's. Its mean density is very low, much less than that of water and less than that of any known solid except the rare element lithium; and yet it has been shown by the motion of the satellites that the inner core of the planet is denser than the outer layers, which must be very light indeed. Both the ball and the rings, and also some of the satellites, have high albedos, and the English astronomer Hepburn has suggested thay they may be composed of ice or loosely packed snow.

Dimensions and Constitution of the Rings.—The dimensions of the rings are given in Table 12. The middle ring, *B*, which is much the widest, is also much the brightest. None of the rings is quite opaque; the outline of the ball is easily seen through the crêpe ring and is visible, though less easily, through Ring *A*; and stars have been seen even through Ring *B*. Moreover, when the Earth and Sun are on opposite sides of the plane of the rings, as happens for a short time every fifteen years, the rings do not completely disappear but are visible in large telescopes by sunlight that has passed through them.

TABLE 12. THE RINGS OF SATURN

Radius of outer limit of ring system	86,300	miles
Width of outer ring (A)	11,100	"
Width of Cassini's division	2,200	"
Width of Ring B	18,000	"
Width of crêpe ring (C)	11,000	"
Distance from inner edge of Ring C to surface of Saturn	6,000	"
Thickness of ring . . . less than	100	"

It was shown theoretically by Laplace that, if the rings were solid, they must collapse like an overloaded bridge under the force of Saturn's attraction; and in 1857 by Clerk-Maxwell that they could be neither solid nor fluid and must therefore consist of a swarm of independent bodies like meteors.

In 1895 Keeler verified the meteoric constitution of the rings by a beautiful application of the spectroscope and of Kepler's laws. He photographed the spectrum of Saturn by forming an image in such a way that the slit of the spectrograph cut through both ansæ and through the center of the disk, as in

Fig. 141. Each spectral line was thus divided in three parts, consisting of the image of an east-west strip of the planet in the middle and of a strip of the ring at each end. The middle portion was inclined because of the rotation of the planet (page 250), the west end being displaced toward the red; but the end portions, though shifted bodily, were inclined the other way (Plate 28), showing that the inner particles of the ring were moving faster than the outer ones. Keeler found that the velocities of the different parts were in accordance with Kepler's harmonic law (page 203).

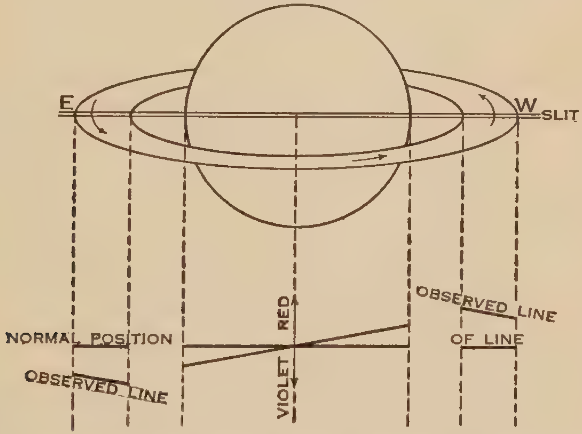


Fig. 141. Spectroscopic Proof of the Meteoric Constitution of Saturn's Rings

Cassini's division lies at a distance from Saturn where, if a particle revolved, its period would be half that of Mimas, the nearest satellite, and commensurable with the periods of three others, and is thus analogous to Kirkwood's gaps in the asteroid belt (page 261). In Ring *A* is a much less prominent division, known as Encke's, where a revolving particle would have a period one-third that of Mimas, and Lowell reported a number of other divisions, mostly in Ring *B*, where the period would be simply commensurable with those of various satellites.

The Satellites of Saturn.—Saturn has nine satellites, which are described in Table 13. They are not so conspicuous as the Galilean satellites of Jupiter, being more distant and mostly smaller, but several of them, particularly Titan, may usually be seen with a moderate-sized telescope. The orbits of the five inner ones are circular and lie in the plane of the planet's equator and of the rings. Titan and Hyperion also revolve nearly in that plane, but the orbit of Japetus is inclined about 10°, while Phœbe revolves *backward* almost in the plane of Saturn's orbit. Eclipses and occultations can occur only when the rings are placed nearly edgewise to the Sun and the Earth.

Uranus and its Satellites.—The discovery of Uranus, by William Herschel in 1781, caused great excitement, since the discovery of a new planet was then a thing unheard of. Herschel was knighted, and was provided by the king of England with means for building great telescopes with which he subsequently did much important work.

Table 13. The Satellites of Saturn

Name	Discovery	Distance, Miles	Period	Diameter, Miles
Mimas	W. Herschel, 1789	117,000	$22^{h}.6$	600 ±
Enceladus	W. Herschel, 1789	157,000	1^{d} 9^{h}	800 ±
Tethys	J. Cassini, 1684	186,000	1 21	1,200 ±
Dione	J. Cassini, 1684	238,000	2 18	1,100 ±
Rhea	J. Cassini, 1672	332,000	4 12	1,500 ±
Titan	Huyghens, 1655	771,000	15 22	3,000 ±
Hyperion	G. P. Bond, 1848	934,000	21 7	500 ±
Japetus	J. Cassini, 1671	2,225,000	79^{d}	2,000 ±
Phœbe	W. H. Pickering, 1898	8,000,000	546^{d}	200 ±

The discovery did not take place in quite the way which might be inferred from Keats's lines, written years later:

> "Then felt I like some watcher of the skies
> When a new planet swims into his ken."

Herschel was "sweeping" the sky in the constellation Gemini with a seven-inch reflector of his own construction when he perceived an object which he distinguished from the stars by its disk. He thought it was a comet and so reported it, and the fact that it was a planet did not become known until nearly a year later, when Lexell, in Russia, computed its orbit, which he found to be nearly circular and to lie beyond Saturn. Herschel wished to call the planet "Georgium Sidus" in honor of King George III, while many other astronomers called it Herschel; but the name Uranus, proposed by Bode, was finally adopted.

Uranus is barely perceptible to the naked eye on a moonless night. In the telescope it presents a little apple-green disk.

It is only under the best conditions that vague belts can be perceived upon it, and there are no markings from which a rotation period can be found. Slipher has determined the period spectroscopically as about ten and two-thirds hours, and L. Campbell, at Harvard, has found that the planet's brightness varies in this period, suggesting that it is unequally bright in different longitudes. The rotation appears to take place in the plane of the satellite orbits, which is inclined at the extraordinary angle of 82° to the plane of the planet's orbit, and the motion, like that of the satellites, is retrograde.

The satellite system is described in Table 14.

TABLE 14. THE SATELLITES OF URANUS

Name	Discovery	Distance, Miles	Period	Diameter (?), Miles
Ariel..........	Lassell, 1851	120,000	$2^{d}\ 12^{h}.5$	500
Umbriel.......	Lassell, 1851	167,000	4 3.5	400
Titania........	W. Herschel, 1787	273,000	8 17	1,000
Oberon........	W. Herschel, 1787	365,000	13 11	800

Neptune and Its Satellite.—The discovery of Neptune has already been discussed (page 236). This distant body appears of the eighth stellar magnitude, quite invisible to the unaided eye. In the telescope it shows a little greenish disk without any markings except, under the finest conditions, vague belts. Its rotation, determined spectroscopically by Moore and Menzel in 1928, is direct and has a period of about sixteen hours.

Neptune has one satellite, discovered by Lassell in 1846. Though the diameter of this satellite is estimated at 2,000 miles, it is so faint as to be very difficult to see. It revolves at a distance of 222,000 miles in five days twenty-one hours, in a plane inclined about 35° to the ecliptic, and in the *retrograde* direction—a circumstance which makes the direct rotation of the planet somewhat surprising. Flammarion has suggested the appropriate name of **Triton** for Neptune's satellite, but no name for it is generally used.

The Planet Beyond Neptune.—Little is as yet known about the newly discovered planet beyond Neptune (page 237) since its orbital elements have not yet been determined from observations of the planet itself. Lowell's computations, based on the perturbations of Uranus, indicate that its distance is about 43 astronomical units; and with this distance, according to Kepler's third law, its period must be about 282 years. Lowell inferred that its mass was between that of the Earth and that of Neptune, and the body's extreme faintness would lead to the conclusion that its diameter cannot greatly exceed that of the Earth.

Illustration of Dimensions in the Solar System.—In order to obtain a clear idea of the relative dimensions and distances of the planets, it is useful to imagine the model of the Solar System described by Sir John Herschel in his *Outlines of Astronomy* and widely quoted elsewhere, particularly by Young in his excellent textbooks.

"Choose any well-leveled field. On it place a globe, two feet in diameter; this will represent the Sun; Mercury will be represented by a grain of mustard-seed, on the circumference of a circle 164 feet in diameter for its orbit; Venus a pea, on a circle 284 feet in diameter; the Earth also a pea, on a circle of 430 feet; Mars a rather large pin's head, on a circle of 654 feet; Juno, Ceres, Vesta, and Pallas, grains of sand, in orbits of from 1,000 to 1,200 feet; Jupiter a moderate-sized orange, in a circle nearly half a mile across; Saturn a small orange, on a circle of four-fifths of a mile; Uranus a full-sized cherry, or small plum, upon the circumference of a circle more than a mile and a half, and Neptune a good-sized plum on a circle about two miles and a half in diameter."

The reader may readily apply these distances to familiar objects in his own neighborhood.

EXERCISES

1. Could we ever see an occultation of Mars by Jupiter? By Venus?

2. As seen from Saturn, what would be the apparent diameter of the Earth when in transit upon the disk of the Sun? What would be the Earth's greatest elongation?

Ans. 2″; 6°

3. When Mercury is seen as a black dot in transit upon the Sun, is its apparent motion across the disk in the same direction as that of the Moon during a solar eclipse?

4. Suppose that Venus's rotation period is 224 of our days, its revolution period being 225. What would be the length of the solar day on Venus?

Ans. 50,400 of our days, or about 138 years

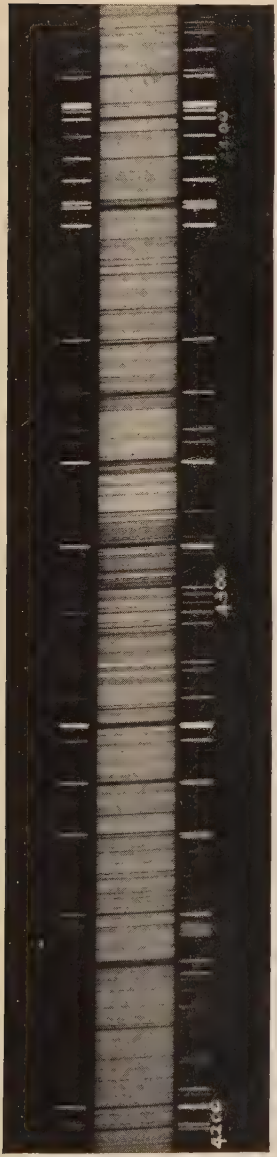
5. In that case, what region of the planet would be best adapted to habitation?

Ans. That of the great circle dividing the bright half of the planet from the dark. As this would revolve around the planet once in about 138 years, the population would perhaps be migratory.

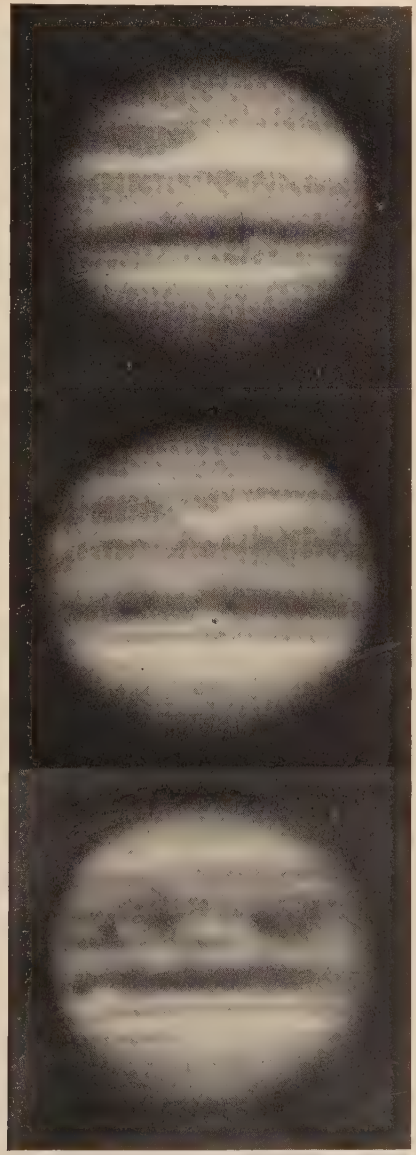
6. Why are not eclipsed satellites of Jupiter dimly visible, like the Moon during a total lunar eclipse (page 142)?

7. When Jupiter is at east quadrature (evening star) do the satellites precede or follow their shadows as they transit across the planet's disk?

PLATE 26. STUDIES OF JUPITER MADE AT LOWELL OBSERVATORY



Spectrum of Jupiter, photographed with slit parallel to planet's equator. Inclination of lines is due to planet's rotation.



Three photographs of the same face of Jupiter, taken in blue light, 1928 Sept. 14, Oct. 6, and Nov. 30. Showing progressive fading of Red Spot and darkening of south tropical zone.

Plate 27. Saturn and Its Rings, Drawn by Percival Lowell, 1912 February 6

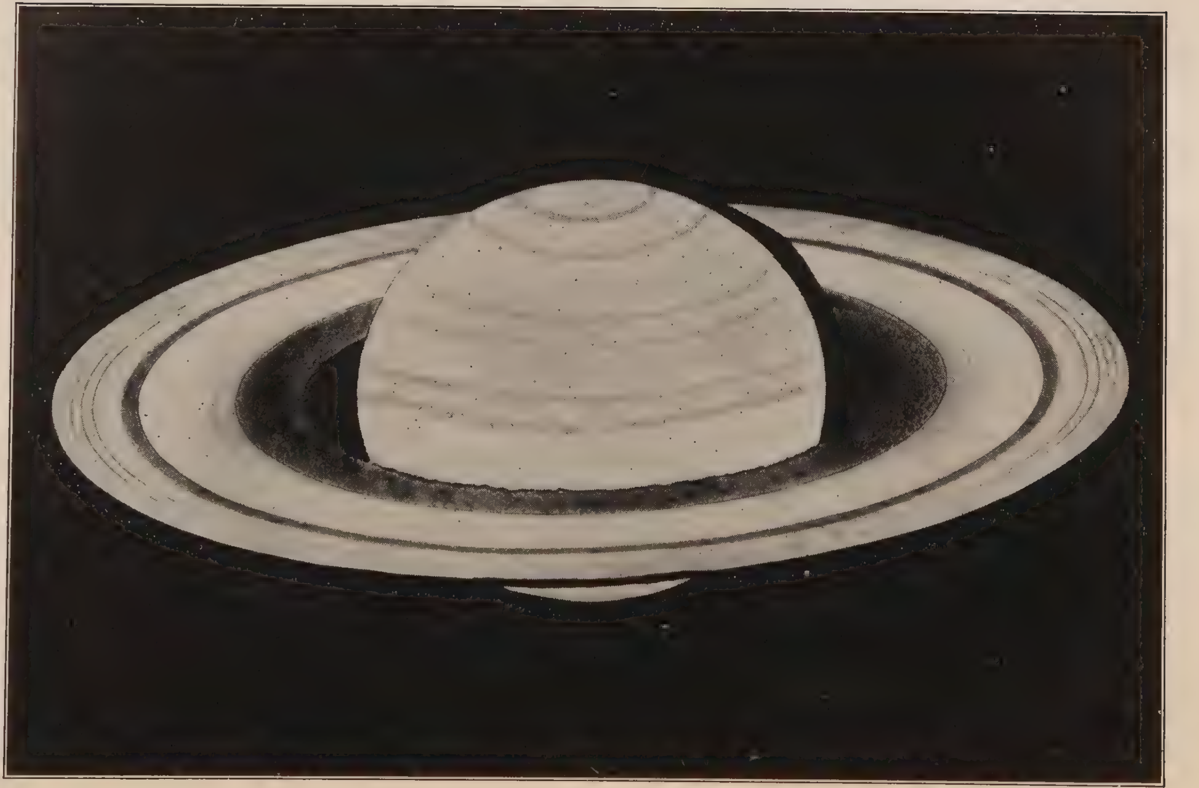
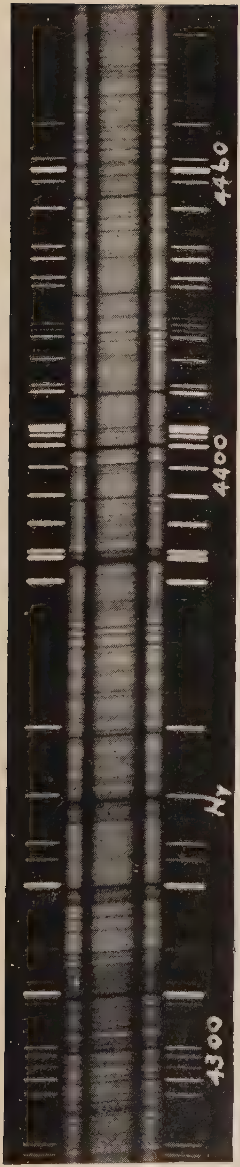
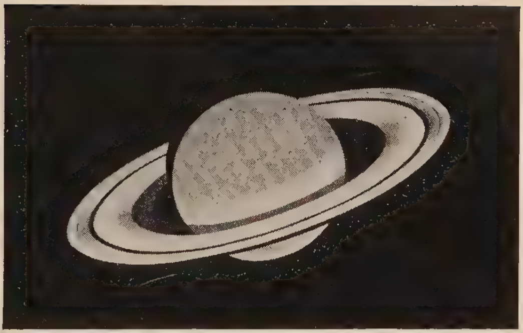
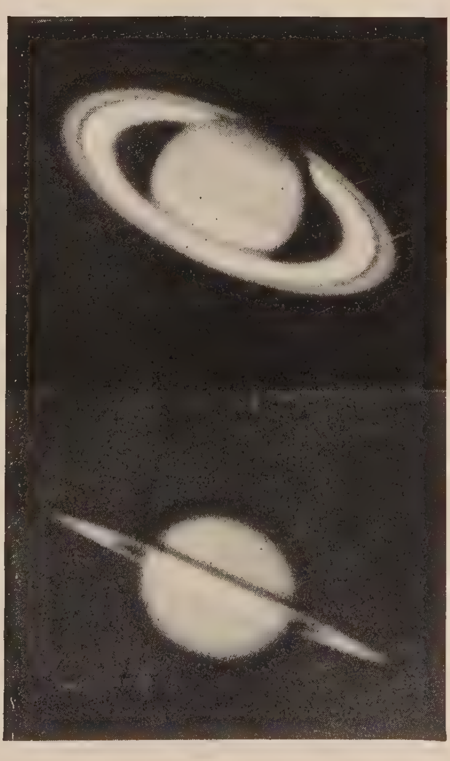


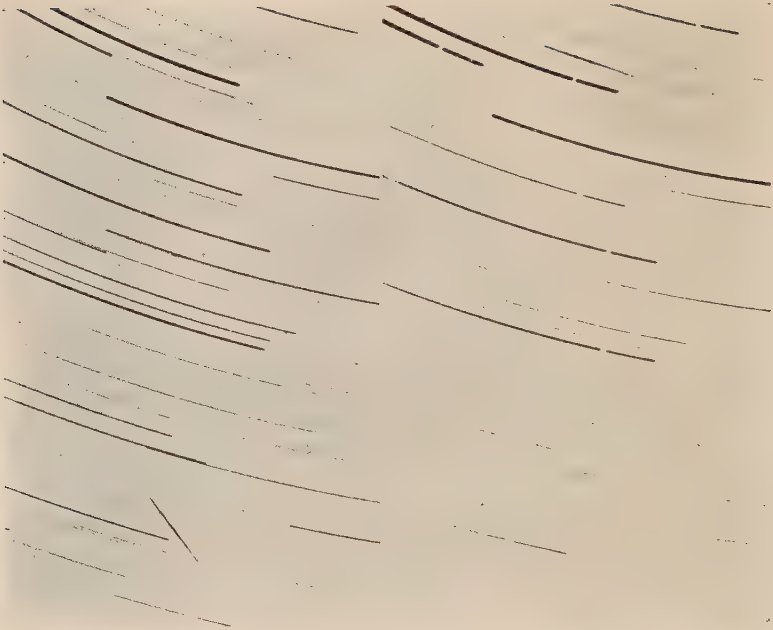
PLATE 28. SATURN



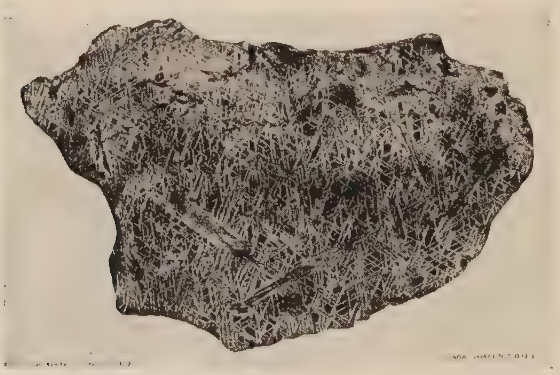




Left—Spectrum by V. M. Slipher, Lowell Observatory, showing distortion of lines due to motion. See Figure 140
Upper right—Drawing by Proctor
Middle—Photograph by E. C. Slipher, 1916 February 11
Lower—Photograph by E. C. Slipher, 1922 May 24



Meteor and polar star trails photographed simultaneously by J. Klepešta at Prague (left) and J. Sýkora at Ondřejov Czechoslovakia, 1925 August 9. In the Ondřejov plate the meteor is near the top, almost parallel to the star trails



Etched surface of a meteorite, showing the Widmannstätten figures

CHAPTER XII

METEORS AND COMETS

General Description of Meteors and Comets.—A few minutes' watch of any part of the sky on a clear, moonless night is sufficient to detect a number of the sudden, brief flashes which are popularly called **falling stars.** Occasionally one of these appears as bright as Jupiter or Venus, and perhaps leaves behind it a phosphorescent **train** which may float slowly over the sky, remaining visible for minutes. Very rarely, a blinding flash occurs, accompanied by a loud noise, and a heavy body falls to the ground. Such a body, after its fall, is called a **meteorite,** but while moving through space the bodies which cause the flashes are called by astronomers **meteors.**

The word meteor, in its older and more general sense, means any natural object or phenomenon seen high in the air—a flash of lightning, a hailstorm, and a rainbow are all "meteors" in this sense—and the science of **Meteorology** deals with phenomena of the weather, but has nothing to do with meteors in the astronomic sense. Until near the end of the eighteenth century, the study of meteors was not considered to belong properly to astronomy at all; their flashes were believed to be atmospheric phenomena, while the falling of stones from the sky was regarded as a miracle or discredited as impossible.

A well-developed **comet** (*ἀστήρ κομήτής*, long-haired star) consists of a small bright **nucleus** which appears star-like except with high magnification, surrounded by a diffuse, misty envelope or **coma,** which is greatly extended on the side opposite the Sun to form the **tail.** The tails of some comets attain an apparent length of many degrees and a real length of many millions of miles. Many comets, however, lack both nuclei and tails and are visible only in the telescope, appearing merely as little hazy patches of light which are distinguishable from faint nebulæ only by their motion. Comets appear usually at unpredicted times and positions, cross the sky with a motion which is perceptible from night to night, and fade from view after a few weeks or months. On the average about four or five comets are seen each year; the greatest number yet recorded for any year is eleven, in 1925.

Bright comets have throughout human history attracted a great deal of attention because of their sudden and extraordinary appearance, and have been regarded by the ignorant and the superstitious as omens of "famine, pestilence, and war." Like meteors, they were supposed to be situated in the Earth's atmosphere until Tycho Brahe proved that the parallax of a great comet which appeared in 1577 was less than that of the Moon and that therefore the comet could not be "sublunary."

The study of comets shows that although, like the planets, they move under the control of the Sun's gravitation, they are contrasted with the planets in almost every other respect. The planets are compact, opaque, and nearly spherical bodies moving all in the same direction in orbits which are regularly spaced, almost circular, and nearly in the same plane. The comets move on highly eccentric orbits, most of which are either parabolas, or ellipses of such size and eccentricity that their observed portions appear parabolic. The planes of their orbits are inclined to the ecliptic at angles ranging from 0 to 90°, and a large proportion of the comets move in the retrograde direction. The comets themselves are bodies of strange form, immense size, and insignificant mass which change shape and dimensions greatly and rapidly (Plates 32, 33), which shine partly by reflected sunlight and partly by light of their own, and which exhibit evidence of repulsive as well as of attractive forces.

The study of meteors proves that they are tiny solid bodies which move around the Sun like the comets but are never seen by man until they enter the Earth's atmosphere and become luminous by the transformation of the energy of their rapid motion into heat. It is known certainly that many meteors, possibly all, are connected with comets, and many are probably in fact the remains of comets which have disintegrated.

Determination of the Path and Speed of a Meteor Through the Air.—For the observation of meteors the telescope is of little use because of its narrow field. Observations are best made with the naked eye by plotting the apparent path on a star-map, or with a wide-field camera which records photographically the trails of bright meteors among the stars. The time of the flash should be noted and the duration of visibility estimated or timed with a stop-watch.

Observation of a meteor from a single place is not sufficient to determine its real position, but if the same meteor be seen simultaneously by two observers stationed a few miles apart a difference of apparent place results from parallax (Plate 29), and from the two observations the actual height and location of the meteor may be found. The positions of meteors were first thus determined by triangulation in 1798, by two young students, Brandes and Benzenberg, at Göttingen.

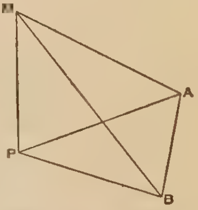


FIG. 142. DETERMINATION OF THE HEIGHT AND DISTANCE OF A METEOR

The plot of the apparent path on the star-map (or the photographic trail) gives the right ascension and declination of each point on the meteor's visible path. From the recorded time and the right ascension, the hour angle may be found, and the hour angle and declination may be transformed into altitude and azimuth, either by trigonometric calculation or by the use of a celestial globe. Neglecting the curvature of the Earth, let *A* and *B* (Fig. 142) be the two observers, *M* a point of the meteor's path, say the point of first appearance, and *P* the point of the Earth's surface vertically beneath *M*. In the horizontal triangle *ABP*, the side *AB* is the distance between the observers, and each of the adjacent angles *PAB* and *PBA* is the difference of azimuth of the meteor and one observer as seen by the other. These three quantities being known, the triangle may be solved and the sides *AP* and *BP* computed. In the vertical right triangles *APM* and *BPM* the side *PM* is the vertical height of the meteor and the angles *MAP* and *MBP* are its apparent altitudes at the two stations; hence, in either triangle, the base and its adjacent angle are known and the height can be computed. From the distances *AP* and *BP* the point *P* may be directly located on a terrestrial map. Having these data for both the beginning and the end of the path, and the time during which the meteor was visible, its entire path and the velocity with which it traveled may be determined.

Meteors seldom appear at heights exceeding one hundred miles, and most of them disappear before descending lower than a height of thirty miles. Their velocities average about twenty-six miles (forty-two kilometers) a second, which is the parabolic velocity due to the Sun's attraction at the distance of the Earth (page 229). This velocity is what would be expected if meteors, like most comets, move around the Sun on parabolas or highly eccentric ellipses. The actual observed velocity, which is the resultant of the velocities of the meteor and the observer, ranges between wide limits.

Source of the Light of Meteors; Their Size.—The luminosity of meteors is readily explained by the high temperature which must be produced by the conversion of their kinetic energy into heat. The kinetic energy of a body of mass m moving with a velocity of v centimeters per second is equal to $\frac{1}{2}mv^2$ ergs. In free space, the velocity of a meteor is many hundreds of thousands of centimeters per second, vastly greater than that of any terrestrial projectile, and, as the energy is proportional to the *square* of the velocity, it is inconceivably great. When the meteor encounters the atmosphere, its motion is checked and its kinetic energy is transformed into heat. From the observed brightness and velocity of meteors their mass can be estimated, and it turns out that the ordinary shooting star is a mere speck, having a mass of only a few milligrams. The temperature produced by the meteor's violent rush through the air is enormous—several thousand degrees—and this explains the surprising brilliancy of the light, and also the fact that none of these tiny meteors are ever seen to reach the ground; they are vaporized by the intense heat, and upon cooling condense into impalpable dust.

Meteorites.—Thousands of meteorites, many of which were actually seen to fall, are preserved in museums. They are classified roughly as stone and iron meteorites. Of meteorites contained in museums, the irons are the more numerous, but this is because they are the more easily identified; many more stones than irons have been seen to fall. The largest meteorite in any collection is an iron weighing about 36 tons, which was brought by Peary from Greenland to the Museum of Natural History in New York City. A block of meteoric iron which lies near Grootfontein, South West Africa, is estimated by Luyten to weigh not less than 50 tons.

Near Cañon Diablo, in northern Arizona, is a crater known as Coon Butte, about three-fourths of a mile in diameter and with walls about 600 feet high, of which the floor is lower than the surrounding plain, as in the lunar craters. On the plain around the crater great quantities of meteoric iron have been found, the largest pieces weighing several hundred pounds each, and competent authorities are of the opinion that the crater was made by the impact of an enormous meteorite, of which these are fragments.

What may have been a still larger meteorite or group of meteorites is reported to have fallen on July 30, 1908, in a sparsely inhabited region of northern Siberia, and to have destroyed forests over an area several miles in radius.

Iron meteorites are nearly pure iron with usually a small admixture of nickel. Very often the iron is crystallized in a peculiar fashion so that when cut, polished, and etched it exhibits characteristic markings which are known as Widmannstätten figures (Plate 29). Stone meteorites also contain some iron, and about thirty other elements have been identified in them, but none not already known in terrestrial substances. Many meteorites contain large quantities of occluded hydrogen, helium, and other gases.

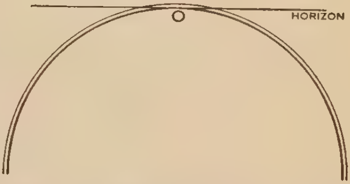


FIG. 143. WE CAN SEE HALF THE CELESTIAL SPHERE BUT NOT HALF THE METEORS

If discovered soon after its fall, before the surface has become weathered, a meteorite is usually found to be covered with a thin black crust formed by the melting of its surface during its passage through the air.

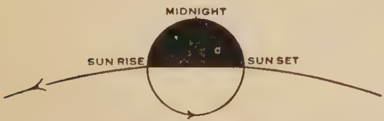


FIG. 144. WHY METEORS ARE MORE NUMEROUS AFTER MIDNIGHT

Number of Meteors.—Although an observer may, from a single point of view, see half the celestial sphere, he can see only a small fraction of the meteors in it, because of the relative shallowness of the air (Fig. 143). Moreover, as every telescopic observer knows, there are many meteors too faint to be perceived by the naked eye. Taking these facts into account, it has been estimated from the number of meteors actually seen that the total number which enter the air daily must be many millions. Their dust, settling through the air, is slowly—but very slowly—adding to the mass of the Earth.

It may be noticed, by anyone who takes the trouble to watch the sky all night, that the meteors seen after midnight are about twice as numerous as those seen before. This is because, in the latter half of the night, we are riding on the front side of the Earth as it moves along its orbit (Fig. 144) and receive meteors from all directions, whereas in the earlier half we see none of those which the Earth meets "head on." For the same reason, the meteors of the morning hours have a greater apparent velocity and are bluer because of the greater intensity of heat generated.

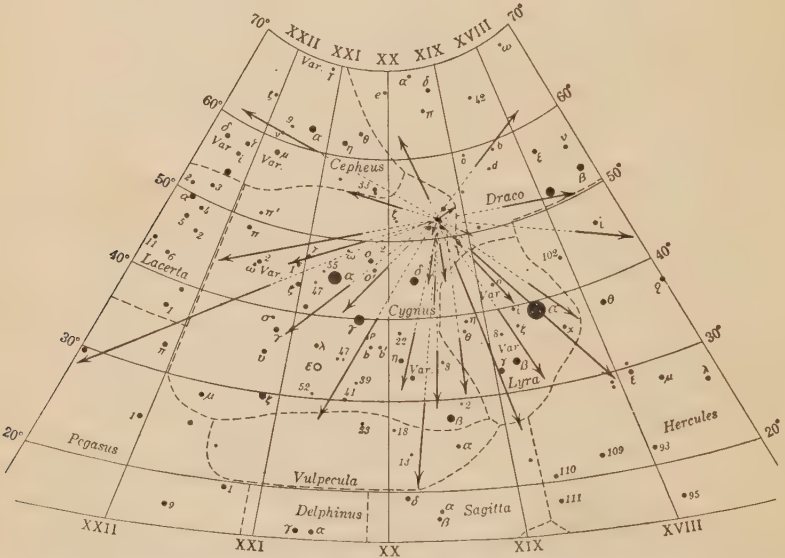

70°
XXII
XXI
XX
XIX
XVIII
70°
60°
60°
50°
50°
40°
40°
30°
30°
20°
20°
Cepheus
Draco
Lacerta
Cygnus
Lyra
Hercules
Pegasus
Vulpecula
Delphinus
Sagitta
Var.
XXII
XXI
XX
XIX
XVIII

FIG. 145. THE CYGNIDS OF 1922 AUGUST 15–26, OBSERVED BY R. M. DOLE (FROM *Popular Astronomy*)

Meteor Swarms and Showers.—When the apparent paths of all the meteors observed on a given night are plotted on a star-map, it is often found that many of them radiate from a single small spot in the sky; that is, if the lines representing the observed paths are extended backward, many of them intersect at nearly the same point (Fig. 145). This point is called the **radiant** of those particular meteors. The meteors seem to fly in all directions from the radiant, and it is not unusual for a "falling star" to move apparently upward instead of downward. Those seen nearest the radiant have the shortest apparent paths, and a meteor exactly at the radiant seems stationary, simply appearing and vanishing without apparent motion.

The radiant has the same place among the stars for observers at different stations, and therefore must be infinitely distant, while the apparent divergence of the meteor paths is an effect of perspective, the meteors concerned moving, in fact, along parallel lines as in Fig. 146. Meteors which move in this way are said to belong to a **meteor swarm,** which is named for the constellation in which the radiant appears; thus, the swarm of **Perseids** has a radiant in Perseus, the **Leonids** one in Leo, etc. Sometimes the Earth encounters a swarm so dense that thousands of meteors are seen in a single night, as happened with the Leonids in 1833 and 1866, and the Andromids in 1872. A display of many meteors from the same swarm is called a **meteoric shower.**

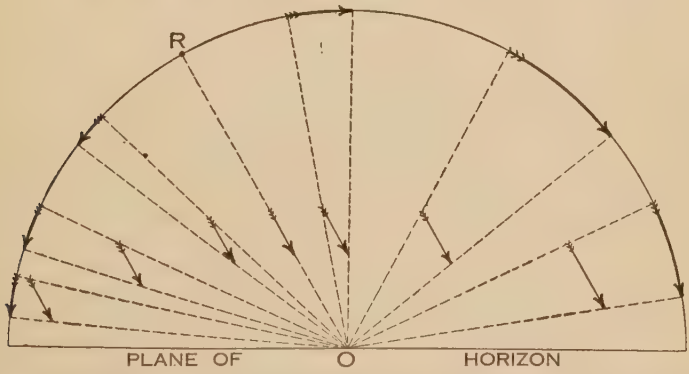


Fig. 146. The Radiant of a Meteor Shower is an Effect of Perspective

The words "swarm" and "dense" are far from applicable if taken with their usual connotation. Even when meteors appear on the same path at the rate of four or five a second, they must still be several miles apart, and it is only by hyperbole that pin-heads or dust-specks several miles apart can be said to constitute a dense swarm.

If the swarm be large, the Earth may require several days to cross it, so that a shower may last several nights; but, since the Earth can encounter a given swarm at only one place in its orbit, the shower cannot be repeated oftener than once a year.

Orbits of Meteor Swarms.—The position of the radiant is determined by the direction and magnitude of the relative velocity of the meteors and the observer, and this depends both upon the orbital and diurnal motions of the Earth and upon the orbital motion of the meteors. The motions of the Earth can be readily allowed for, and from the resulting motion of the meteors the elements of their orbits can be computed. Meteor swarms have thus been found to move around the Sun in elliptic orbits, and many of these have been proved to be identical with the orbits of known comets, thus establishing a close connection between the two classes of bodies.

The first such identification was made by Schiaparelli in 1866, between the Perseids and Tuttle's comet of 1862. A famous case is that of Biela's comet and the Andromids. The comet, discovered in 1826 by Biela, was shown to move on an elliptic orbit with a period of about 6.6 years, and was identified with a comet seen as early as 1772 by Montaigne and in 1805 by Pons. There was nothing especially remarkable about it until 1846, when it divided in two. When next seen in 1852 the two components had moved farther apart, and they have not been seen since; but in several years when the comet was due to appear, particularly in 1872, there have been notable showers of Andromids. Here the meteors seem to be products of the comet's disintegration; in some other cases the comet seems to be accompanied by the meteors, being the most conspicuous body of the aggregation.

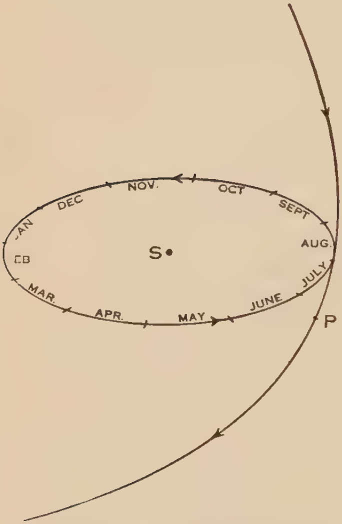


FIG. 147. THE ORBITS OF THE EARTH AND THE PERSEID METEORS, PROJECTED ON THE PLANE OF THE METEOR ORBIT

The most important meteor swarms are listed in Table 15. The Perseids are evidently distributed fairly equally in a ring which extends all the way around their orbit, for a noticeable shower occurs every year. While a few stragglers of the other swarms also are seen each year, the main shower generally occurs only once in the meteors' period, showing that these swarms are more compact.

Discovery and Designation of Comets.—Large comets which come near the Sun and the Earth are spectacular objects, some of which have been visible in broad daylight and readily command attention. Records exist of about four hundred which appeared before the invention of the telescope. The number discovered since that event is greater, but most of these later comets have been faint. In most cases neither the time nor the place of their first appearance can be predicted, and they are usually found by searchers with small, wide-field telescopes, or, in recent decades, upon celestial photographs.

The news of such discoveries is communicated to the principal observatories by telegraph and cable through central stations at Harvard and Copenhagen. When a comet is found, its right ascension and declination at a designated time on the night of discovery are telegraphed to the central station, which relays the news to the subscribing observatories. Measurements of the comet's position are then made and communicated by various observers, and as soon as positions on three nights are made known a preliminary orbit is computed and the elements and a short ephemeris are sent in. This prompt procedure has probably saved many comets from being "lost."

TABLE 15

METEOR SWARMS

Name	Approximate Date of Shower	Period	Appearance of Meteors	Associated Comet
Lyrids.....	April 20	415 years	Swift	1861 I
Perseids...	August 10	120	Swift, bright, with trains	1862 III
Orionids...	October 19		Swift	
Leonids....	November 14–15	33.3	Very swift, blue	1866 I
Andromids	November 17–27	6.6	Slow, with trains	Biela's
Geminids..	December 11–12		Swift, white, short	

In the preliminary computation the orbit is assumed to be parabolic, as this assumption simplifies the work and is usually at least accurate enough to provide a "finding ephemeris" good for several days. When more observations have accumulated, they are used for correcting the preliminary elements to obtain a more accurate orbit. For the determination of a parabolic orbit the method developed by Olbers in the early part of the nineteenth century is probably the most convenient; but for an elliptic orbit, and especially for modifying a preliminary parabolic orbit to agree with later observations, the method developed by Leuschner from the older method of Laplace is often shorter and better. Much of the computation of the orbits of comets and asteroids is done at the Students' Observatory of the University of California, of which Leuschner is Director.

Comets are temporarily designated by a letter and year number in the order of their discovery; later, when the elements of the orbit have been well determined, the letter is superseded by a Roman numeral following the year number, the order being that of perihelion passage. A comet is often known also by the name of its discoverer or of a person who has made an important research upon it. Thus Morehouse's comet, Comet *c* 1908, and Comet 1908 III are identical.

The Orbits of Comets.—After Tycho had proved that comets are more distant than the Moon (page 274), Kepler made observations of two comets of his time, and came to the conclusion that they moved freely through the planetary orbits, with a motion not far from rectilinear. Hevelius ("a noble emulator of Tycho Brahe," as Halley calls him), supposing that Kepler was right, "complained that his calculations did not agree perfectly with the matter of fact in the heavens," and became aware that the path of a comet was curved toward the Sun. In 1680 appeared a great comet whose orbit was shown by Doerfel and by Newton to be a parabola.

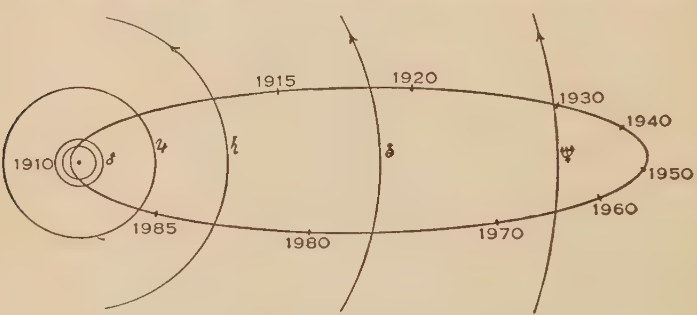


FIG. 148. THE ORBIT OF HALLEY'S COMET

Later, by using the gravitational principles developed by Newton, and "by a prodigious deal of calculation," Halley determined parabolic orbits for twenty-four bright comets which had appeared from 1337 to 1698. Noticing that the orbits of the comets of 1531, 1607, and 1682 were very similar, he concluded that these comets were the same, and that the orbit was really a long ellipse with a period of about seventy-five years. He accordingly predicted the return of this comet in 1758 and, although he did not live to see it, his prediction was fulfilled; and the comet, which has ever since been famous as **Halley's Comet,** was seen again promptly in 1835 and 1910. By taking account of planetary perturbations, Cowell and Crommelin have been able to determine its path with such accuracy as to identify it with recorded comets at every appearance save one since 240 B.C. Fig. 148 is a diagram of its orbit with the distances of the planets shown for comparison.

About three-fourths of all the cometary orbits which have ever been computed appear to be parabolic. If they are strictly so, the mathematical significance is that these comets fall from rest at an infinite distance from the Sun, pass around it once, and recede again to infinity. Since, however, the parabola is the limiting case between the ellipse and the hyperbola, it is highly probable that the orbits which seem parabolic are either slightly elliptic or slightly hyperbolic; and since no distinctly hyperbolic orbits have been found, astronomers are mostly of the opinion that the orbits of these comets are really long, eccentric ellipses with periods of many centuries or millenniums.

Some of these comets have very close perihelia; for instance, the great comets of 1668, 1843, 1880, 1882, and 1887, the orbits of which were similar to one another, passed within a few hundred thousand miles of the Sun's surface and must have gone right through the corona. Their periods are at least several centuries, but they swung through 180° of their orbits near perihelion in a few hours, moving at a speed of 300 miles a second or over. Near aphelion, where such comets spend most of their time, their motion is quite slow. The majority of observed comets have perihelia within the orbit of the Earth, and all but one approach more closely to the Sun than the orbit of Jupiter; but it is very likely that many comets have greater perihelion distances and escape detection for this reason, for, in general, comets are very inconspicuous objects until they are well within the orbit of Mars.

The exceptional comet referred to is 1925 II, which was discovered by Schwassmann and Wachmann in 1927, two years after its perihelion passage. Its perihelion distance is 5.6 astronomical units and its aphelion distance 7.2, so that its orbit lies wholly between those of Jupiter and Saturn. Its period is about sixteen years and the eccentricity of its orbit only 0.14—less than that of the orbit of Mercury. But for the hazy appearance of this object, it might well have been classed as an asteroid instead of a comet.

Many comets are known definitely to move in elliptic orbits with periods less than a century. Most of these are faint, Halley's comet being the only "great comet" among them. The shortest known period is that of Encke's comet, about 3.3 years. About fifty periodic comets have their aphelia at about the distance of Jupiter from the Sun, and are said to belong to Jupiter's **comet family.** Saturn and Uranus have families of two comets each, much less certainly established, and Neptune has a family of seven, one of which is Halley's comet. Halley's comet, the comet of 1827 (which also belongs to Neptune's family), Tempel's comet of the Leonid meteors, and Tuttle's comet of the Perseids all have retrograde motion; all the other short-period comets move directly. This is in contrast to the case of the parabolic comets, about as many of which have retrograde motion as direct.

The existence of so numerous a family of comets as that of Jupiter could not well have come about by chance. According to the **capture theory,** these comets moved originally in parabolic orbits which were changed to ellipses by the perturbative action of Jupiter. This would happen if Jupiter and the comet approached each other in such a way that Jupiter's attraction caused the comet to lose about half of its velocity. It was shown by Chandler by computations on the orbit of Comet 1889 V that in 1886 it had passed within the orbit of Jupiter's closest satellite, and that the action of Jupiter was such as to have reduced the comet's period from twenty-seven to six years. Barnard observed in 1889 that this comet was double, and that the two parts were separating at such a rate that they must have been together in 1886, at the time of the encounter. However, an approach to Jupiter is as likely to result in acceleration as in retardation of a comet, and acceleration of parabolic velocity would change the orbit to a hyperbola. Crommelin shows on the theory of probability that only one comet in half a million would pass close enough to Jupiter to have its velocity reduced by half, and is inclined to agree with a theory proposed by Proctor, who suggested that the Jupiter comets originated in the planet, having been emitted by volcanic or analogous action.

Dimensions, Mass, and Density of Comets.—Comets are the bulkiest objects in the Solar System. The nucleus, when one is present, may have any diameter up to ten thousand miles; the coma, a diameter from ten thousand to several hundred thousand miles; and the tail, a length of many million miles. The tail of the great comet of 1843 was said to be more than five hundred million miles long—longer than the radius of the orbit of Jupiter.

The mass of comets, on the other hand, is so small that it has never been detected at all, for comets have no perceptible perturbative action on other bodies. Even when Comet 1889 V went through Jupiter's satellite system it produced no appreciable derangement of the satellites. It is fairly certain that no comet's mass exceeds one one-hundred-thousandth the mass of the Earth.

The mean density of a comet must, therefore, be exceedingly low—certainly thousands of times lower than that of air at sea-level. It is therefore not surprising that comets are transparent. Faint stars are often seen through the heads of comets thousands of miles in diameter. This low mean density does not signify, however, that a comet is a tenuous gas throughout; it is more likely that the head, at least, contains much solid matter in the form of small bodies like meteors, separated by distances which are large compared to the diameters of the bodies, probably with highly rarefied gas between.

The Spectra of Comets.—The spectrum of a comet which is faint and distant from the Sun appears to be continuous and is probably due wholly to reflected sunlight. As the comet nears the Sun, bright bands make their appearance and increase in intensity as the distance diminishes. These bands are certainly due to light emitted by the comet itself. With increasing brightness of the spectrum, the Fraunhofer lines of the solar spectrum may also be discerned upon the continuous background. The brightest bands are identical with those of the Swan spectrum (page 154), which are due to hydrocarbons and may be seen in the blue light of a Bunsen gas flame. The principal Swan bands have sharply defined edges or "heads" at wave-lengths 4737, 5165, and 5635 Angstroms, and each fades out gradually on the side toward the violet. Cyanogen, a compound of carbon and nitrogen, is also indicated in the spectra of comets' heads, principally by a band at λ 3883. Bright comets which approach near the Sun show bright D lines of sodium. The great comet *a* 1910 was bright yellow in color, due to the predominating brilliancy of these lines. The golden color of Skjellerup's great comet of 1927 so enhanced its contrast with the blue sky that the comet was easily observed and even photographed at the Lowell Observatory at midday within a few degrees of the Sun. When near perihelion, the great comet of 1882 is said to have shown, in addition to the Swan bands and the D lines, bright lines of iron.

Plate 31 consists of photographs of comet spectra. The objective-prism spectrogram of Halley's comet was made with the edge of the prism placed parallel to the comet's tail, so that the tail itself may be said to have served as a slit. Each image of the double-tailed comet is thus a bright spectral line or band. The tailless image of the head at the left end of the spectrum is due to cyanogen, which evidently did not extend into the tail. The bright horizontal streaks are the spectra of stars.

The spectra of comets *a* 1915 and *b* 1914, which in Plate 31 are placed together for comparison, show the Swan bands and a cyanogen band at λ 4215, and also the background of continuous spectrum with weak Fraunhofer lines. The spectrogram of the great comet of 1927 was made in full daylight, early in the afternoon of December 19, the day after perihelion passage. The comet was then some 20,000,000 miles from the Sun, but so nearly in the Sun's direction that it would have been lost in the bright sky save for its yellow color and great brilliancy (several times the brilliancy of Venus). Most of the broad central spectrum shown in the photograph is due to sunlight reflected from the sky, but the bright central D lines belong to the comet. Their displacement relative to the corresponding comparison lines is a Doppler-Fizeau effect due to the comet's recession at the rate of 90 km./sec.

The emission (bright-band) spectra of comets appear at a distance from the Sun far too great to be caused by incandescence due directly to the Sun's heat, and the variation of their intensity with distance is also out of accordance with such an effect. Apparently the energy of the Sun's radiation is absorbed by the cometary material, to be re-emitted wholly in the particular wave-lengths of the comet spectra. The problem resembles that of the luminescence of nebulæ which seems to be derived from the radiation of neighboring stars (page 398).

The Nature of a Comet's Tail.—The tail of a comet does not trail behind it all the way, but extends always in a direction nearly opposite that of the Sun, lagging a little behind the prolongation of the radius vector (Fig. 149). It cannot be a fixed appendage, for if so, the tail of the comet of 1843, for example, must have traveled with a velocity comparable with that of light as the comet swept around the Sun. Photographic studies of numerous comets since 1890 have shown condensations in the tail which moved away from the head with velocities of several miles a second, thus proving that the tail is a flowing stream of matter. It is evident that the Sun exerts upon this matter a repulsion which is stronger than its gravitational attraction.

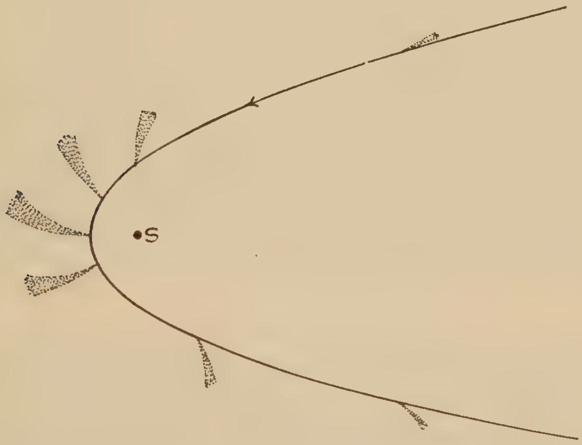


FIG. 149. POSITIONS OF THE TAIL OF A COMET IN DIFFERENT PARTS OF ITS ORBIT

Probably this repulsion is due to the pressure of the Sun's radiation (page 163), and the particles of the tail must be very small. Radiation pressure, however, is not sufficient to explain all the motions of the matter within a comet. The nuclei of some great comets—for example, Donati's comet of 1858 (Plate 30) and Morehouse's of 1908—have been surrounded at times by a number of **envelopes** of approximately parabolic form which were observed to dilate as if composed of matter expelled in all directions from the nucleus, the particles sent toward the Sun being driven back by the Sun's repulsion. The pressure due to the radiation of the nucleus is certainly not powerful enough to expel these envelopes, and it is more likely that the repulsive force here is electrical, the particles being ionized by the action of the Sun.

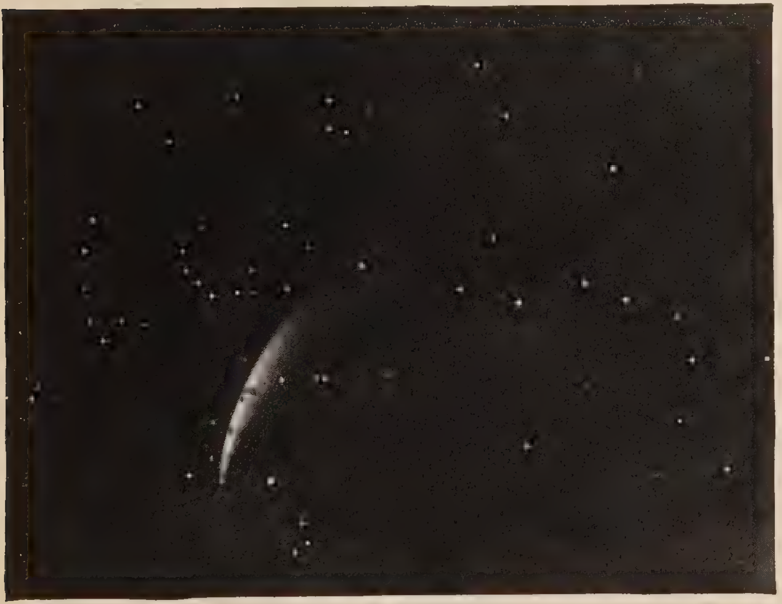
The Disintegration of Comets.—The material which flows outward in the tail of a comet can never rejoin the head. Being repelled by the Sun, each particle must travel on a hyperbolic orbit which is convex to the Sun and has the Sun in its outer focus, while the main body continues in its elliptic or parabolic path which is concave to the Sun. The tail-forming material of a comet must therefore waste away, although there is no certain observational evidence that the tails grow smaller or fainter at successive returns.

In addition to this wastage, a comet which comes very near to the Sun or to a large planet is subjected to enormous disruptive tidal forces which must be very effective in separating its loosely connected parts. Biela's comet and the fifth comet of 1889 are known to have divided in two, while the great comet of 1882, after its close perihelion passage, was attended by a number of smaller cometary bodies and its nucleus was divided into five. There is evidence also that comets have been disarranged by encounters with invisible obstacles, probably meteor swarms, as was the case of Brooks's comet of 1893, photographed by Barnard. This disintegration of comets affords a plausible explanation of the origin of metecr swarms.

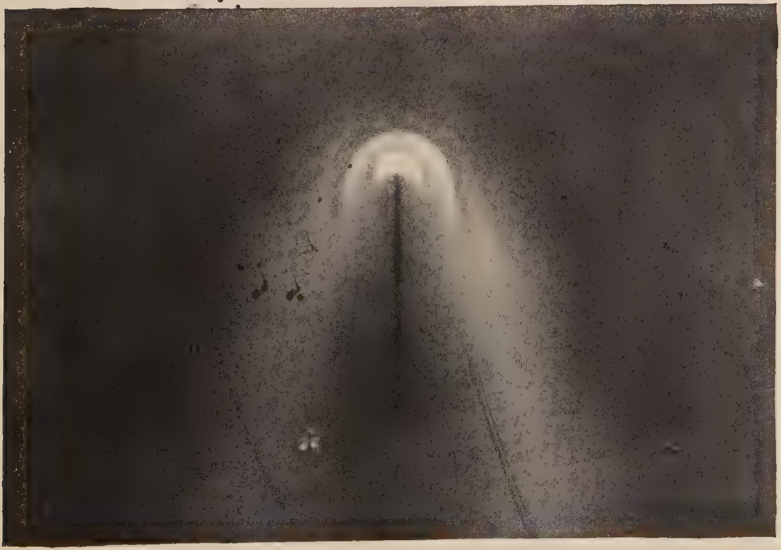
Encounters of the Earth with Comets.—A direct collision of the Earth with the head of a comet is not at all impossible, since many comets cross its orbit. The necessary condition is that the comet and the Earth arrive at the node at the same time. It has been computed from the theory of probability that such a collision is likely to happen about once in 15,000,000 years. Passages of the Earth through the tails of comets should be much more frequent since the tails are of such immense size.

On May 18, 1910, the head of Halley's comet crossed the plane of the Earth's orbit directly between the Earth and the Sun, thus performing a transit like the transits of Venus and Mercury and changing from a morning to an evening object. Its distance from the Earth was about 15,000,000 miles, and the tail is known to have been considerably longer than this and to have extended nearly in the Earth's direction. On the mornings of May 17 and 18 the tail, though faint, was a magnificent object which extended like the beam of a searchlight across the sky a distance of about 120°. A broader and fainter secondary tail involved a long arc of the ecliptic, and

PLATE 30. DONATI'S COMET OF 1858
Drawings by G. P. Bond, 1858 October 6. From *Annals of Harvard College Observatory*, Volume 3

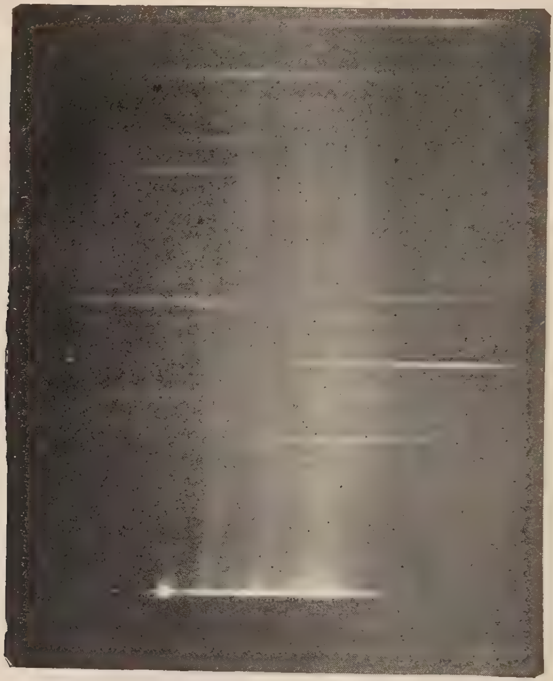


Appearance to the unaided eye

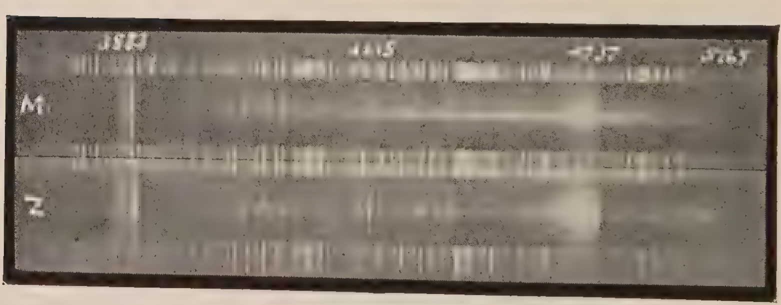


Appearance of the head in the 15-inch Harvard refractor

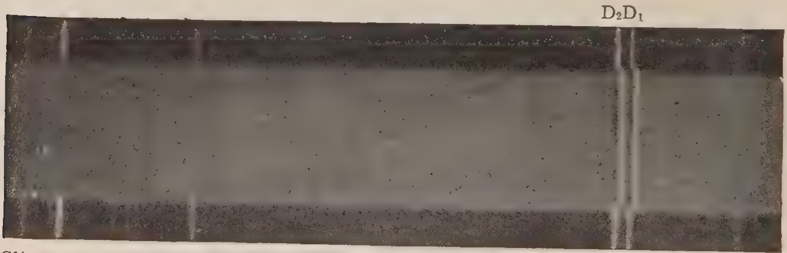
Plate 31. Spectra of Comets, Photographed by V. M. Slipher, Lowell Observatory



Halley's Comet, 1910 May 6. Objective prism spectrogram

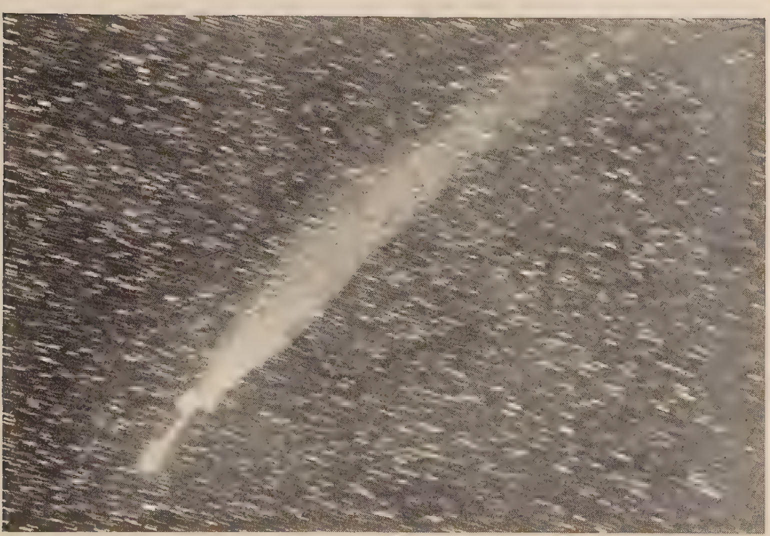


Slit spectrograms of Mellish's Comet *a* 1915 (M) and Zlatinsky's Comet *b* 1914 (Z)

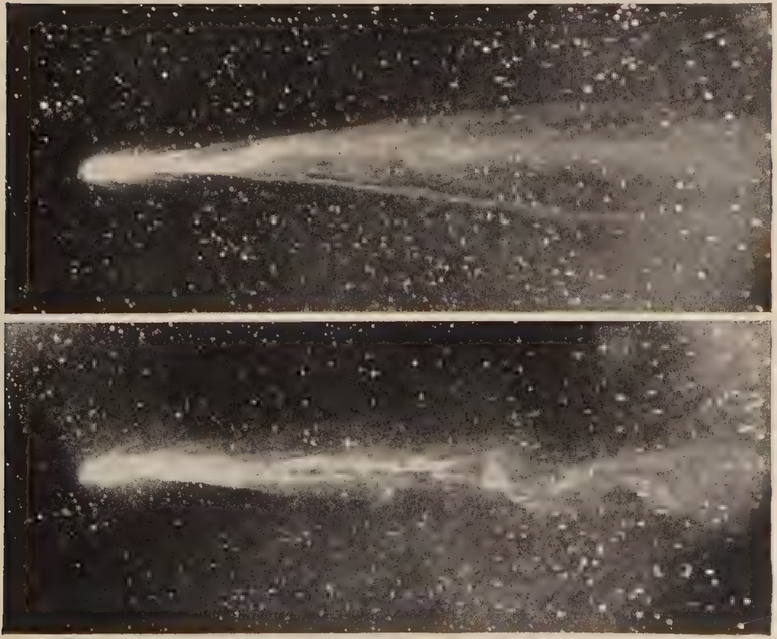


Slit spectrogram of the great Daylight Comet of 1927 (Skjellerup's Comet)

PLATE 32. PHOTOGRAPHS OF MOREHOUSE'S COMET, *c* 1908



October 15, by Barnard, Yerkes Observatory. The long star trails are due to the comet's motion during the exposure of 90 minutes. The enlargement in the tail was receding from the head at 3'.3 an hour.



November 17 (upper) and November 28 (lower) by E. C. Slipher, Lowell Observatory

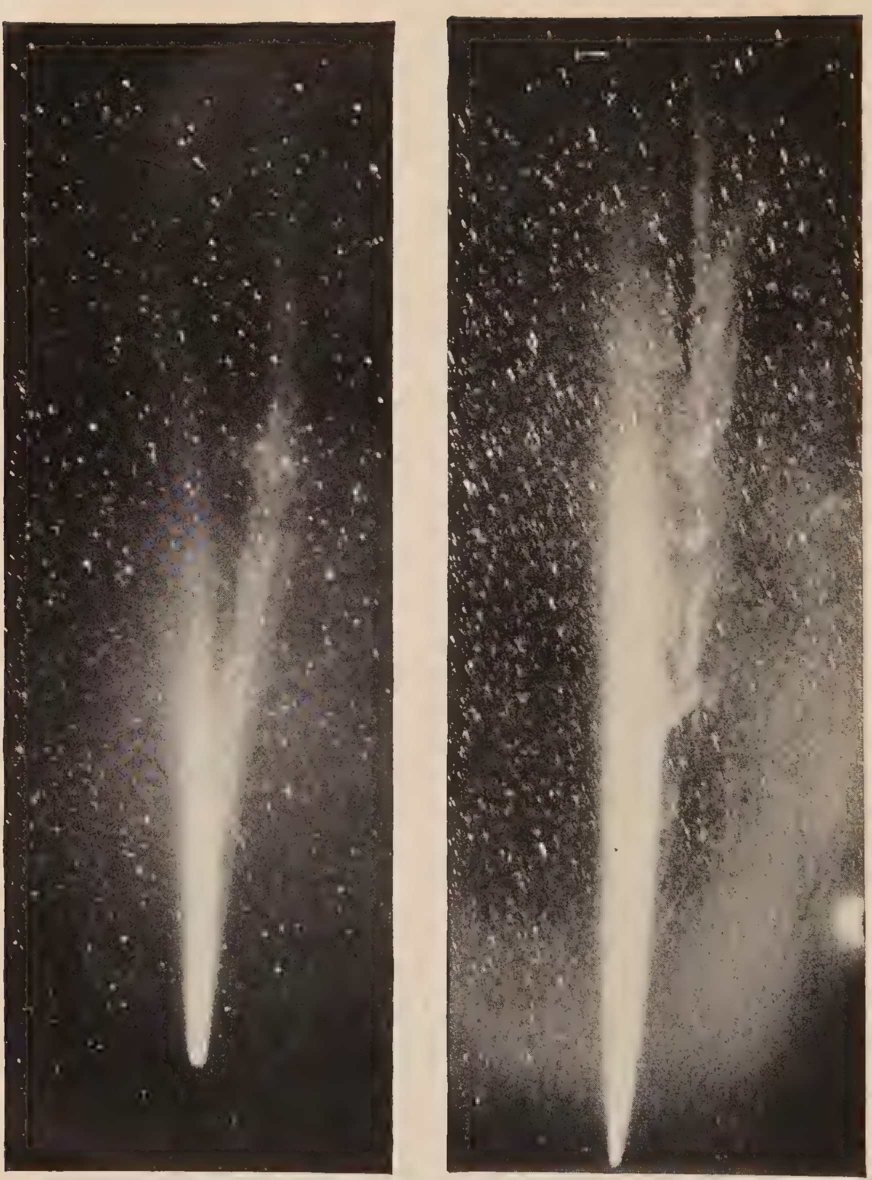


PLATE 33. PHOTOGRAPHS OF HALLEY'S COMET IN 1910
By C. O. Lampland, Lowell Observatory
Left—May 5. Length of tail 30°. Spectrum of right fork showed carbon monoxide bands, that of left fork was continuous. *Right*—May 13. Length of tail 50°. Venus appears at lower right edge, meteor trail 7° from head

there is little doubt that the Earth passed through this secondary tail soon after the transit of the head. Nobody was inconvenienced, although certain newspapers had caused some apprehension by describing the possible dire effects of the poisonous gases of the tail. It is, in fact, inconceivable that so rare a gas could do any harm. The only effect observed near that time which might be attributed to the comet was a slight iridescence of the sky noticed by Barnard and others during daylight on May 19. A similar encounter with a comet took place on June 30, 1861. An aurora was observed, but there is no special reason for connecting it with the comet.

EXERCISES

1. The Earth passes through about 80,000,000,000,000 cubic miles of space every twenty-four hours. Verify this statement. If it sweeps up 80,000,000 meteors per day, how much space, on the average, is allotted to each meteor?

Ans. 1,000,000 cubic miles

2. Suppose the Sun's repulsive force upon a particle in a comet's tail just equals the Sun's gravitational attraction. What path does the particle pursue relative to the Sun?

Ans. A straight line which is tangent to the comet's orbit at the point where the particle left the head

3. In a very eccentric orbit such as that of the comet of 1882, the aphelion distance is practically twice the mean distance. What is the greatest distance from the Sun attained by such a comet if its period is 1000 years?

Ans. 200 astronomical units

CHAPTER XIII

THEORIES OF THE EVOLUTION OF THE SOLAR SYSTEM

Regularities of the Planetary System.—So far as the law of gravitation is concerned, a planet might revolve in any direction in any plane passing through the Sun, and in a conic section of any size and of any form from the circle (ellipse of eccentricity 0) to the straight line (hyperbola of eccentricity ∞). The character of its orbit depends only on the speed, direction, and distance from the Sun at which the planet was launched. If, then, the world were a "fortuitous concourse of atoms," we should expect a great variety and even a great confusion among the planetary orbits. It is true that there would be no hyperbolic or parabolic orbits of bodies belonging permanently to the system; but the elliptic orbits would be of a great range of size and eccentricity, their planes would be inclined to each other at a great variety of angles, and about as many planets would revolve in one direction as in the other.

Instead of the random arrangement which would thus exist if the bodies of the Solar System had been brought together by chance, we observe a neat and orderly system which is characterized by the following regularities:

1. The system is relatively compact, the distance of the nearest star being thousands of times that of the farthest planet.
2. The orbits of all the principal planets and of the great majority of the asteroids lie nearly in the same plane.
3. All the planets and asteroids revolve around the Sun in the same direction.
4. The orbits of all the principal planets and of most of the asteroids are nearly circular.
5. The Sun rotates in the direction of the planets' revolution.
6. The plane of the Sun's equator lies near the planes of the planets' orbits.

7. There is a fair degree of regularity in the distances of the planets, expressed approximately by Bode's law.

8. The principal satellites of several of the planets form, with their respective primaries, systems which possess regularities similar to those of the planetary system itself.

Men of science are united in attributing this remarkably simple and orderly state of the planetary system to the working of some process of evolution.

The Meaning of Evolution.—The word **evolution** means an unrolling or unfolding. As used in modern science, it means usually a passage from unorganized simplicity to organized complexity. An example is the development of the semi-fluid, relatively homogeneous contents of an egg into a highly organized and complex animal.

To the ancients, the world consisted of a stationary Earth and a rotating firmament which carried stars fixed on its surface, and Sun, Moon, and planets which followed definite, re-entrant curves. Such simple motions as were observed among the heavenly bodies repeated themselves indefinitely. Even after Copernicus's description of the Solar System was accepted, the motions of the heavenly bodies were seen to be so orderly and the system of the world seemed so permanent, that men long assumed that the universe had been created in its present perfect state at some epoch in the not inconceivably remote past and that it would continue in this state until its final cataclysmic destruction at the hand of its Creator. However, the smaller works of nature with which we are familiar come to maturity not by sudden creation, but by a process of growth; and many of the facts that have been set forth in the preceding pages lead astronomers to believe that in the universe at large the order is that of gradual, ceaseless change—of evolution.

A theory of evolution is not a theory of creation. To say that stars have evolved from a nebula or that a nebula has originated in the close approach of two stars merely postulates the existence of an earlier state of the material in question which is different from the present state. The hypotheses which will be mentioned in the following pages make no attempt to explain the ultimate origin of the material.

Growth of the Theory of Evolution.—Thomas Wright's theory of the Milky Way (page 413) turned the attention of the brilliant German philosopher Kant to the subject of the origin of celestial systems, and in 1755 he published a book in which he advocated the theory that the Solar System had evolved from a nebula. Kant's work, however, attracted little attention and had no important influence on science.

In 1796 the great French mathematical astronomer Laplace published a work on general astronomy entitled *Exposition du Système du Monde*, at the end of which, in the last of seven brief appended notes, he outlined a hypothesis of the evolution of the Solar System which became famous as the Laplace **Nebular Hypothesis.**

Laplace's hypothesis is in many ways similar to Kant's, but there is no reason to believe that it was inspired by Kant, for Laplace states that the only previous attempt with which he was acquainted to account for the origin of the planets was that of Buffon, who had suggested their genesis in a collision of a comet with the Sun—a suggestion which Laplace disposes of in three paragraphs. The greater influence of Laplace's hypothesis is doubtless due in part to the great prestige of his mathematical work and in part to its having appeared just after the French Revolution, when his country was prepared for the introduction of new ideas.

Though presented by its author, as he said, "with that diffidence which everything ought to inspire which is not the result of observation or of calculation," the Nebular Hypothesis had an incalculable effect upon the scientific, philosophic, and religious thought of the following century. It furnished geologists with an account of the pre-geologic history of the Earth and encouraged them to view the changes which the Earth had undergone as continuous rather than cataclysmic. Biologists came to view the different forms of life, which are made evident by fossil remains in the strata of the Earth's crust, as forming a continuous chain from the lowest (simplest) animal or plant to the highest yet evolved—which may be man. These ideas were given definite form in Charles Darwin's book *Origin of Species*, published in 1858, which stirred up a storm of opposition on theological grounds which has not even yet everywhere subsided.

During the nineteenth century, as new information accumulated, the Nebular Hypothesis was modified or added to by a number of investigators. In 1900 the astronomer Moulton and the geologist Chamberlin, both of the University of Chicago, offered as a substitute for the Laplace hypothesis the **Planetesimal Hypothesis,** according to which the Solar System originated in a spiral nebula which itself was engendered by the close approach of two stars. More recently, the Englishmen, Sir James Jeans and H. Jeffreys have proposed what they call the **tidal theory,** which resembles the planetesimal theory in postulating a close approach of two stars, but differs in the subsequent details of the development of the planets and satellites. The planetesimal and tidal theories are sometimes referred to as hypotheses of **dynamic encounter.**

The student should bear in mind that none of these theories rest on any such secure foundation of observation or calculation as do most of the results of astronomical research. The changes in the universe for which it is their purpose to account took place so many millions of millions of years ago that it is doubtful that any such theory can ever be securely established. For this reason it does not seem profitable to enter into the subject extensively here, and in this chapter the discussion is limited to a brief summary of the Laplace hypothesis and the hypotheses of dynamic encounter.

The Laplace Nebular Hypothesis.—Laplace reasoned that, to impart to the planets the motions which they now possess, the Sun must once have been surrounded by an atmosphere which, due to its excessive heat, extended to a distance equal to that of the farthest planet. The body would then have resembled some of the nebulæ which appear as bright nuclei surrounded by nebulosity. He imagined that, previous to this state, the nucleus was less luminous and the nebulosity more diffuse, and so, reasoning backward, arrived at a state wherein the nebulosity was so diffused as to be invisible. He suggested that every star had evolved by the condensation of such a nebula under the mutual gravitation of its parts, and that double stars and groups like the Pleiades had been the product of condensation around two or more nuclei.

As the nebula contracted, any rotatory motion which it possessed must have increased in swiftness, and so the centrifugal force at the equator must have increased. Laplace conjectured that the nebula would flatten into an oblate spheroid and that, as the centrifugal force grew still greater, there would come a time when it equaled gravity, and a ring of matter would then be left in equilibrium at the equator while the main body continued to shrink inside it.

The ring thus formed would probably be denser or thicker at one point than at others, and the material here would gather to itself, by gravitational attraction, the material in the remainder of the ring and form a planet. The main body would meanwhile go on contracting and might abandon other rings to form other planets, the final globe being the Sun.

Any planet thus formed would probably be given a rotation by the oblique impact of some of the particles of its ring; it would contract as it cooled, and might abandon rings of its own, which might form satellites in a miniature system or might remain in the form of rings such as those actually seen around Saturn.

The Laplace hypothesis is in agreement with many of the observed facts of the Solar System, but there are some that it fails to explain and there are also serious theoretical difficulties. The origin of the Moon, with its great mass and its high inclination, and perhaps also that of the inner satellite of Mars, which revolves more rapidly than its planet rotates, are better explained on the tidal theory of G. H. Darwin (page 244). The retrograde motion of the tiny outer satellites of Jupiter and Saturn cannot be accounted for by the Laplace theory. No nebulæ such as Laplace postulated are known, but that fact is not strong evidence against the hypothesis, for if such a nebula were at a distance of 200 light-years or greater, and were no larger than Neptune's orbit, its disk would be less than 1″ in diameter and it might easily have escaped notice.

Probably the most serious difficulty is that of the moment of momentum or, as it is now more often called, the angular momentum. This quantity may be defined as follows: Suppose a particle of mass m moving with velocity v along a line from which the perpendicular distance of a given point or center is p. The *angular momentum* of the particle with respect to the point is the product mvp. A rotating body or system may be considered as made up of particles. The angular momentum of the entire system is the sum of the moments of all its particles, taken with respect to the axis of rotation, and is represented by the summation Σmvp. The angular momentum of the present Solar System can easily be computed, and so can that of the hypothetical nebula at the time when it abandoned the ring which developed into Neptune. Now, it is a principle of dynamics that in a system which is not acted on by outside forces the total angular momentum is constant. If, for a given particle of mass m, p diminishes, v must increase; this is in accord with Laplace's theory that the nebula rotated faster as it contracted. But the angular momentum of the present system is less than 1/200 that of the hypothetical nebula before Neptune was born, and, moreover, the planet Jupiter now has more than 95 per cent of the total amount of angular momentum, which is inconsistent with an earlier uniform distribution. The recent discovery of a planet beyond Neptune makes the case against the Laplace hypothesis all the stronger.

The Planetesimal Hypothesis.—According to the hypothesis of Chamberlin and Moulton, the Solar System owes its present state to the close approach of two stars which swept toward each other with a speed greater than the parabolic velocity (page 229), so that the relative orbit was a hyperbola and their association was brief. We are concerned only with what happened to one of the stars, which is now the Sun and which we shall call S, while the other may be called S'. In the vicinity of S there may have been a quantity of scattered material, some of it ejected from the Sun, and S was, perhaps more than at present, in a condition to eject more on slight provocation. As S' approached, it raised enormous tides on S and so encouraged the ejection of material in two opposite directions, toward and away from S'; then, as S' swept rapidly through perihelion, its attraction for this material acquired a component at right angles to the line SS' and imparted angular momentum to the particles.

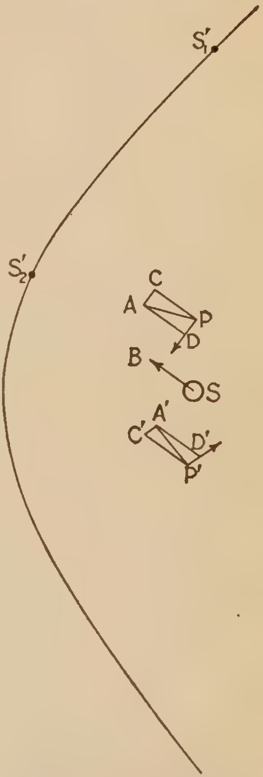


Fig. 150. Tidal Effects of an Encounter of Two Stars

To make this clear, let S_1' (Fig. 150) be an early position of S' on its hyperbolic relative orbit, and let P and P' be two particles, ejected from S in directions toward and away from S_1'. These particles have departed almost radially from S, and if left to themselves would fall directly back like a bullet shot vertically into the air. But S' continues swiftly in its orbit and soon occupies the position S_2'. Its attraction upon P and P' will then be in the directions PA and $P'A'$ respectively, and its effect upon the motion of these particles relative to S may be determined by the kind of reasoning which we employed in discussing perturbations (page 233). Thus, draw PC and $P'C'$ equal and parallel to SB, the acceleration produced by S' in S. Complete the parallelograms $PCAD$ and $P'C'A'D'$ upon the diagonals PA and $P'A'$, taken of such length as to represent the accelerations produced by S' in P and P'. The sides PD and $P'D'$ will then represent the perturbative action, and since $PA > SB > P'A'$, PD must be directed

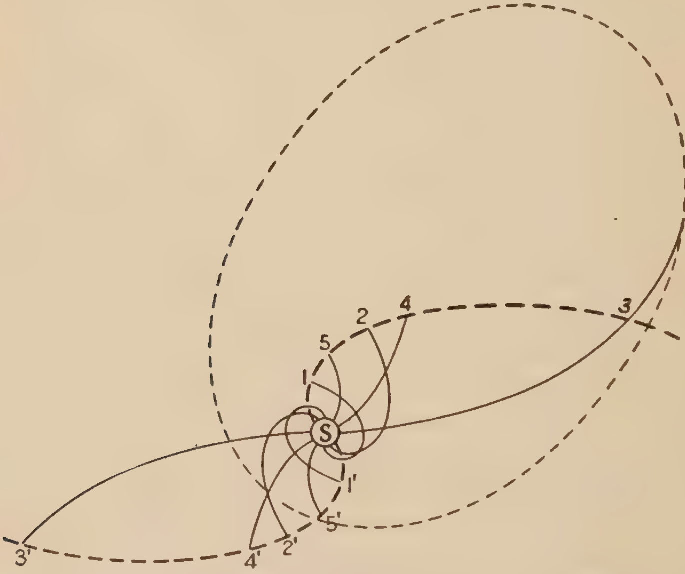


Fig. 151. Formation of a Spiral from Planetesimals

toward a point between S and S' while $P'D'$ is directed to a point beyond S. That is, the action of S' in its new position is to give to the matter earlier ejected from S an angular momentum in the direction of the orbital motion of S'. S' moved rapidly away from the scene of action, and is now indistinguishable from other stars if, indeed, it is visible at all; and P and P' were left revolving in elliptic orbits under the attraction of the Sun.

As S' swung round the Sun, material was ejected from S at frequent intervals or continuously, and all the particles were acted upon in the way described above but in different directions and by different amounts. In Fig. 151 the curves S_1 and S_1' represent the paths followed by the particles P and P', while the curves S_2 and S_2', S_3 and S_3', . . . represent the paths followed by particles which were ejected later. The dotted ellipse is the orbit finally pursued by particle No. 3 after S' had receded so far that its influence was unimportant. The heavy, dotted spiral arms represent the locus of all the particles at an epoch closely following the encounter of the two stars. The result of this encounter, according to the theory, was thus to produce from the débris ejected from the Sun a double-armed spiral of a form (but not size) which is common among the extra-galactic nebulæ (Chapter XVIII).

The motion of the particles was not along the spiral arms, but at large angles with them, and the orbits of the numerous particles crossed each other, also at large angles. Collisions were inevitable, and as the particles collided, their attraction held them together to form **planetesimals** which grew by accretion in further collisions until they formed the planets as they now exist. It may be shown that one effect of the collisions would be to decrease the eccentricity of the orbits of the nuclei, and thus is explained the present approximate circularity of the planetary orbits. Not all the planetesimals were swept up, and the process is still going on (at a greatly diminished rate), as evidenced by the meteors which daily enter the atmosphere of the Earth.

The angular momentum imparted to some of the material was insufficient to make it clear the Sun at the next perihelion passage, but in falling back it struck slantingly, thus giving the Sun itself an angular momentum in the direction of the motion of S'. This persists in the Sun's rotation and equatorial acceleration. The fact that the Sun's equator is somewhat inclined to the orbits of the planets is explained by supposing that S had already a rotation about a different axis, which was modified but not completely obliterated by the new angular momentum.

The planetesimal hypothesis attributes the satellites to secondary nuclei associated with the primary nuclei from the beginning or captured by them later; views the comets as survivors of an earlier system of planetesimals, or as matter ejected from the visiting star, or as matter ejected from the Sun on other occasions and prevented from falling directly back by the perturbations of other stars; and gives plausible explanations of the asteroids, the zodiacal light, and the rotation of the planets.

It should be emphasized that, though the hypothetical result of the encounter of *S* and *S′* was a spiral aggregation similar in shape to many known spiral nebulæ, it was not even remotely similar to them in size. If any spirals of the type described by Moulton and Chamberlin exist, they are, like any possible Laplacian nebulæ, too small to be recognized as such.

The Tidal Hypothesis.—According to Jeffreys and Jeans, the tidal action of the visiting star *S′* caused two long filaments of gas to be shot out on opposite sides of the Sun, and the perturbing action of *S′* upon the particles of this gas started them revolving in eccentric ellipses and may have caused the filaments to curve into the form of the letter *S*, similar to the arms of the spiral nebula N. G. C. 7479 (Plate 62). (Jeans finds it possible that only one filament was ejected, and the subsequent theory works just as well with only one.) The filament was thickest and densest near its middle, that part having been formed when *S* and *S′* were nearest each other. Condensations formed in the ejected gas, and the smaller condensations soon dissipated into surrounding space while the larger increased by attracting to themselves the material around them, and the filament broke up into detached masses which later condensed into planets. Jupiter and Saturn were formed from the middle of the filament and are therefore the largest. The orbits of the planets were rounded up, as in the planetesimal hypothesis, by the resistance of the scattered material through which the planets moved. At its first perihelion passage, each gaseous planet was affected by the tidal action of the Sun as the Sun had been acted upon by the star *S′*, with similar results—the planets gave birth to satellites as the Sun had brought forth the planets. The particular case of the Earth-Moon system, however, is admitted to be best explained by the tidal hypothesis of Darwin (page 244).

Age of the Solar System and of the Earth's Crust.—There is no doubt that the Solar System is exceedingly old. Jeffreys has based an estimate of its age on the eccentricity of the orbit of Mercury, assuming that this orbit was rounded up by the resistance of scattered material, the remaining relic of which is the material of the zodiacal light. He reasons that the time needed for this rounding up is about the same as that required for the disappearance of the resisting medium; for, if the latter interval were much the shorter, the medium would have gone before the eccentricity was much reduced, while if it were much the longer the eccentricity would now be practically zero instead of its actual value of 0.2. He concludes that the time elapsed since the encounter of the two primeval stars is between 1,000 millions and 10,000 millions of years.

Various methods have been employed for estimating the age of the Earth's crust. The earliest of these was suggested by Halley in 1715 and is based on the saltness of the sea.

The water carried to the sea by rivers contains a minute quantity of salt. The total amount of water in the sea remains practically constant because it evaporates as fast as it is delivered. The vapor thus produced condenses in the form of rain, to be again carried to the sea by the rivers. During the evaporation, however, the salt is left behind, so that the salinity of the oceans must be slowly increasing. If we knew the rate at which the salt is carried by all the rivers, the present saltiness of the ocean would permit a calculation of its age. Of course, our knowledge of this rate is very uncertain, but modern data, according to Jeans, suggest that the oceans must be many hundreds of millions of years old.

Probably the most reliable method is that of Russell, which is based on the study of **radioactivity,** the marvelous process by which the heavy elements uranium and thorium slowly and spontaneously disintegrate into elements of lower atomic number. The final product of this disintegration is lead. Uranium lead can be distinguished from ordinary lead by its atomic weight, which is exactly 206, while that of ordinary lead is 207.18. In their natural places in the rocks, neither of these substances—uranium and uranium lead—is ever found without the other; and so it is reasonable to assume that all the uranium lead has been produced from the uranium since the rocks were formed. The rate at which uranium is transformed into lead is known: one per cent of any given quantity of uranium is thus transformed in 66,000,000 years. The ratio of the amount of uranium lead to the amount of uranium found in ancient rocks indicates an age of the Earth's crust of over 1300 millions of years, which is of the same order of magnitude as Jeffreys's estimate of the age of the Solar System.

The Solar System Probably a Rare Type.—The Laplace hypothesis is ruled out by modern mathematical theory, and the only plausible substitutes so far offered to explain the origin of the Solar System postulate the close approach of two stars. If this is the only manner in which a star can acquire a family of planets, such systems must be rare. In the present arrangement of stars in our neighborhood, if all were similar to the Sun, the chance of a close encounter in a period of 6 million million years is computed by Jeans to be about one in fifty thousand. However, in its earlier and more tenuous stages a star is more likely to be broken up, and the chances of encounter were greater in the past if, as may be true, the stars of the Galactic System were then more closely packed than now. While planetary families are doubtless rare, there is not good reason to believe that the Solar System is unique.

Bibliography.—Those who wish to pursue the subject of this chapter in detail may find material in the following books:

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T. C. Chamberlin, *The Two Solar Families.* The University of Chicago Press, 1928

J. H. Jeans, *Astronomy and Cosmogony.* Cambridge University Press, 1929 (Second edition)

J. H. Jeans, *The Universe Around Us.* Cambridge University Press, 1929

H. Jeffreys, *The Earth.* Cambridge University Press, 1929 (Second edition)

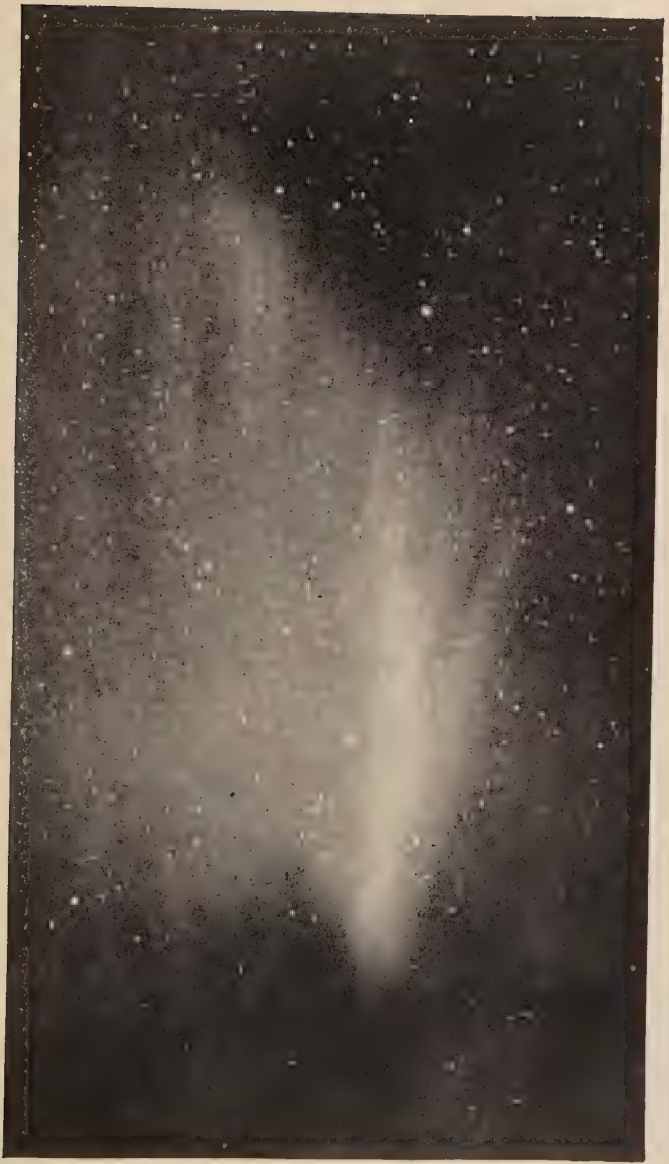


PLATE 34. TAIL OF THE GREAT COMET a 1910
Photographed by Lampland at Lowell Observatory, January 27. The head was concealed by the pine forest west of the Observatory. Length of tail about 20° or 40,000,000 miles. This was by far the brightest comet of the first quarter of the twentieth century. It had passed its perihelion on January 17

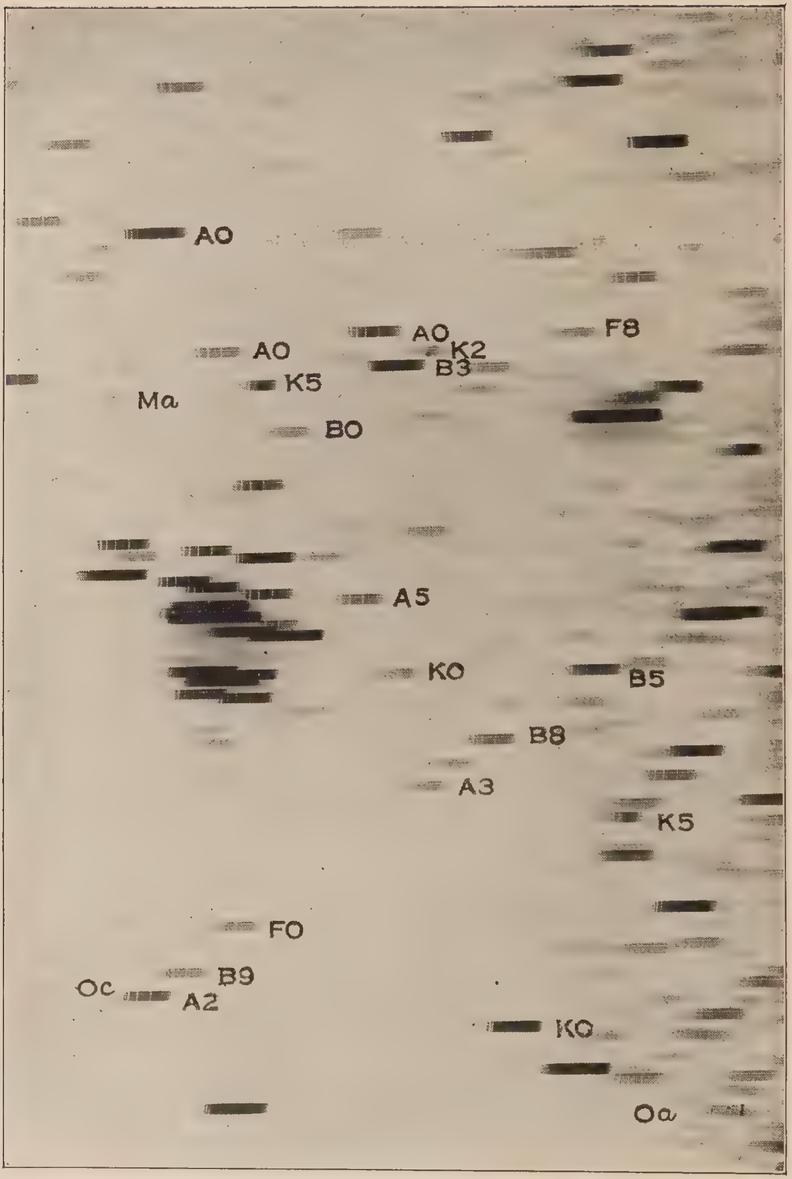

A0
A0
K2
F8
A0
B3
K5
Ma
B0
A5
K0
B5
B8
A3
K5
F0
B9
Oc
A2
K0
Oa

Plate 35. Objective-prism Spectrogram of a Star Field in Carina
Photographed at Harvard College Observatory (Arequipa station)

CHAPTER XIV

THE STARS—THEIR SPECTRA, DISTANCES, AND MAGNITUDES

Distant Suns.—Each star of the multitudes that adorn the sky is a hot, self-luminous ball of gas—a body of the same nature as the Sun. As seen from one of them, the Sun—or rather the whole Solar System—would itself appear as a star, in no way pre-eminent among the others; the planets and comets would be hopelessly lost in the light of their primary.

That the stars are suns is proved beyond doubt by their spectra; but, long before the spectroscope was first used, their sunlike nature was made evident by their great distances combined with their apparent brightness. In order to be seen at all at such distances, the stars must shine by their own light and must be comparable with the Sun in real brightness. The present chapter is devoted partly to the analysis of star light by means of the spectroscope, partly to the determination of the distances of the stars, and partly to the classification of stars according to the intensity of their light.

The Study of Star Light.—It is a noteworthy fact that our knowledge of the heavenly bodies has come to us entirely by way of a single sense—the sense of sight. We cannot touch them, or hear them, or taste or smell them. In the case of the stars, even the sense of sight is limited as compared with the case of the bodies of the Solar System, for the latter are near enough to be observed in detail by means of the telescope, while the stars are so distant as to appear only as points. The stars, however, afford an advantage in being self-luminous; and the study of the quality and quantity of their light reveals many facts concerning their nature.

Instruments of Stellar Spectroscopy.—On account of the faintness of the stars, a spectroscope to be used for their study must be attached to an astronomical telescope, the objective of which serves to concentrate the light into a small, bright image. Probably the simplest form of stellar spectroscope is the **ocular spectroscope,** which is simply a small prism, or more usually a direct-vision combination of prisms (page 151), that may be attached to the eyepiece of a telescope. The one shown in Fig. 152, a common form, is only a little more than an inch long. No slit is necessary for this instrument; the diminutive image of a star, formed by the telescope objective, takes the place of the slit, the eyepiece forms the collimator, and no view-telescope is needed since the pencil of light that emerges from the prism is small enough already to enter the pupil of the eye. The resulting spectrum is as narrow as the image of the star—a mere line of colored light—and in order that dark lines may be seen it is necessary to broaden the spectrum by means of a small concave cylindric lens placed with the axis of the cylinder at right angles to the refracting edge of the prism. This, of course, diminishes the intensity of the spectrum, and so cannot be applied to very faint stars. If the spectrum contains bright lines, these can be well seen without the cylindric lens as bright dots which are monochromatic images of the star. The only advantage of the ocular spectroscope over the types next to be discussed lies in the smallness of the prism, which renders the instrument light and inexpensive.

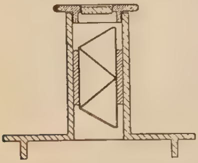


FIG. 152. THE OCULAR SPECTROSCOPE

The oldest form of star spectroscope is the **objective prism,** which was first used in 1823 by Fraunhofer. It is simply a prism placed in front of the objective of an astronomical telescope, the latter taking the place of the view telescope of an ordinary spectroscope. No slit or collimator is needed, since the star is only a point of light and is so distant that its rays are already parallel. When the instrument is used visually, as Fraunhofer used it, the spectrum is broadened by a cylindric lens placed over the eyepiece. Much more commonly it is used photographically, in which case the refracting edge of the prism is placed parallel to the equator (the telescope being mounted equatorially) and the driving clock is given a rate differing slightly from the sidereal rate, so that the spectrum is trailed upon the plate in a direction parallel to its lines. This broadens the photograph by any desired amount. When used with a photographic objective covering a large field, this type of spectrograph is extremely useful, as it records the spectra of many stars together on the same plate, so that they may be conveniently and rapidly classified (Plate 35).

The chief drawback in the use of the objective prism is the fact that, as in the ocular spectroscope, no comparison spectrum can be employed. For the accurate observation of star spectra for the purpose of determining the wave-length of lines or the displacement due to radial velocity, a complete **star spectrograph,** with a slit, is used in connection with an astronomical telescope. This instrument is similar to the prism spectrograph described in Chapter VII, but it must be constructed with great rigidity to prevent flexure of its parts and guarded against changes of temperature during the long exposure necessary for photographing the spectra of faint stars, and it must also be provided with a device for guiding the telescope so as to keep the star's light within the slit.

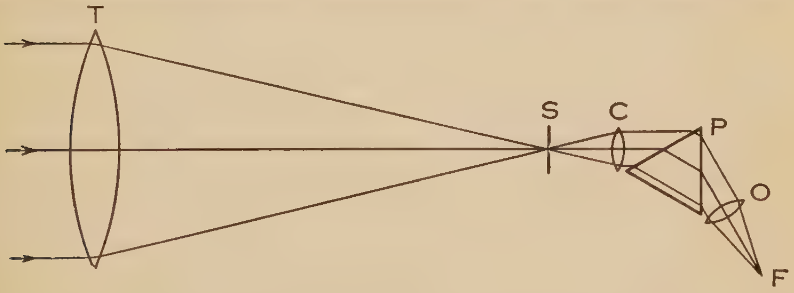


FIG. 153. THE STELLAR SPECTROGRAPH

The arrangement of the optical parts of a star spectrograph and of the telescope to which it is attached is shown diagrammatically in Fig. 153. *T* is the objective of the astronomic telescope, *S* the slit, *C* the collimator, *P* the prism (two or more prisms are sometimes used in a train), and *O* the objective of the camera which focuses the spectrum upon the photographic plate at *F*. The objective *T* forms no part of the spectrograph, but performs the important duty of gathering a large amount of star light into a small image. In order that a maximum amount of the star's light may pass through the slit, it is necessary that the slit be kept accurately in the focal plane of the objective *T*. The slit might at first thought seem to be unnecessary, since the star image is so small; but without the slit-jaws the wandering of the star image due to "bad seeing" and imperfect guiding would, during an exposure of any length, blur the lines of the spectrum. Moreover, the slit is necessary for photographing the comparison spectrum. A change of temperature during an exposure would result in a change of the refractive index of the prism, and in contraction or expansion of the parts of the spectrograph, which would cause the spectrum to move on the plate and blur the spectral lines; and so the instrument is ordinarily inclosed in a felt-lined case which is provided with an electric thermostat that holds the temperature constant within a small fraction of a degree. For "guiding" the telescope, the jaws of the slit are made of polished metal and are so placed as to reflect into a small observing telescope placed just above them any part of the star's light that does not enter the slit. It is the observer's duty so to control the main telescope as to keep this unused light a minimum throughout the exposure of the plate.

Light for the comparison spectrum is produced by an electric arc or spark between metallic terminals, or by a vacuum tube. The source of light is placed beside the slit, and its light is reflected into the spectrograph by diminutive prisms placed over the ends of the slit. The star's light falls upon the middle of the slit, between the prisms, and in the resulting photograph the star spectrum appears between two identical impressions of the comparison spectrum. Measurement of the relative position of star and comparison lines is made with a high degree of accuracy upon the finished photograph under a microscope. The great advantage of the slit spectrograph over all other forms of instrument for studying star spectra lies in the fact that it provides a comparison spectrum.

In Plate 36 is shown the star spectrograph of the Lick Observatory, and, beneath it, a spectrogram of the star Arcturus made with this instrument, with the titanium spark as comparison, enlarged to four times its original dimensions.

Historical Outline of Stellar Spectroscopy.—Fraunhofer observed the spectra of a few of the brightest stars, and recognized that some of them, like Arcturus and Capella, were like the spectrum of the Sun, while others, such as Sirius and Castor, showed only a few dark lines. In 1864, soon after Kirchhoff and Bunsen had identified the spectra of many of the chemical elements, the study of stellar spectra was taken up extensively by Sir William Huggins in England and by Father Angelo Secchi in Italy. Huggins used the slit spectroscope in a detailed study of about a hundred stars, identifying in their spectra the lines of many known terrestrial elements, and so established the fact that these substances were widely distributed throughout the visible universe. Secchi, using an ocular spectroscope, examined the spectra of about four thousand stars in less detail, and divided them into four classes according to spectral type, a classification which is still found useful. In 1885, the photographic study of star spectra with the objective prism was begun by E. C. Pickering at the Harvard College Observatory, where it has been pursued assiduously ever since. A classification based on this study was formed by Miss Antonia Maury, and has since been superseded by one due chiefly to Miss Annie Cannon, who is responsible for the classification of about a quarter of a million stars.

The Doppler-Fizeau principle was first applied to stars in 1866 by Huggins, who derived an approximation to the radial velocity of Sirius by visual measurements of the lines of its spectrum. In 1888 Vogel began making measurements of much greater accuracy by photography, and a few years later the photographic method was developed and applied extensively by Campbell at the Lick Observatory. Since 1900 the study of stellar spectra in its different branches has been carried on vigorously by a number of large observatories in America and Europe, and the value of this study has been greatly enhanced by the application of modern theories of the atom and of radiation.

Secchi's Classification.—Secchi's four spectral types may be briefly described as follows:

Type I. Spectra continuous (as seen in Secchi's instrument) except for four prominent dark lines, which have since been identified with hydrogen. Red part of spectrum relatively faint, so that the stars appear bluish or greenish. About half the stars examined by Secchi belong to this type, including Sirius, Rigel, Vega, and Castor.

Type II. Spectra very similar to that of the Sun, containing many fine dark lines due mostly to metals. Stars of this type are yellow in color and very numerous, embracing almost the other half of the stars examined. Capella and Arcturus are examples.

Type III. Orange or reddish stars, having spectra in which the hydrogen lines are inconspicuous and the metallic lines stronger than in Type II, while superposed upon these lines are a number (at least eight) of dark bands, each of which is strong on the edge toward the violet end of the spectrum and fades out gradually toward the red. Secchi observed twenty-five of these stars, of which α Herculis and Betelgeuse are examples.

Type IV. Very red stars, showing the metallic spectrum with three dark bands superposed (in the visible part of the spectrum); these bands, unlike those of Type III, are sharp on the redward side and fade out toward the violet.

Secchi catalogued seventeen examples, of which the brightest is only barely visible to the unaided eye; it is a blood-red star in the constellation Canes Venatici which, on account of its color, Secchi called Superba; but it is more commonly known by the less romantic name of 152 Schjellerup, from its number in the catalogue of red stars prepared by the Danish astronomer.

To these four classes Pickering, in 1891, suggested the addition of Type V, to include the bright-line nebulæ and the stars which, after Secchi's time, were discovered to exhibit bright lines.

The Harvard Classification.—In the system evolved at Harvard, which is the one most commonly used by astronomers, the principal classes are designated by the letters O, B, A, F, G, K, M, N, R, S, which were originally arranged in alphabetical order, but which, as ideas of stellar evolution were modified, were transposed to the order just given. Each division contains a number of subdivisions designated by small letters or numbers, as Ob and F2, and the complete description of the system as published in the *Annals of the Harvard Observatory* covers nearly thirty quarto pages. It may be outlined as follows:

Class O. All subdivisions except one (Oe5) show faint continuous spectra upon which are superposed bright bands. The hydrogen and helium lines are bright at the beginning of the class, but dark in the later subdivisions. Toward the end of the class appear the dark lines of the "Pickering" or "ζ Puppis" series due to ionized helium. The bright line stars of this type are rare and are found only in the Milky Way. They are sometimes referred to as Wolf-Rayet stars.

Class B. Sometimes called Orion or helium stars. The most prominent features are the dark lines of hydrogen and helium;

the H and K lines of ionized calcium are present but very weak. Examples: Spica, Rigel, and the Belt stars in Orion.

Class A. Hydrogen dark lines very strong, helium absent, H and K and the metallic lines increasing in prominence throughout the class. Examples: Sirius, Vega, Fomalhaut.

Class F. H and K grow stronger than the hydrogen lines, and the metallic lines increase in prominence. Procyon is an example.

Class G. The solar type of spectrum, characterized by numerous metallic lines and very strong H and K.

Class K. Hydrogen and metallic lines weak, arc lines strong; dark bands appear faintly. Spectrum similar to that of sun spots. Examples: Arcturus and Aldebaran.

Class M. Identical with Secchi's third type, characterized by the bands noted by Secchi, each of which is sharp on the side toward the violet and fades out toward the red. These bands have been identified with titanium oxide. In subdivision Me, bright hydrogen lines appear upon the continuous spectrum among the dark lines. All stars of subdivision Me are variable in light.

Class N. Identical with Secchi's Type IV, and characterized by dark superposed bands that are sharply defined on the redward side and are now known to be due to carbon monoxide and cyanogen—a reversal of the "Swan" spectrum produced by the blue base of a Bunsen flame. Violet end of spectrum extremely weak, making the stars very red.

Class R. Lines and bands similar to those of Class N, but without the general darkening of the violet. "Blue N-stars." Only a few examples are known, and they are faint.

Class S. Spectra like those of Class M except that bands of zirconium oxide replace those of titanium oxide. Stars of this class are very few and very faint.

The gaseous nebulæ, which have spectra consisting of bright lines only or of bright lines on a continuous background, are placed in a class by themselves, denoted by P.

The characteristics of the different spectral classes may be best seen from a study of the spectra themselves, as illustrated in Plates 35, 37 and 38.

Properties Which Progress Regularly Through the Series of Spectral Classes.—A study of the spectra reproduced in Plates 37 and 38 will disclose certain characteristics that progress regularly throughout the series from the "late" O type through B, A, F, G, K and M. Chief among these characteristics may be mentioned:

1. The intensity of the dark Balmer lines of hydrogen. These lines are weak in spectra of late O and B types, are very strong in A, and gradually fade from A to M.

2. The intensity of the dark H and K lines of ionized calcium, which increases from O through G and reaches a maximum in K, after which it decreases slowly.

3. The number of the dark lines of the metals, which begins at zero in the B type and increases throughout the series.

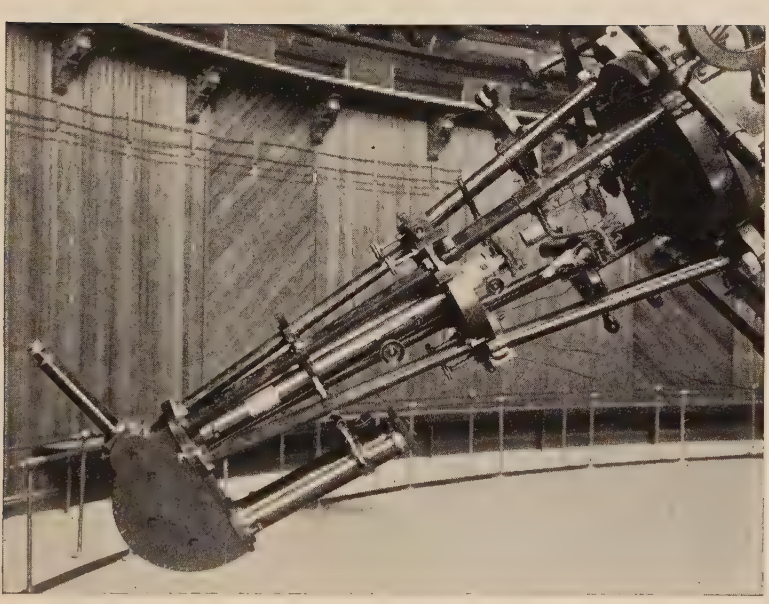
4. The relative intensity of the red and violet portions of the continuous background. The violet grows relatively weaker throughout the series from O to M, and the color of the stars corresponds with their spectral type, the O stars being bluest and the M stars reddest of the series. The effective temperature, as shown by various methods of investigation (page 351), also varies with the spectral type from about 20,000° C. for the late O stars to about 3,000° for the late subdivisions of M.

The relationship of the different types as regards spectral properties is exhibited graphically in Fig. 154, and the relation of spectral type to color and temperature in Fig. 155. The progression of spectral characteristics from class to class is indicative in all respects of a progression in the degree of atomic excitation (page 165) in the stellar atmospheres where the spectra originate. It is believed that the observed differences in stellar spectra are due to differences of surface temperature, and that *the chemical composition of all stars is essentially the same.*

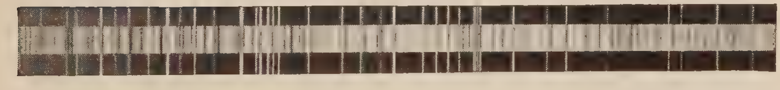
The Wolf-Rayet Stars and Stars of Classes R, N, and S.—The vast majority of stars whose spectra are known belong to the classes from B to M, the properties of which were discussed in the preceding section. The others are grouped as follows:

1. The Wolf-Rayet stars, belonging to the lower subdivisions of Class O, and characterized by bright bands of great strength as compared with the

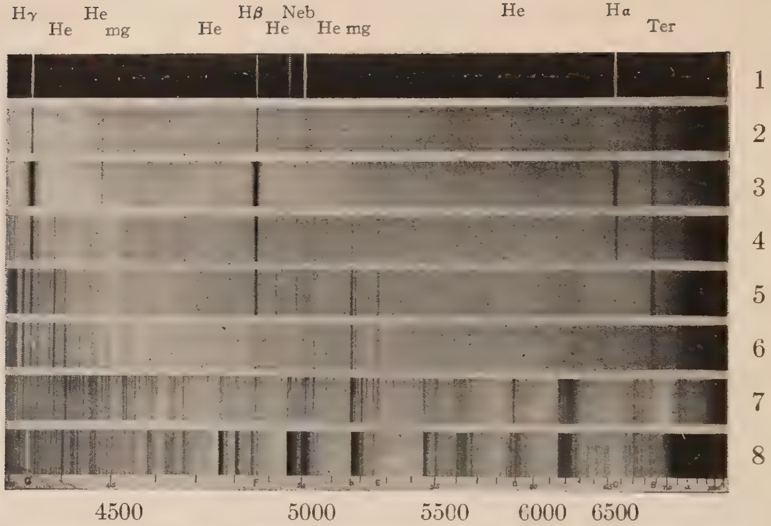
PLATE 36



The Mills Spectrograph attached to the 36-inch Lick Refractor



Spectrogram of α Bootis (Arcturus) with titanium comparison. Made with the Mills Spectrograph (enlarged 4 times)


Hγ
He
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mg
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Neb
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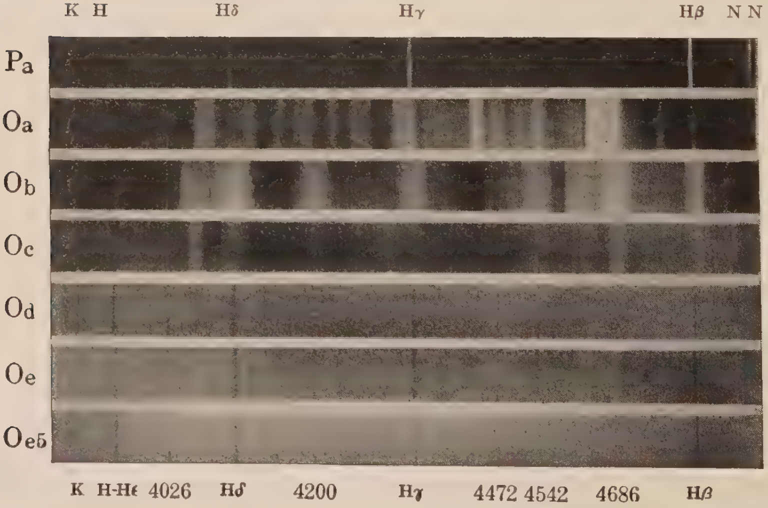
He mg
He
Hα
Ter
1
2
3
4
5
6
7
8
4500
5000
5500
6000
6500

Nebular and Stellar Spectra, photographed by V. M. Slipher at Lowell Observatory

1, Orion Nebula; 2, Rigel (B8); 3, Sirius (AO); 4, Procyon (F5); 5, The Sun (Go); 6, Arcturus (Ko); 7, Betelgeuse (M2); 8, Mira (M6)


K H
Hδ
Hγ
Hβ
N N
P_a
O_a
O_b
O_c
O_d
O_e
O_{e5}
K
H-Hε
4026
Hδ
4200
Hγ
4472
4542
4686
Hβ

P-type and O-type Spectra photographed at Detroit Observatory

Plate 37

continuous background. They are set apart from the other stars by the existence of these bands and also by their very high temperature, which Plaskett has placed for certain of these stars at 40,000°. Wolf-Rayet stars are often found at the centers of the planetary nebulæ.

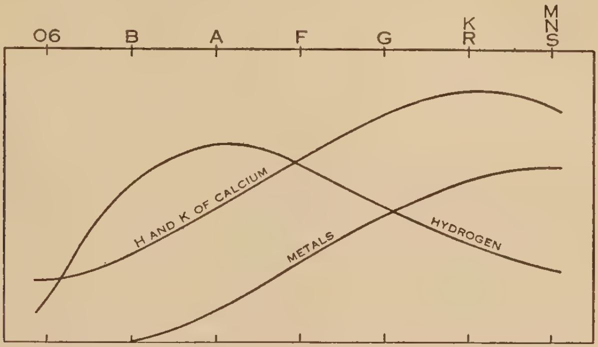


FIG. 154. CHARACTERISTICS OF THE SPECTRAL TYPES

2. Stars of Classes R and N, which are similar to those of Classes K and M, respectively, except that they show the dark bands of cyanogen and carbon monoxide. They may be regarded as belonging to groups which branch off from the main series.

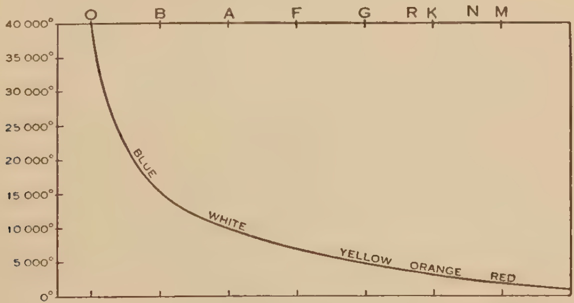


FIG. 155. CORRELATION OF COLOR AND TEMPERATURE WITH SPECTRAL TYPE

3. Stars of Class S, which form an additional branch, and differ from the M stars in the possession of bands of zirconium oxide instead of titanium oxide.

The Distances of the Stars Determined by Their Heliocentric Parallax—The relation between the distance of a star and its heliocentric parallax was pointed out on page 91. The **heliocentric parallax** may be most simply defined as the angle subtended at the star by the semidiameter of the Earth's orbit, and this angle is inversely proportional to the star's distance.

One second of arc is the angle subtended by any line at a distance 206,265 times its length. A star which has a parallax of one second is therefore at a distance of 206,265 astronomical units, one with a parallax of one-half a second is twice as far away, and, in general, a star with a parallax of p seconds is at a distance of $\frac{206265}{p}$ astronomical units. A star's distance may therefore be determined by measuring its parallax, the method being in fact a triangulation (page 84) in which the base line is the radius of the Earth's orbit. Great as this base line is, it is very small compared with the distance of even the nearest star, and the determination of star distances is very difficult because of the slenderness of the triangle.

Success in detecting parallactic displacement in a star was first attained almost at the same time—about 1838—by three different observers: Bessel, in Germany, who measured 61 Cygni; Struve, in Russia, who chose Vega; and Henderson, at the Cape of Good Hope, who observed α Centauri. Henderson measured the star's right ascension and declination with a meridian circle at intervals throughout the year, and studied the variation of these co-ordinates; but the corrections for precession, nutation, and aberration are enormous as compared to the quantity which he sought, and moreover the instrumental corrections are mingled with the parallax because, like it, they have a yearly period. The alternative method, which was used by Bessel and Struve and is now adopted by all parallax observers, consists of measuring the apparent distance of the star under investigation from a number of comparison stars which are very near it in the sky. Since all these stars are affected by precession and the like in the same way, and since the great majority of stars which are likely to be used as comparison stars are so far away that their parallax is insensible, this "differential" method gives the parallactic displacement free from the troublesome corrections to which the "absolute" method of Henderson is subject.

At the bottom of Fig. 156 is represented the orbit of the Earth. A, B, and C are neighboring stars which, seen upon the background of very distant stars at the top of the drawing, seem to reflect the Earth's motion in tiny parallactic ellipses. The nearest star, A, appears to move in the largest ellipse. The background stars are so remote that their parallactic ellipses dwindle to points. Of course the size of the Earth's orbit relative to the distance of the stars is, in the drawing, grossly exaggerated.

Throughout the nineteenth century parallax measurements were made either with the filar micrometer or with the *heliometer*, an instrument of great accuracy devised by Fraunhofer for measuring the diameter of the Sun. These methods were extremely laborious, and at the beginning of the twentieth century the distances of less than a hundred stars were known. Parallaxes are now determined almost entirely by measurements of photographs taken with long-focus telescopes, a method introduced by Schlesinger at the Yerkes Observatory

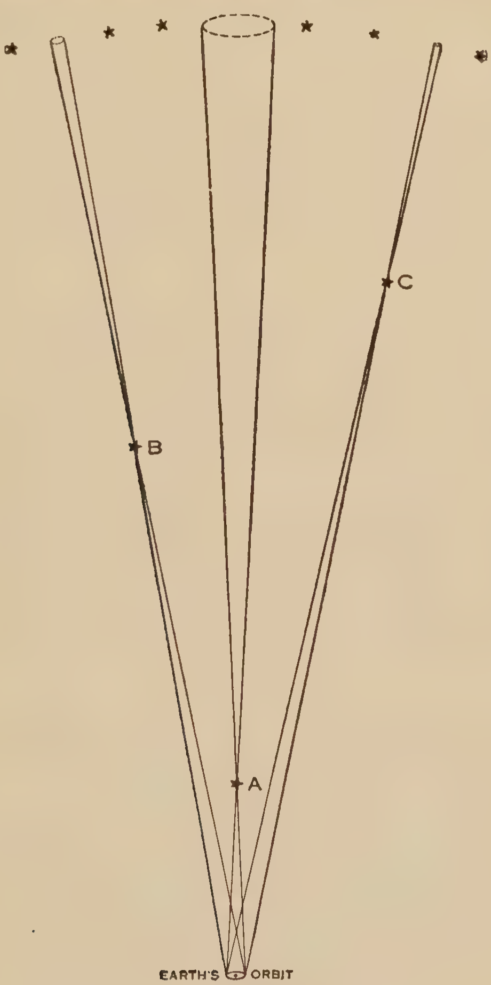


FIG. 156. HELIOCENTRIC PARALLAX

about 1903. At the present time (1930) over two thousand parallaxes have been measured with considerable certainty.

Units of Stellar Distance.—The Earth's mean distance from the Sun, which is convenient as an astronomic unit of distance when applied to the Solar System, is much too small for use between stars. A unit which brings vividly to mind the colossal distances involved is the **light-year,** which is the distance over which light travels in a year. Since light travels 186,000 miles a second and there are 31,560,000 seconds in a year, the product of these two numbers gives the number of miles in a light-year. It equals about 63,000 astronomic units, and bears about the same ratio to the astronomic unit as the mile to the inch. If any star had a parallax of exactly one second, its distance would be $206,265 + 63,000 = 3.26$ light-years; but the largest stellar parallax known is only $0''.78$, and so the distance of the nearest known star is $3.26 + 0.78 = 4.2$ light-years. The distance of any star whose parallax is p seconds is $3.26/p$ light-years.

For many purposes, a more convenient unit is the distance which a star would have if its parallax were $1''$. As a name for this unit, English astronomers have invented the monstrous word **parsec.** The Germans call it *Sternweite.* A parsec is equal to 3.26 light-years. The distance of a star whose parallax is p seconds is simply $1/p$ parsecs.

Stellar Distances.—Table 16 is a list of the twenty nearest known stars with their positions, magnitudes, spectral types, parallaxes, and distances (six are double stars, and for these the magnitude and spectral type are given for each component). Most of these stars are very faint, and have no other designations than their numbers in star catalogues or the names of astronomers who have investigated them; such are Barnard's star and Wolf 359. Three, however—α Centauri, Sirius, and Procyon—are among the brightest stars in the sky. The nearest star of all (called Proxima for that reason) is so remote that its light requires more than four years to reach us, whereas the light of the Sun is only eight minutes on the way. The distance of this star is more than 264,000 astronomical units, and the Solar System is thus seen to be a mere speck in the

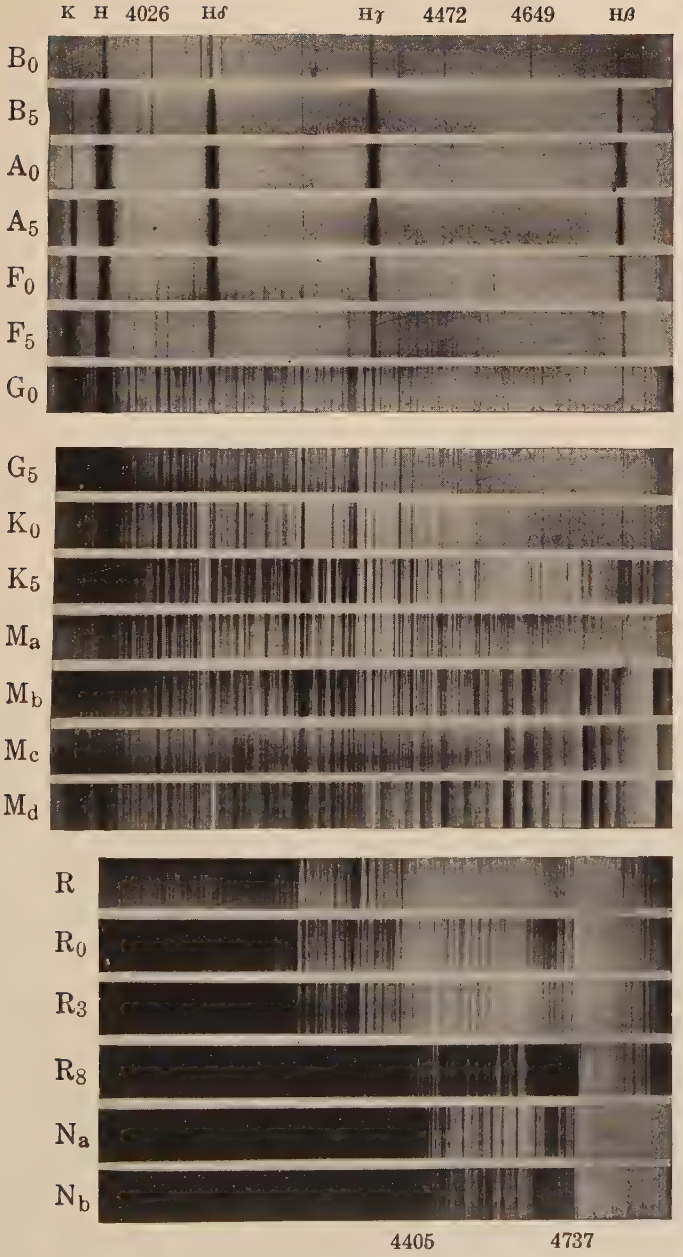
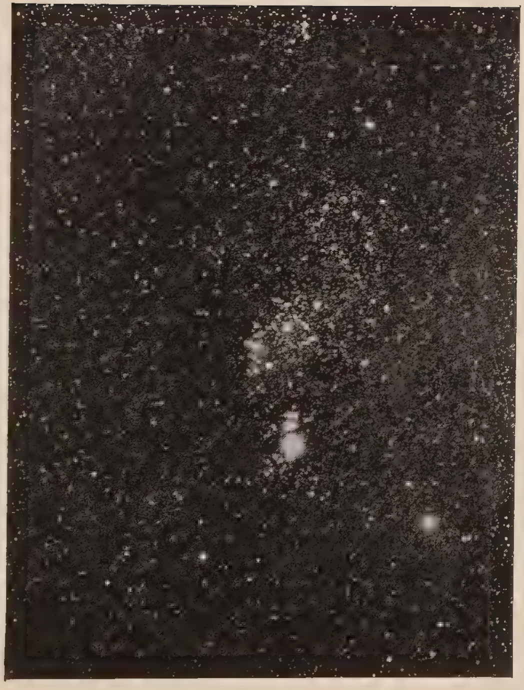

K
H
4026
Hδ
Hγ
4472
4649
Hβ
B0
B5
A0
A5
F0
F5
G0
G5
K0
K5
Ma
Mb
Mc
Md
R
R0
R3
R8
Na
Nb
4405
4737

Plate 38. Star Spectra of different Types
Photographed at the Detroit Observatory

PLATE 39



The Constellation Orion
Photographed with 1.5-inch lens at Wellesley College.
Exposure 1 hour

Map of the Constellation Orion showing relative visual brightness of the stars visible to the naked eye. (Reproduced from Schüller's Atlas by permission of Jos. Klepešta, Publisher, Prague, 1926.)

immensity of space, separated from its nearest neighbor by thousands of times its own diameter. The known stellar population within four parsecs of the Sun averages one star to about 520 cubic light-years, but it is likely that the actual star density is somewhat greater than this, for there may be a number of faint stellar neighbors yet undiscovered.

Proxima is a near neighbor of α Centauri, separated from the latter star by a mere step over which light passes in about ten weeks. If the Sun be represented by a plum an inch in diameter, α Centauri will be a pair of plums a fraction of a mile apart and 500 miles away, and Proxima will be a grain of sand about 25 miles from the pair of plums. The most distant stars in the table will on this scale be about 1,500 miles away.

TABLE 16

THE NEAREST STARS

Star	α 1900	δ 1900	Mag.	Spec.	Parallax	Distance, parsecs	Distance, light-years
Proxima Centauri	14^h23^m	− 62°0	10.5	M	0″78	1.28	4.2
α Centauri	14 33	− 60.4	0.3 1.7	G0 K5	.76	1.32	4.3
Barnard's star	17 53	+ 4.5	9.7	M5	.56	1.79	5.8
Wolf 359	10 52	+ 7.6	13.5	M6e	.41	2.44	8.0
Lalande 21185	10 58	+ 36.6	7.6	M2	.39	2.56	8.3
Sirius	6 41	− 16.6	− 1.6 + 8.4	A0 A5	.37	2.70	8.8
BD−12° 4523	16 25	− 12.4	11.5	M5	.35	2.86	9.3
Innes's star	11 12	− 57.0	12	?	.34	2.94	9.6
τ Ceti	1 39	− 16.5	3.6	K0	.32	3.12	10.2
Cordoba 5^h243	5 8	− 45.0	9.2	K2	.32	3.12	10.2
Procyon	7 34	+ 5.5	0.5 13	F5 ?	.31	3.23	10.5
ε Eridani	3 28	− 9.8	3.8	K0	.30	3.33	10.8
61 Cygni	21 2	+ 38.3	6.3 5.6	K8 K7	.30	3.33	10.8
Pos. Med. 2164	18 42	+ 59.5	8.7 9.4	M4 M4	.29	3.45	11.2
Lacaille 9352	22 59	− 36.4	7.4	M0	.29	3.45	11.2
ε Indi	21 56	− 57.2	4.7	K5	.29	3.45	11.2
Groombridge 34	0 12	+ 43.4	8.1	M2	.28	3.57	11.6
Krüger 60	22 24	+ 57.2	9.6 11.3	M3 M	.26	3.85	12.6
Van Maanen's star	0 44	+ 4.9	12.3	F0	.26	3.85	12.6

Heliocentric parallaxes of 0".01 or smaller cannot be reliably measured, and so the distances of stars beyond 300 light-years cannot be satisfactorily determined in this way. Not one star in a thousand of those which can be observed with large telescopes is so near us as this, but the distances of many of these remote stars can be determined by indirect methods which will be discussed later (pages 331, 332, 350, 360, 381, 387, 388). The light by which we see most of the stars left them long before we were born, while that by which the faintest stars are photographed left them before the dawn of civilization or even before the appearance of mankind on the Earth.

Star Magnitudes.—A star's **magnitude** may be defined as a number in an arbitrary scale that expresses the apparent brightness of the star as compared with that of other stars. In the scale used by astronomers the brightest stars are given the smallest numbers. Hipparchus and Ptolemy classified the stars visible to the unaided eye into six "magnitudes," describing about twenty of the brightest stars as of the first magnitude and the stars just visible to the naked eye as of the sixth. This classification is preserved in modern celestial photometry, although there seems no better reason for six magnitudes of lucid stars than for five, or seven, or another number. The word magnitude, it should be noticed, here refers to brightness and not to size. Two stars of the same size may differ greatly in magnitude if their temperatures or their distances from the Earth are very different.

Little attention seems to have been paid to the subject of the brightness of the stars from the time of Ptolemy until the nineteenth century; for although the system of magnitudes was extended to the telescopic stars, there was no general agreement among different observers. In 1827 Sir John Herschel compared the light of faint stars, observed with a reflector of eighteen inches aperture, with that of bright stars observed with smaller apertures, and concluded that the average first-magnitude star sends us about one hundred times as much light as a star of the sixth magnitude. Pogson of Oxford, in 1854, by noting the smallest aperture with which certain stars could be seen and comparing together the average "magnitudes" of these stars as estimated by various well-known observers, became convinced that each "magnitude" is about two and one-half times as bright as the next.

The relation between the number expressing a star's magnitude and the amount of light that the eye receives from the star is an example of the **psychophysical law** which was suggested to the German psychologist Fechner by the results of Herschel and Pogson. This law states that if the intensity of a sensation varies in arithmetic progression, then the intensity of the stimulus that produces it must vary in geometric progression. Thus, if one listens to three sounds, a, b, c, and judges the *difference* of loudness between b and c to be the same as that between a and b, then in reality the *ratio* of the intensity of c to that of b must be the same as the ratio of intensity of b to a. The law holds for each of the other senses as well as for those of sight and hearing, but is not rigorously true when applied to very feeble or very powerful stimuli.

The brightnesses of stars of the sixth, fifth, . . . first magnitude *appear* to be separated by a constant difference; measurement shows that they really bear a constant *ratio*, which Pogson found to be about 2.5. Let the exact value of this ratio be ρ. Then a fifth-magnitude star sends us ρ times as much light as a sixth; a fourth-magnitude star sends us ρ times as much as a fifth, or ρ^2 times as much as a sixth; a third-magnitude star ρ^3 times as much as a sixth, and so on. In general, if m and n represent the magnitudes of two stars, m referring to the brighter, so that the difference of magnitude is $n - m$, then the ratio of their brightness is $\rho^{(n-m)}$; or, representing the light sent to the Earth by the two stars by l_m and l_n, respectively,

$$\frac{l_m}{l_n} = \rho^{(n-m)}.$$

But Herschel had found that a difference of five magnitudes corresponds to a ratio of 100; and this being a convenient relation, it was adopted as exact in the **Pogson scale** of magnitudes, which is now universally used, so that $\rho^5 = 100$ and $\rho = \sqrt[5]{100} = 2.512 \ldots$

The most convenient expression for the relation between magnitude and actual brightness is found by taking the logarithm of each side of the above equation, thus:

$$\log \frac{l_m}{l_n} = (n - m) \log \rho.$$

But the logarithm of $\sqrt[5]{100}$ is exactly 0.4, hence,

$$\log \frac{l_m}{l_n} = 0.4\ (n - m)$$

or $$\boldsymbol{n - m = 2.5 \log \frac{l_m}{l_n}} = 2.5\ (\log l_m - \log l_n).$$

The following examples illustrate the use of this formula.

1. Find the difference of magnitude of two stars, one of which is twice as bright as the other. Here $l_m/l_n = 2$, $\log l_m/l_n = 0.30$, hence $n - m = 2.5 \times 0.30 = 0.75$; or, the brighter star outshines the fainter by three-fourths of a magnitude.

2. How does the brightness of Sirius, magnitude -1.58, compare with that of β Tauri, magnitude $+1.78$? Here $n - m = 3.36$, and so $\log l_m/l_n = 1.344$; hence, $l_m/l_n = 22.08$—that is, Sirius is 22.08 times as bright as β Tauri.

Fractional and Negative Magnitudes.—Since there are stars of practically all gradations of brightness from Sirius down to the faintest star that can be perceived, it is of course necessary to express the magnitudes of most of them by fractions. The scale of magnitudes is so adjusted that about half of Ptolemy's "first magnitude" stars are brighter and half are fainter than what is now the standard first-magnitude star, which is represented very nearly by Aldebaran and Altair. A star 2.512 times as bright as this is of the zero magnitude, and the magnitudes of the two stars Sirius and Canopus, which are brighter still, are negative. The magnitudes of the planets Venus and Jupiter and, at times, those of Mercury and Mars, must also be expressed by negative numbers; Venus becomes a little brighter than the −4th magnitude, which means that she is then more than one hundred times as bright as Aldebaran. The magnitude of the Sun is, according to Russell, −26.72.

The Forty Brightest Stars.—The twenty brightest stars, with their visual magnitudes and spectral types, are listed in Table 17.

TABLE 17

STARS BRIGHTER THAN MAGNITUDE 1.50

	Mag.	Sp.		Mag.	Sp.
α Canis Majoris (Sirius)	−1.58	A0	α Orionis (Betelgeuse)	1.0 to 1.4	M2
α Argûs (Canopus)	−0.86	F0	α Crucis	1.05	B1
α Centauri	+0.06	G0	α Tauri (Aldebaran)	1.06	K5
α Lyræ (Vega)	0.14	A0	α Virginis (Spica)	1.21	B2
α Aurigæ (Capella)	0.21	G0	β Geminorum (Pollux)	1.21	K0
α Bootis (Arcturus)	0.24	K0	α Scorpii (Antares)	1.22	Ma
β Orionis (Rigel)	0.34	B8	α Piscis Australis (Fomalhaut)	1.29	A3
α Canis Minoris (Procyon)	0.48	F5	α Cygni (Deneb)	1.33	A2
α Eridani (Achernar)	0.60	B5	α Leonis (Regulus)	1.34	B8
β Centauri	0.86	B1			
α Aquilæ (Altair)	0.89	A5			

The twenty stars which follow those of the above list in order of brightness are given in Table 18. These two tables contain all the stars which are brighter than visual magnitude 2.00.

TABLE 18

STARS OF VISUAL MAGNITUDE 1.50 TO 2.00

	Mag.	Sp.		Mag.	Sp.
β Crucis	1.50	B1	β Argûs	1.80	A0
α Geminorum (Castor)	1.58	A0	α Trianguli Australis	1.88	K2
γ Crucis	1.60	Mb	α Persei	1.90	F5
ε Canis Majoris	1.63	B1	η Ursæ Majoris	1.91	B3
ε Ursæ Majoris	1.68	A0	ζ Orionis	1.91	B0
γ Orionis (Bellatrix)	1.70	B2	γ Geminorum	1.93	A0
λ Scorpii	1.71	B2	α Ursæ Majoris	1.95	K0
ε Argûs	1.74	K0	ε Sagittarii	1.95	A0
ε Orionis	1.75	B0	δ Canis Majoris	1.98	F8
β Tauri	1.78	B8	β Canis Majoris	1.99	B1

Magnitude of Faintest Star Visible in a Telescope of Given Aperture.— Observation shows that the faintest star visible in a telescope of *one-inch* aperture is a star of the ninth magnitude. Since the light-gathering power of a telescope varies as the square of the aperture, this star is a^2 times as bright as the faintest that can be seen in a telescope of a inches aperture. Hence, by substituting 9 for m, and a^2 for l_m/l_n in the equation connecting magnitude and brightness, we may obtain the magnitude, n, of the faintest star visible in a telescope of a inches aperture:

$$\begin{aligned} n &= 9 + 2.5 \log a^2 \\ &= 9 + 5 \log a. \end{aligned}$$

For example, since the logarithm of 10 is 1, the *minimum visibile* of a ten-inch telescope is $9 + 5 = 14$. The logarithm of 100 being 2, the *minimum visibile* of a one-hundred-inch telescope is $9 + 10 = 19$. Of course, although these numbers represent the magnitudes of the faintest stars that can be *seen* in the respective instruments, much fainter stars can be *photographed*, and some of the plates taken with the one-hundred-inch telescope of the Mount Wilson Observatory show stars down to about the twenty-second magnitude.

Enormous Range in Apparent Brightness of Stars.—Since a difference of five in the magnitudes of two stars corresponds to a hundredfold ratio of brightness, a first-magnitude star sends to the Earth 100 times as much light as a star of the sixth magnitude, 100 × 100 or 10,000 times as much as one of the eleventh, 100 × 10,000 or 1,000,000 times as much as one of the sixteenth, and 100 × 1,000,000 or 100,000,000 times as much as one of the twenty-first. It would require at least 3,000,000,000 of the faintest stars photographed with the Hooker telescope to give us as much light as does Sirius. On the other hand, since the Sun is more than twenty-five magnitudes brighter than Sirius, the light of 100^5, or 10,000,000,000, stars as bright as Sirius would be no brighter than sunlight.

Determination of Magnitude by the Method of Argelander.—Probably the simplest method of observing stars for magnitude is that known as the method of Argelander, from the name of the German astronomer who, in the middle of the nineteenth century, compiled the great star catalogue known as the *Bonn Durchmusterung*, giving the positions and magnitudes of about 324,000 stars. The method consists of forming a "sequence" of the star to be measured and at least two other stars of known magnitude. The observation may be made with or without a telescope, but the stars should be at about the same altitude and as near together in the sky as possible. It is best to find one comparison star just perceptibly brighter and one just perceptibly fainter than the star to be measured, and in any case the proportion of brightness of the three stars is noted. From the known magnitudes of the comparison stars the desired magnitude can easily be determined by this method with an accuracy of a tenth of a magnitude.

Photometers.—For the more accurate measurement of the light of stars, a number of different instruments known as photometers are in use. One of the simplest is the **wedge photometer,** which consists essentially of a wedge of dark glass inserted in the focal plane of the telescope, or between the eyepiece and the observer's eye. The observation is made by pushing the wedge into the beam of light until the star is extinguished or made equal to an artificial "star" formed by a small lamp shining through a pin hole. The star to be measured and a comparison star of known magnitude are thus observed in turn, and readings are made on a scale which gives the thickness of dark glass traversed by the light. It can be shown on elementary physical principles that the difference of magnitude between the comparison star and the star under measurement is proportional to the difference of the scale readings.

Other important types are the **polarizing photometer,** with which the great photometric star catalogues of the Harvard and Potsdam observatories were formed, and in which the apparent brightness of two stars (one of which may be artificial) is equalized by passing the light of one or both through a system of polarizing prisms; and the **photo-electric photometer,** which is based on the principle that, when light falls upon the surface of certain metals such as sodium, potassium, and cæsium, the metal gives off negative electrons and so acquires a positive charge which can be measured with an electrometer. The photo-electric photometer is capable of great accuracy, being reliable to about a hundredth of a magnitude.

Photographic Photometry.—The use of star photographs for determinations of relative brightness was begun in 1857 by G. P. Bond at the Harvard Observatory, and the method has been extensively used at Harvard and other large observatories. It has the advantage that the resulting magnitudes are not affected by individual peculiarities of vision, and the great further advantage that it can reach much fainter stars than the visual methods.

The most commonly used method is the one developed by E. C. Pickering at Harvard and known as the method of **sequences.** A list of stars in some readily accessible region, such as the Pleiades or the vicinity of the north pole, is chosen so that their magnitudes form a graduated series, and these magnitudes are determined with great care. A photograph of these stars is then compared with a photograph of the stars to be measured, preferably taken on the same plate, and the unknown magnitudes are found by interpolation in the sequence. In making the comparisons, both the size and the density of the images are taken into account. The standard North Polar Sequence formed at the Harvard Observatory contains ninety-six stars, ranging from the second to the twenty-first magnitude, and located within two degrees of the 1900 position of the North Pole.

Color Index—Since the eye is most sensitive to yellow light while the ordinary photographic plate is most sensitive to blue, a red star appears fainter, relative to a white one, on a photograph than in the sky. This is very noticeable in the photograph of the constellation Orion reproduced in Plate 39, where any one familiar with only the visual appearance of the constellation may have some difficulty in identifying the red star α Orionis because of its faintness, although to the eye it appears quite comparable to Rigel. Because of this difference of sensitiveness to different colors, it is necessary to express magnitudes determined photographically and those determined visually on different scales. The number found by subtracting the visual magnitude from the photographic may be regarded as a measure of the redness of the star, and is called the **color index.** The color index, of course, varies with the spectral type, and it is generally agreed to consider the color index of the white Ao stars as zero and that of the K stars as unity; the resulting values for the principal types are:

B	A	F	G	K	M
−0.24	0.00	+0.28	+0.56	+1.00	+1.35

Photovisual Magnitudes.—By using special plates, sensitive to the yellow and red, together with a suitable screen, a combination may be formed for making photographs in which the relative sensitiveness to light of different colors is about the same as that of the eye. Magnitudes determined from such photographs are known as **photovisual magnitudes.** The method has the advantages of the regular photographic method, and at the same time the results are expressed in terms of the visual scale. The approximate spectral type of stars too faint for observation with the spectrograph may be determined by comparing their photographic and photovisual magnitudes, which gives their color index.

The Measurement of Total Radiation.—The most complete way to compare one star with another is to measure their total radiation, including ultra-violet, visible, and infra-red rays. Many attempts have been made to do this with different forms of **radiometers,** and the greatest success has been attained by the use of the **vacuum thermocouple** in the hands of Coblentz and Lampland at the Lowell Observatory and of Nicholson and Pettit at Mount Wilson. The thermocouple consists of a junction of two metals such as bismuth and platinum. If the free extremities of the two pieces of metal are connected through a galvanometer, an electric current may be detected in the circuit when radiation falls upon the junction, and the strength of this current is proportional to the intensity of the radiation, which may therefore be measured by reading the galvanometer deflection. In the instrument as used for the study of the stars, the couple is made very small—the receiver upon which the light falls is only a small fraction of a millimeter in diameter—and is inclosed in a vacuum chamber to prevent loss of heat by convection in the surrounding air, and the receiver is placed in the focal plane of a large telescope, The telescope must be a reflector since the glass lenses of a refractor absorb radiation of certain wave-lengths more strongly than that of others. As used with the 100-inch Hooker telescope, the instrument is sufficiently sensitive to detect the heat of a candle 120 miles away.

The difference of the "radiometric" and visual magnitudes of a star is called the star's **heat index.** The heat index of A0 type stars, like their color index is taken as zero. That of the red stars, whose radiation is relatively more intense in the infra-red, amounts in some cases to several magnitudes.

Star Catalogues.—The stars are so numerous that progress in our knowledge of them depends largely upon statistical studies. For this purpose, it is necessary that the results of observations of many stars be gathered together in a single work. A **star catalogue** is a list of stars giving for each its apparent position (usually expressed as right ascension and declination), its magnitude, and such other particulars as may serve the special purpose of the compiler.

The earliest star catalogue is believed to have been made by Hipparchus about 150 B.C. No copy of it is now in existence, but the catalogue of Ptolemy, contained in the Almagest, is supposed to be derived from it. Ptolemy's catalogue contains 1,030 stars and gives their magnitudes, their latitudes and longitudes, and their positions in the constellation figures. The latitude or longitude of some of the stars is in error by as much as a degree, yet the work is invaluable as a description of the ancient constellations. Other catalogues of about the same extent as Ptolemy's were made by the Persian Al-Sufi during the tenth century, by Ulugh Beg of Samarcand during the fifteenth, and by Tycho Brahe in 1580. A supplement to Tycho's catalogue, which extended it to the South Pole, was published by Halley in 1677. In 1601 Bayer, who introduced the Greek-letter designations of stars now so commonly used, published a set of maps of the constellations with a list of the stars in each constellation on the back. Hevelius, in 1690, published the first catalogue containing telescopic stars.

Modern star catalogues are very numerous and extensive. Most of them give the visual magnitudes of the stars, their right ascensions and declinations referred to a designated epoch, and values of the precession and sometimes of its rate of change; and some give other data, such as spectral type, photographic magnitude, etc.

One of the most important is that known as the *Bonn*

Durchmusterung (abbreviation B. D. or D. M.) published by Argelander, which contains more than 324,000 stars down to magnitude 9.5 between the North Pole and −2° declination, arranged in zones corresponding to every degree of declination. The right ascensions are given to tenths of seconds of time and the declinations to tenths of minutes of arc, and all positions are referred to the equinox of 1855.0. This work is continued to declination −22° in Schönfeld's *Southern Durchmusterung*.

A much more accurate catalogue is the *Preliminary General Catalogue* of Boss, which gives, for the epoch 1900.0, the positions of 6,188 stars. The right ascensions are given to the thousandth of a second of time, and the declinations to the hundredth of a second of arc. These stars lie in all parts of the sky from the North Pole to the South.

Among catalogues giving information of special kinds may be mentioned the *Henry Draper Catalogue* published by Miss Cannon of the Harvard Observatory, which gives the visual and photographic magnitudes and the spectral types of 225,000 stars in all parts of the sky, together with their approximate positions for 1900.0; Burnham's *Catalogue of Double Stars*, which contained over 14,000 double stars when published by Burnham of Yerkes in 1906, and which is being about doubled in size by Aitken of Lick; and Dreyer's *New General Catalogue* of nebulæ and star clusters (abbreviated N. G. C.) which, together with its appendices, the *Index Catalogues*, gives positions and descriptions of more than 13,000 nebulæ and star clusters.

EXERCISES

1. Show that, in the spectrograph illustrated in Fig. 153, in order that neither star light shall be wasted nor any part of the prism remain unused, the collimator C must have the same ratio of aperture to focal length as the telescope objective T.

2. Verify the distances given for the first five stars of Table 16.

3. How does the brightness of Venus, when her magnitude is − 4, compare with that of a sixth-magnitude star?

Ans. Venus is 10,000 times the brighter

4. How does the brightness of Venus, magnitude − 4, compare with that of a star of the 16th magnitude? With that of a star of the 21st magnitude?

5. If the distance of an eighth-magnitude star were reduced to 1/10 its present value, what would be the magnitude of the star?

Ans. 3

6. What is the photographic magnitude of a K-type star of visual magnitude 2?

Ans. 3

NOTE: For the following exercises, a table of logarithms or a slide-rule is needed. The three-place logarithmic table printed below is of ample accuracy.

7. How does the brightness of Aldebaran compare with that of β Canis Majoris (Tables 17 and 18)?

Ans. Aldebaran is 2.4 times the brighter

8. What is the magnitude of a double star whose components are each of the fourth magnitude?

Ans. 3.25

9. What is the magnitude of a double star whose components are of magnitudes 2 and 3?

Ans. 1.64

10. What is the magnitude of the faintest star visible in a 12-inch telescope?

Ans. 14.4

11. What is the magnitude of the faintest star which will be visible in the 200-inch telescope mentioned on page 51?

Ans. 20.5

TABLE 19

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9
1	000	041	079	114	146	176	204	230	255	279
2	301	322	342	362	380	398	415	431	447	462
3	477	491	505	519	531	544	556	568	580	591
4	602	613	623	633	643	653	663	672	681	690
5	699	708	716	724	732	740	748	756	763	771
6	778	785	792	799	806	813	820	826	833	839
7	845	851	857	863	869	875	881	886	892	898
8	903	908	914	919	924	929	934	940	944	949
9	954	959	964	968	973	978	982	987	991	996
10	000	004	009	013	017	021	025	029	033	037

CHAPTER XV

THE MOTIONS OF THE STARS

Proper Motion.—The stars, which are so often called "fixed stars" to distinguish them from the planets, in reality possess rapid motions which are evident in different ways. When observations of the right ascension and declination of a star, carefully made on two or more different dates, are compared, it is always found that the apparent position has changed. A large part of this change is due to precession, nutation, and aberration, which, being common to all stars in a given small region of the sky, are sometimes called "common motions." These changes, and also the annual parallactic displacement which is perceptible in the nearest stars, are of course really due to the motion of the Earth. When all these effects have been accounted for, however, there remain small and usually random changes of apparent position which increase steadily with the passage of years, and which, being proper or peculiar to individual stars, are called **proper motions.**

Proper motion must necessarily be expressed in units of angular measurement. It is in all cases small, only a few seconds per century for most of the stars in which it has been detected. It is determined by comparing star catalogues of widely separated epochs or photographs made with the same instrument at intervals of several years. The greatest proper motion known is 10″.3 a year, that of a tenth-magnitude star in Ophiuchus which is usually referred to as "Barnard's proper-motion star," having been discovered photographically by Barnard at the Yerkes Observatory in 1916. The only bright stars with a proper motion of more than 2″ a year are α Centauri, 3″.7, and Arcturus, 2″.3. These motions are so slow that hundreds of years must elapse before the stars change their alignment with other stars by an amount noticeable to the unaided eye.

Components of a Star's Motion.—Proper motion, being a change of *apparent* position, depends upon three factors: the actual speed of the star, its distance from the Solar System, and the direction of its motion. Let the Solar System be at O and let the star move in one year from A to B, as in any of the three cases presented in Fig. 157. The distance AB, which may be called the star's **space motion,** cannot be determined directly, but only as the resultant of the **radial motion,** AH, and the **cross motion,** AK, which are the projections of the space motion along and perpendicular to the line of sight OA. The proper motion will be the projection, *ab*, of the space motion upon the celestial sphere and will be measured by the angle AOB or μ.

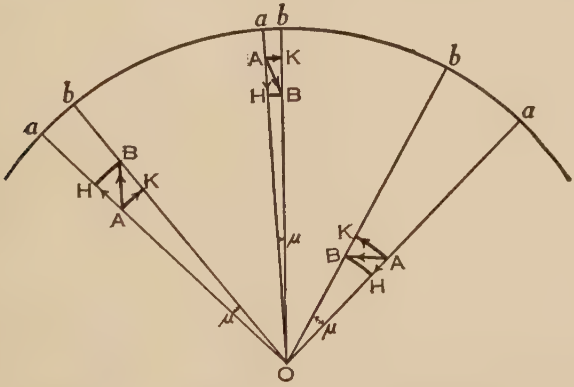


Fig. 157. Components of a Star's Motion

The cross motion may be computed from the proper motion and the heliocentric parallax if these are known. If D be the distance of the star from the Solar System expressed in astronomic units, and if the proper motion μ be expressed in seconds of arc, the yearly cross motion in astronomic units is $\mu D/206,265$. D is related to the heliocentric parallax p by the equation

$$D = \frac{206,265}{p};$$

hence, the yearly cross motion in astronomic units is equal simply to μ/p. It is often desirable to have the cross motion in kilometers per second. Denoting this number by x, we have

$$x = \frac{\mu}{p}\,\frac{149{,}500{,}000}{365\frac{1}{4} \times 24 \times 60 \times 60} = 4.74\frac{\mu}{p},$$

as the reader may verify.

The radial motion is determined in kilometers per second by measurements of the Doppler-Fizeau displacement (page 167) in the lines of the star's spectrum. To the observed radial velocity must be applied a correction, known as the **reduction to the sun,** for the motion of the observer; the result is the radial motion of the star referred to the center of the Solar System and represented by the line *A H* in the figure. With existing spectrographs the radial velocity of a star can be determined with an uncertainty of only about one kilometer per second if the lines of the spectrum are numerous and sharp, as they are in the case of the G-type or redder stars; the velocities of the hottest stars, which have broad, diffuse lines, are not so accurately known.

The reduction to the Sun is the projection of the observer's heliocentric velocity upon the line of sight to the star, and comprises two parts, one due to the Earth's rotation and the other to its revolution. The former is zero for a star at the pole of rotation (and would be zero also for an observer at the Earth's pole, wherever the star might be); for an observer in latitude 0° and a star in declination 0°, it varies from $-$ 0.47 km./sec. at hour-angle 6^h to $+$ 0.47 km./sec. at hour-angle 18^h. The annual part of the reduction to the Sun is zero for a star at the pole of the ecliptic, while for stars on the ecliptic it varies from 0 to ± 30.27 km./sec., depending on the longitude of the star and the time of year.

The space motion of a star is the square root of the sum of the squares of its cross motion and its radial motion. From the foregoing discussion it appears that its determination depends upon three distinct kinds of observation: that of proper motion, by comparison of the apparent positions of the star at dates many years apart; that of the distance of the star, most directly determined by the heliocentric parallax derived from observations of position about six months apart; and that of radial velocity, from an observation of the spectrum.

Proper motions have been studied since 1718, when Halley detected the motions of Sirius, Arcturus, and certain other stars by comparing their observed places with the positions given in Ptolemy's catalogue. Distances have been known for some of the stars, as we have seen (page 310), since 1838. Radial velocities were first measured by Huggins in 1866; but his observations, being visual, were very inaccurate. Accurate photographic determinations of radial velocity date from the work of Vogel and Scheiner at Potsdam about 1890 and that of Campbell at the Lick Observatory in 1892.

Magnitude of Stellar Velocities.—The great majority of known velocities of stars relative to the Solar System are less than fifty kilometers a second, and so are quite comparable to the orbital velocities of the planets. A few stars move much more rapidly than this, and about fifty are known to have speeds of 100 km./sec. or more. Table 20 gives the velocities with respect to the Solar System of some of the most rapidly moving stars. Slightly higher velocities than these are found in the globular star clusters and Magellanic clouds, and much higher ones in the extra-galactic nebulæ. As the great distances of these objects preclude accurate determinations of their cross-motions, our information consists of radial velocities only.

The columns in Table 20, at the right of that which gives the star's declination, contain the following data: the annual proper motion, μ; the position angle ρ (page 335) of its direction; the heliocentric parallax; the radial velocity, V; the cross motion, χ; and the space motion, S.

As early as 1903, Frost noted that the B stars moved more slowly than others, and in 1910 it was discovered by Campbell from radial velocities and by Boss from proper motions that the average velocities of stars increase in passing through the spectral series from B to M, being about 6 km./sec. for the B stars and about 17 for the M stars. As might be expected, the dwarf stars of each type have higher velocities than the giants of the same type.

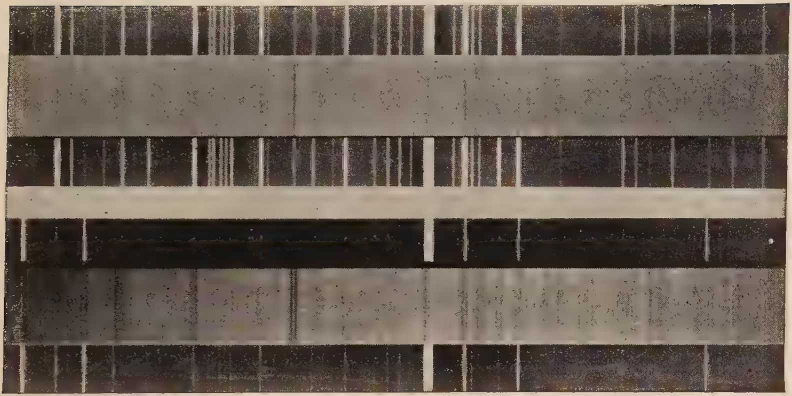
The Motion of the Solar System.—If all the stars were known to be fixed relative to one another, as by a framework, their apparent motions would necessarily be explained by a motion of the observer relative to the frame. Since the apparent displacement of a star produced by the Earth's *orbital* motion is in a closed curve, progressive changes such as proper

TABLE 20

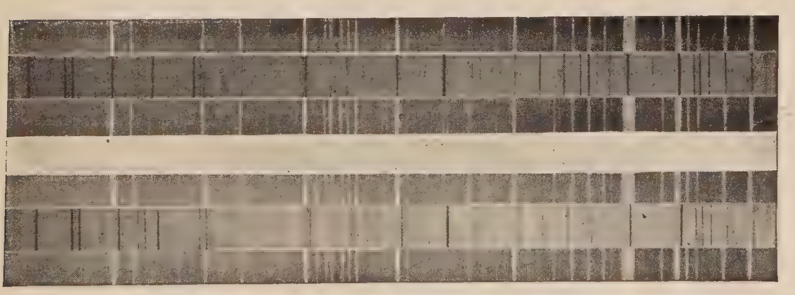
STARS OF RAPID MOTION

Name	Mag.	Spec.	α1900	δ1900	μ	ρ	Parallax	V km./sec.	x km./sec.	S km./sec.
Van Maanen's star	12.34	F0	0^h 44	+ 4°.9	3″.01	155°	0″.24	+ 240	59	247
Helsingfors 956	7.8	G0	1 3	+ 61.0	0.64	85.5	0.03	− 325	100	340
μ Cassiopeiæ	5.26	G5	1 2	+ 54.2	3.76	114.5	0.15	− 97	118	153
e Eridani	4.30	G5	3 16	− 43.2	3.16	76.4	0.15	+ 87	100	133
BeB 1366	8.9	F2	4 9	+ 22	0.54	124.9	0.10	+ 339	26	340
o_2 Eridani	4.48	G5	4 11	− 7.8	4.08	212.7	0.21	− 42	92	101
CZ5^h243 (Kapteyn's star)	9.2	K2	5 8	− 44.8	8.76	130.5	0.32	+ 242	130	279
Wolf 359	13.5	M6e	10 52	+ 7.6	4.84	232	0.41	....	56	....
22H Camelopardalis	7.60	M2	10 58	+ 36.6	4.78	186.8	0.41	− 87	54	102
Bo 7899	8.9	Ma	11 0	+ 44	4.52	282.1	0.21	+ 65	100	119
Innes's star	12.5	......	11 12	− 57.0	2.69	291	0.34	....	37	....
Groombridge 1830	6.46	G5	11 47	+ 38.4	7.05	145.3	0.11	− 97	303	318
Wolf 489	13	......	13 32	+ 4	3.94	252	......	....	....	....
Proxima Centauri	10.5	M	14 23	− 62.3	3.85	282.9	0.78	....	23	....
α Centauri	0.33	G0	14 33	− 60.4	3.68	281.4	0.76	− 22	23	32
OA$_s$ 14320	9.9	G5	15 5	− 15.9	3.68	195.7	0.03	+ 307	583	660
Barnard's star	9.67	M5	17 53	+ 4.4	10.3	355.9	0.56	− 117	87	149
61 Cygni	5.57	K5	21 2	+ 38.3	5.27	52.1	0.31	− 64	81	103
ϵ Indi	4.74	K5	21 56	− 57.2	4.70	123.4	0.29	− 39	77	89
Cordoba 31353	7.44	Ma	22 59	− 36.3	6.90	79.3	0.28	+ 12	117	117
Cordoba 32416	8.3	Ma	23 59	− 37.8	6.11	112.7	0.19	+ 26	152	154

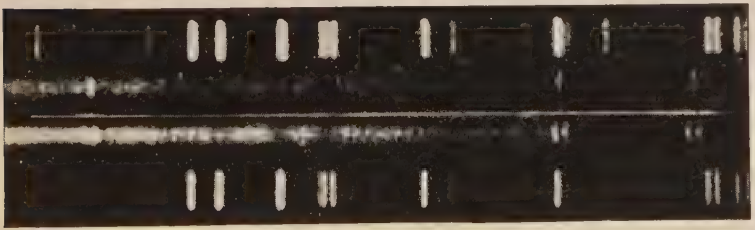
PLATE 40. SPECTRA OF SPECTROSCOPIC BINARIES



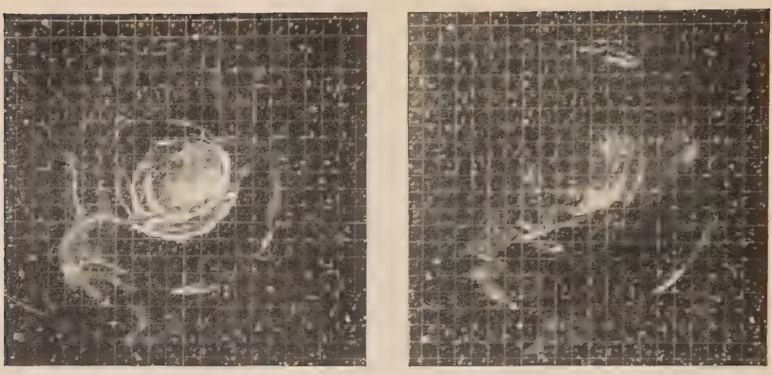
ζ Ursae Majoris—single and double lines. Yerkes Observatory



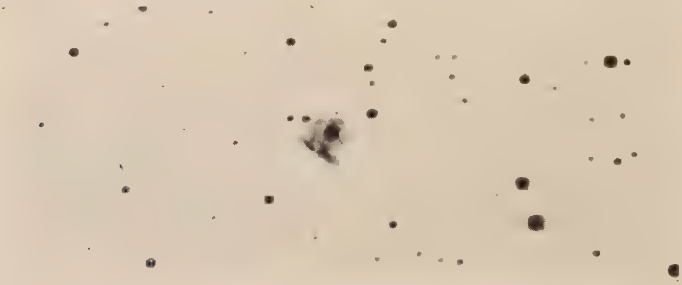
μ Orionis—one bright component only. Yerkes Observatory



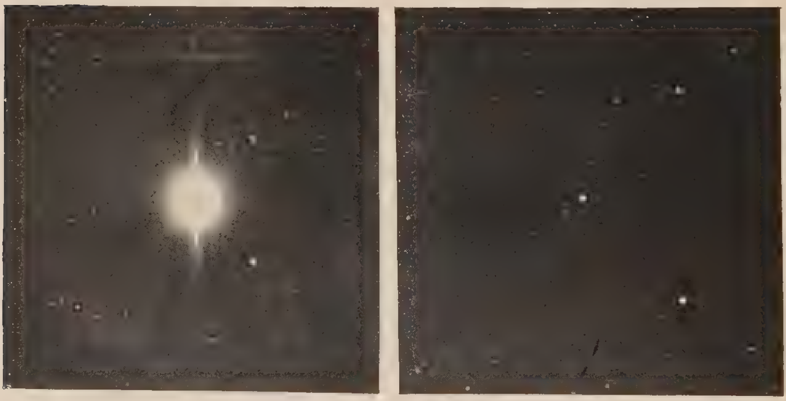
Faint Proper-Motion Companion of Castor—Single and double bright lines
Mount Wilson Observatory



Wolf's nebulous shell around Nova Persei 1901 (page 342). Drawings from photographs with Yerkes 24-inch reflector. *Left*—1901, September 20 (Ritchey); *right*—1902 February 8 (Pease). Ruled lines are 2 minutes of arc apart. Top is north, left east



Barnard's nebulous envelope around Nova Persei, photographed by Ritchey with 60-inch reflector, 1917 October 16. Negative print, greatly enlarged. Diameter of ring was 16″. Top is north, left west



Nova Aquilae 1918, photographed by Duncan at Mount Wilson. *Left*—1918 July 7, one month after outburst (60-inch). *Right*—1926 May 17, 8 years after outburst (100-inch). The original plate of 1926 shows a planetary nebula 16 seconds of arc in diameter around the star, but it is too faint for reproduction motion and radial motion could be produced only by a progressive traveling of the whole Solar System.

Let the circle in Fig. 158 represent the infinite celestial sphere, and let the Sun, carrying the Earth and the other planets with it, move from *S* to *S′* in a hundred years, while the stars remain at rest. The proper motions of stars *B*, *C*, *D*, etc., would be in the directions of the arrows, away from the point *X* toward which the Sun is moving and toward the opposite point *N*. The size of these proper motions would depend upon the speed of the Sun's motion, the star's distance from us, and the star's angular distance from the point *X*, being zero for stars *A* and *E*. The radial velocity of star *A* would be negative and that of *E* positive, of a numerical value equal to the speed of the Sun; while the radial velocities of other stars would be less, depending on their apparent distances from *X* or *N*.

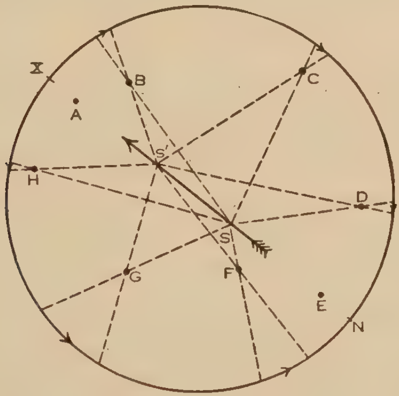


FIG. 158. MOTION OF THE SOLAR SYSTEM

The fact that the stars are not at rest complicates matters, but by taking the average apparent motion (proper or radial) of a number of stars in each of many parts of the sky the solar motion may be determined with respect to the system of stars so chosen. The problem is analogous to that of the motion of a pedestrian through a swarm of gnats; the average of his velocity with respect to a number of the gnats should give his velocity with respect to the swarm, although the individuals of the swarm may be flying in many directions at different speeds.

As early as 1783 Sir William Herschel inferred from the proper motions of thirteen stars—all whose motions were then known—that the Solar System was traveling toward a point in the constellation Hercules; but, not knowing the distances or radial velocities of any stars, he was unable to estimate the speed. Herschel applied the term **apex of the Sun's way** to the point *X* toward which the Sun is moving, and called the opposite point *N* the **antapex.**

Herschel and others improved his result as additional proper motions became known, and when distances also became available estimates of the speed of the Sun were made. During the twentieth century, more accurate determinations of the solar motion have been made by using the radial velocities of stars.

From the radial velocities of 2,034 stars, Campbell and Moore in 1925, at the Lick Observatory, made the following determination of the motion of the Solar System:

Right ascension of apex	$17^h\ 55^m.8$
Declination of apex	+27°.2
Speed	19.0 km./sec.

This result does not differ materially from the results of other careful determinations made from large numbers of miscellaneous stars. The apex is not far from the direction of the star μ Herculis, and the antapex is in the constellation Columba, about 15° south of Rigel. Expressed in other units, the speed of the Sun with respect to the group of stars used in this determination is about twelve miles a second, four astronomic units a year, or one light-year in 16,000 years. Only ten million years ago—not very long, speaking astronomically or geologically—our system was about 600 light-years in the rear of its present position, in the general neighborhood of the bright stars of Orion.

As nearly as may be determined from the observations, the motion of the Sun, and also that of every other isolated star, is in a straight line; the curvature due to the attraction of other stars is too small to be detected.

Motus Parallacticus and Motus Peculiaris.—The apparent motion of a star away from the solar apex or toward the antapex, which it possesses in virtue of the Sun's motion relative to the whole aggregation of stars, is called its *motus parallacticus;* the remaining apparent motion when this is eliminated from the proper motion is called the *motus peculiaris.* Dividing the *motus parallacticus* by the sine of the star's apparent distance from the solar apex gives the **secular parallax,** which may be defined as the apparent motion of the Solar System as seen from a point situated at the star's distance and in a direction at right angles to that of the solar motion. On the assumption that the peculiar motions of all the stars in a group are random, the secular parallax of the group may be calculated from the average of their proper motions; and the motion of the Solar System in a long interval, say a hundred years, affords a long base line which with the secular parallax gives the distance of the group. Although inapplicable to single stars, the method is very valuable in statistical studies, and it was from such studies that Kapteyn made his determination of the size of the discoid Galactic System (page 414).

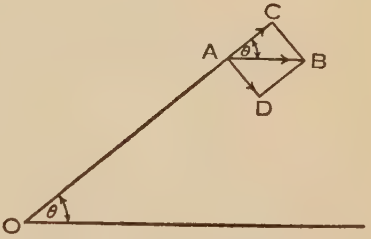


Fig. 159. Determination of the Distance of a Star in a Moving Group

Moving Groups of Stars.—Studies of proper motions have disclosed a number of groups of stars whose proper motions are either nearly equal and parallel or along converging great circles of the celestial sphere. One of the best examples is that known as the **Taurus moving cluster** which comprises thirty-nine stars, many of which belong to the Hyades. L. Boss found that the proper motions of these stars converge toward a point of the sky a little east of α Orionis. Since, from the effect of perspective, apparent convergence results from actual parallelism, it is reasonable to suppose that the space motions of these stars are parallel; and this is confirmed by the computed space motions of a number of the stars for which radial velocities have been obtained. Moreover, the space motions are equal within the probable value of the errors of observation, and it is highly probable that they agree within a small fraction of a kilometer per second; for if they did not, the cluster could not remain compact for many million years and it is unlikely that its age is not far greater than this. In other words, these thirty-nine stars must be moving through space in a squadron while preserving their mutual distances.

If, in addition to the proper motions of the stars in a moving group, the radial velocity of one of their number can be obtained, the cross motions, space motions, distances, and luminosities (page 347) of all the stars of the group may be readily and accurately computed. To explain this, let the Solar System be at O (Fig. 159) and let the space motion of one of the stars of the group be from A to B. Draw the line OX parallel to AB; it will meet the celestial sphere at the "convergent" of the moving star-group—*i.e.*, at the point toward which the proper motions converge, for parallel lines meet at the surface of the celestial sphere. The angle AOX (which is the angular distance of the star from the convergent) equals the angle CAB made by the direction of the star's motion with the line of sight. The radial velocity is represented by the side AC of the rectangle $ACBD$, of which the space motion AB is the diagonal, and the cross motion is the side AD. Let the radial, space, and cross motions be V, S, and χ, respectively, and let the angle AOX or CAB be θ. Then

$$S = \frac{V}{\cos\theta} \text{ and } \chi = S \sin\theta.$$

The heliocentric parallax, proper motion, and cross motion are connected, as we have seen (page 326), by the relation

$$\chi = 4.74\ \mu/p,$$

whence

$$p = \frac{4.74\ \mu}{S\sin\theta} = \frac{4.74\ \mu}{V\tan\theta},$$

from which the parallax, and hence the distance and luminosity, of the star may be found. Since S is common to all the stars of the group, while μ and θ are known for each, a similar computation may be made for every star of the group.

It is known in this way that the stars of the Taurus moving cluster occupy a roughly globular space some 32 light-years in diameter and 130 light-years away; that the stars of the group are from 5 to 100 times as luminous as the Sun;[1] and that their space velocity is about 46 km./sec. Presumably, there are among the stars of the group many interloping stars which do not partake of the group motion. About 800,000 years ago the group passed its perihelion at a distance of about 65 light-years. After about 65,000,000 years more it will have receded so far as to appear as a sparse globular cluster about 20′ in diameter.

Another celebrated moving group is that known as the Ursa Major system, which consists of about a dozen stars, including five stars of the Big Dipper and also Sirius, α Coronæ, β Aurigæ, and β Eridani. These stars are from 7 to 400 times as luminous as the Sun, occupy a disk-shaped region of space 100 to 175 light-years in diameter, and move with a speed of 29 km./sec. toward a point in the Milky Way southwest of Altair.

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[1] In addition to these thirty-nine giants of Boss's original group, a number of dwarfs are now known to belong to it also.

The stars of the Pleiades have a common proper motion, and so do those of Præsepe. Other moving groups are found in Perseus, Orion, and Scorpius. The Pleiades and Perseus appear to be at a distance of about 350 light-years, Præsepe at 400, and Orion at 600 light-years.

Motions of Different Classes of Cosmic Bodies.—When the bodies outside the Solar System are divided into classes, as giant stars of different spectral types, dwarfs of different types, long-period and short-period Cepheids, moving star groups, globular clusters, galactic nebulæ, extra-galactic nebulæ, etc., it is found that the motions of these different classes vary greatly. Strömberg of Mount Wilson has recently made an extensive study of this kind, considering the motions under two aspects: the motion of the individuals of each group and the motion of the group. These motions are like those of gnats in swarms—the individuals buzz about within the swarm and at the same time many swarms may be moving at different speeds and directions across a summer landscape, even interpenetrating without mutual interference; but among the stars the individuals are much farther apart as compared to their size than among the gnats, and interpenetration is more general.

Strömberg obtained two highly interesting results: First, the range of speed of the bodies within the groups is greater, the greater the group-motion. The motions of the stars in most of the classes within their groups are greater than the group-motion and are prevailingly in two opposite directions, a fact which was detected a quarter-century earlier by Kapteyn and attributed by him to two streams of stars flowing in opposite directions. Second, the various swarms or classes of bodies are moving, as if along a great cosmic thoroughfare, toward a point which to us appears in the Milky Way, in α $20^h\ 40^m$, $\delta + 57°$, near the border of Cygnus and Cepheus. The group-velocities along this axis, taken relative to the Sun, vary from 9 km./sec. for the long-period Cepheids to 300 km./sec. for the globular star-clusters and spiral nebulæ.

Binary Stars.—The term **double star** is sometimes applied to any two stars that lie close together on the celestial sphere. Ptolemy applied the Greek equivalent of the term to ν Sagittarii, which consists of a pair of fifth-magnitude stars about 14′ apart. Similarly, Mizar and Alcor, in the handle of the Great Dipper, which are separated by about 12′; α_1 and α_2 Capricorni, about

6′ apart; and ε Lyræ, in which the separation is 3½′, might be called double stars. In the stricter sense in which astronomers use the term, however, it is applied only to pairs in which the separation is not more than half a minute, or is even much less if the stars are very faint. A remarkably large number of stars are shown by a good telescope to be double, and many more have invisible companions which manifest their presence by their effect upon the motion or brightness of their primaries.

Until late in the eighteenth century it seems to have been generally assumed that the two components of a double star were in reality far apart and that they appeared close together merely through being almost in line with the Solar System, one behind the other. In 1789 Sir William Herschel began a careful study of double stars in the hope of detecting the parallactic displacement of the nearer component with respect to the more remote; but instead, after a number of years he found in a few pairs an orbital motion of one star around the other, showing that the two were really near together and subject to their mutual gravitation. To such a pair of physically connected stars Herschel gave the name of **binary star.** It is probable that the majority of the 20,000 or more close visible double stars are binary, but in most cases the orbital motion is so slow that it cannot be detected until after many years' observation.

The presence of an invisible companion is detected in many cases by the spectroscope through the change of radial velocity produced in the visible star by orbital motion; in many others it is revealed by the change of brightness of the visible star when the companion passes before it. Stars whose duplicity is disclosed by the spectroscope are called **spectroscopic binaries;** those whose companions produce variations of brightness by occulting their primaries are called **eclipsing binaries,** or, from the name of their prototype, β Persei, **Algol variables.** This classification, being based on methods of observation, does not represent a difference in the real nature of the stars or of their motions. Many stars of each of the three classes belong also to another.

The Study of Visual Double Stars.—The appearance of a double star is described by the magnitude, color, separation and position angle of its components. The separation (denoted by ρ) is expressed in seconds of arc. The **position angle** (denoted by θ) is defined as the angle between the great circle which bisects the two stars and the hour circle bisecting the brighter, and is measured from the north through the east.

The values of ρ and θ are measured with the filar micrometer (page 47) as follows: At the beginning of the night's work a **trail reading** is taken upon the position circle of the micrometer by turning the telescope, with the driving clock stopped, to an equatorial star and placing the spider-lines so that the star trails, in its diurnal motion, along one of the lines. The lines are then exactly parallel to the equator, and 90° added to that reading of the position circle which corresponds to the exit-point gives the north point or "zero reading." The telescope is then turned to the double star and one of the lines placed so as to bisect both components as at *AA*, Fig. 160, and the reading of the position-circle in the direction of the fainter star is taken. This reading minus the zero reading is the position angle which, in the figure, is about 245°. After a number of settings are thus made and the mean taken, 90° is added to the mean and the micrometer is turned to this setting, which brings the spider lines perpendicular to the line joining the two stars. The separation is then measured by turning the micrometer screw until one line bisects each star, as at *BB*, *CC*, then interchanging the lines, placing the second line on the first star and the first line on the second; the difference between the readings of the screw, multiplied by the micrometer constant, gives *twice* the distance between the stars in seconds. This measurement also is of course repeated several times, and the results, both for position angle and separation, are carefully recorded. In the hands of a skilled observer, the whole operation requires only about ten minutes.

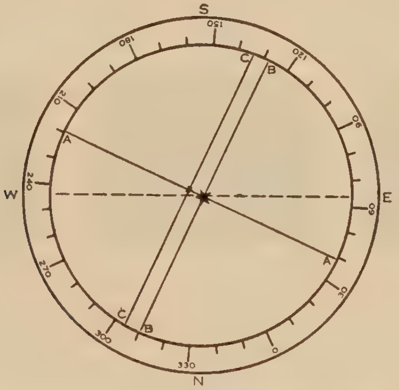

Fig. 160. Measurement of Position-Angle and Separation

Double-star astronomy is one great field in which visual observation has not been largely superseded by photography.

When an observation of this kind is repeated after the lapse of many years, if the star is a binary it will be found that ρ or θ, or more often both, have changed (Fig. 161); and by means of successive repetitions—usually by successive generations of observers, for most binary periods are long—the orbital motion may be followed through a complete revolution. To determine the relative orbit, a graphical method is used in which the brighter star is represented by a fixed point through which passes a fixed line representing the hour circle. A convenient scale being chosen to represent seconds of arc, the observed positions of the companion are then plotted according to position angle and separation. Except in a few cases where the motion is disturbed by a third body, the plotted points are found to lie, within limits set by the errors of observation, upon an ellipse (Fig. 162).

1908 July 21 1915 Sept. 2 1920 July 10

FIG. 161. THE BINARY STAR KRÜGER 60 AND A NEIGHBORING STAR, AS SEEN IN AN INVERTING TELESCOPE. PHOTOGRAPHED WITH THE YERKES 40-INCH REFRACTOR BY BARNARD

While the bright component always lies within the ellipse, it does not, in general, appear at the focus. This is an effect of foreshortening; we do not usually see the binary from a direction perpendicular to the plane of its orbit, but obliquely. The law of areas, however, is obeyed by the line joining the stars in the apparent as in the true orbit, since the projections of the areas on the plane tangent to the celestial sphere are proportional to the areas in the orbit. By mathematical analysis it is possible, from the apparent orbit, to compute all the elements except the *direction* of the inclination; from visual observations alone it cannot be determined whether, at a given point of the orbit, the companion is approaching or receding from the Earth, but this question also can be decided if even a single determination of the difference of radial velocity of the two stars can be secured. The value of the semi-major axis which results from the computation is necessarily expressed in seconds of arc; if the star's parallax is known, this may be transformed into astronomic units, and the sum of the masses may then be found (page 232).

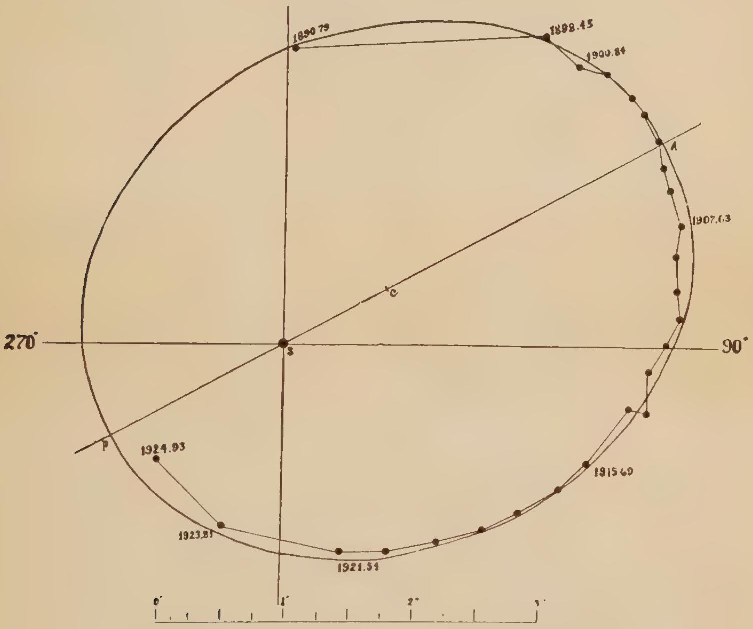

FIG. 162. THE ORBIT OF KRÜGER 60 AS DETERMINED BY AITKEN

While thousands of visual binaries have been discovered, the orbits so far determined number only about 100; the periods of most are centuries long, and they have not yet been observed over a sufficient arc of their orbits.

Proper-motion Companions.—Many cases are known in which two stars, in the same small region of the sky, have equal and parallel proper motions; they thus form small systems similar to the moving groups of stars (page 331). Many of these are doubtless near enough together to produce orbital motion, their mutual attraction being greater than the attraction of other stars for either; but their periods are so long that no curvature of their paths has as yet been found. Besides such pairs of this kind as form close double stars, the most noteworthy case is that of Proxima and α Centauri, which have practically identical proper motions and parallaxes. According to Luyten, the little star 46 Tauri has a space motion equal and parallel to that of the Solar System, and so in a sense forms a very distant companion to the Sun.

The Study of Spectroscopic Binaries.—In the discussion of the problem of two bodies (page 227) it was noted that the center of mass of two mutually gravitating stars is unaffected by their motions and moves uniformly in a straight line, while the stars revolve in similar ellipses with the center of mass at their common focus. Fig. 163 represents the orbits of two equally massive components of a binary whose center of mass is at C. By definition, the straight line joining the two stars must always pass through C; hence, when one star is at A_1 the other is at B_1; when the first is at A_2 the second is at B_2; etc. Suppose the Solar System is in the direction of the right side of the page. From position 3 to position 6 star A will, relatively to the center of mass, recede from the Solar System, while B approaches it; during the remainder of the revolution B will recede and A will approach.

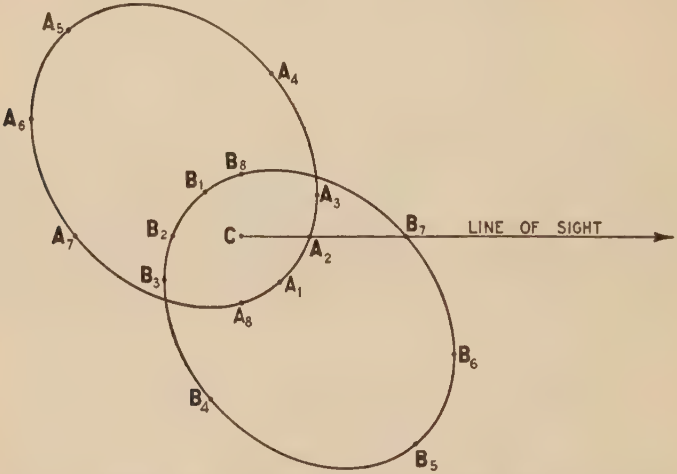

Fig. 163. Orbits of the Components of a Spectroscopic Binary

The binary character of a star may be discovered spectroscopically in one of two ways, depending on the relative brightness of the components. (1) If one star is much brighter than the other, its spectrum alone will impress the photographic plate and its lines will be displaced alternately toward the red and toward the violet from the position corresponding to the velocity of the center of mass as the bright star alternately recedes and approaches. (2) If the two stars are about equally bright they will be represented equally in the spectrum; then, when the lines of one are shifted to the red, the lines of the other will be shifted to the violet, as would occur at points 4, 5, and 8, while at points 3 and 6, where both stars are moving perpendicularly to the line of sight, the two spectra will be superposed. Thus, in case (1), all the lines of the spectrum are displaced together; in case (2), those lines which are common to both spectra appear alternately single and double. The two cases are illustrated in Plate 40.

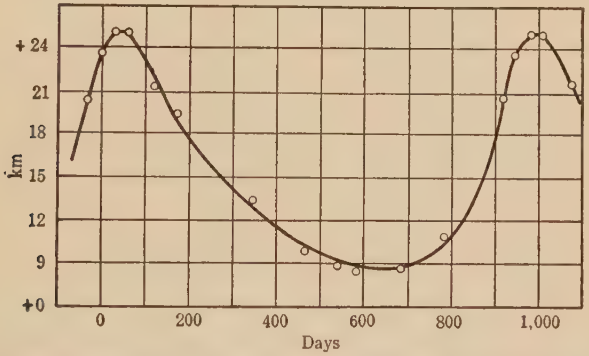

FIG. 164. THE VELOCITY CURVE OF THE SPECTROSCOPIC BINARY 5 TAURI (ONE COMPONENT), DETERMINED BY W. E. HARPER AT THE DOMINION ASTROPHYSICAL OBSERVATORY

Usually, of course, the plane of the orbit is inclined to the line of sight; the observed radial velocity, when reduced to the Sun, is then the projection, upon a plane containing the line of sight, of the resultant of the orbital motion and the motion of the center of mass. If the orbit plane be perpendicular to the line of sight, the motion cannot be detected by the spectroscope.

The nature of the orbital motion of the observed component is revealed by plotting the radial velocity, reduced to the Sun, as ordinates of a curve in which the abscissæ represent time counted from a fixed epoch. This curve, called a **velocity**

curve, is sinuous in form and repeats itself exactly if the motion is truly elliptical. If the orbit is circular, the velocity curve is a simple sine curve; otherwise, the ascending and descending portions are not of equal steepness. In Fig. 164 is shown the velocity curve of a binary of which the spectrum of only one component has been observed; in Fig. 165 are shown the velocity curves of both components of a binary where the two are about equally bright and equally massive. The circles in either figure represent observations, and the height of the dotted line in Fig. 165 represents the radial velocity of the center of mass of the pair of stars.

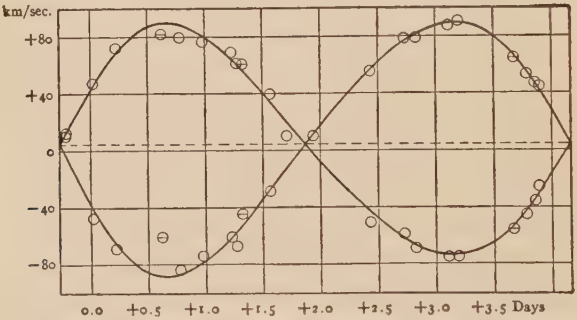

FIG. 165. VELOCITY CURVES OF BOTH COMPONENTS OF THE SPECTROSCOPIC BINARY BOSS 373 AS DETERMINED BY SANFORD AT MOUNT WILSON OBSERVATORY

It may be proved that the top and bottom of the velocity curve correspond to the points where the star crosses the plane through the center of mass and perpendicular to the line of sight; also, that a horizontal line bisecting the area of the curve represents the velocity of the center of mass. By mathematical analysis all the elements of the true orbit may be computed except the separate values of the major axis and the inclination of the orbit to the plane tangent to the celestial sphere; instead of these, the quantity $a \sin i$ may be found in kilometers, which sets a minimum value to the major axis. The mass cannot be found without a knowledge of the inclination, but the quantity $\frac{m^3 \sin^3 i}{M^2}$, (called the **mass function**), where m is the mass of the observed component and M the sum of the masses of both, may always be found.

The components of spectroscopic binaries being in general much closer together than those of visual binaries, their periods are much shorter—in some cases, less than a day—and information concerning their orbits accumulates much faster. Although spectroscopic binaries have been studied only since 1888, more than 1,000 examples are now (1930) known, and the number of computed orbits already exceeds 250.

The Study of Eclipsing Binaries.—Suppose two equal stars, one luminous and the other opaque, revolving in a plane which passes nearly through the Solar System. At each revolution, the dark star must pass in front of the bright one and intercept a part of its light; but during the remainder of the period the bright star would shine with constant luster. Plotting magnitudes as ordinates and time as abscissæ, we should then get

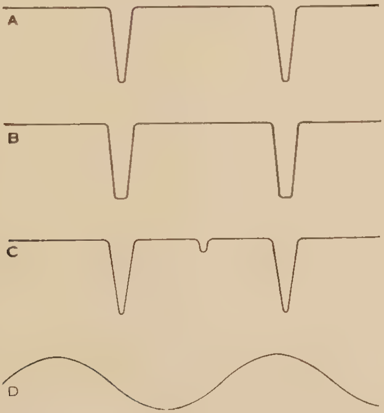

FIG. 166. LIGHT-CURVES OF ECLIPSING BINARIES

a **light-curve** somewhat like *A*, Fig. 166. Now suppose the dark companion to be smaller than the bright star and the inclination of the orbit to be such as to project the companion, at eclipse, wholly upon the surface of the primary. The light at minimum would then be constant for a short time and the light curve would be as at *B*. Thirdly, suppose that the companion were not perfectly dark, but shone faintly by its own light; the companion would then not only eclipse the primary, but would be eclipsed by it, producing a secondary dip in the light curve as at *C*, or as in Fig. 167, which is a light curve derived from actual observations. Finally, suppose the stars to be equal, both in diameter and in luminosity, and that they revolve almost in contact so that the interval between eclipses is short; the light-curve will resemble *D*.[1]

Algol, the eponym of this class of stars, was recognized as a variable star by Goodricke in 1783. Observations with the naked eye result in a light-curve similar to *A* (Fig. 166), and Goodricke proposed the explanation which is now known to be correct, namely, that the star, although appearing single in all telescopes, had a close, dark companion revolving in an orbit turned edgewise to the Earth in the period of the observed light-changes, $2^d\ 21^h$. This explanation was confirmed by Vogel of Potsdam in 1888 by showing the star to be a spectroscopic binary, the maximum velocity of recession occurring a quarter-period before the eclipse and the maximum velocity of approach a quarter-period after, while during eclipse and a half-period later the velocity is equal to that of the center of mass (Fig. 168). See also page 345. All Algol variables which have been investigated spectroscopically have shown similar variations of radial velocity, and there is no question that all variable stars of this type are binaries. About 200 such stars are now known.

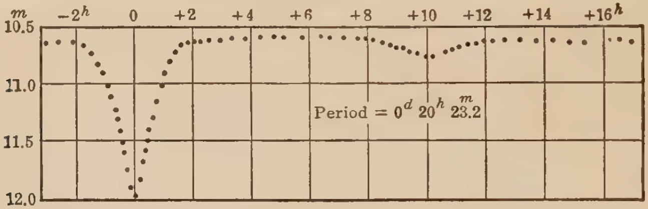

FIG. 167. MEAN LIGHT-CURVE OF RT PERSEI FROM OBSERVATIONS BY R. S. DUGAN, (FROM RUSSELL, DUGAN AND STEWART'S ASTRONOMY), BY PERMISSION OF THE PUBLISHERS, GINN AND COMPANY

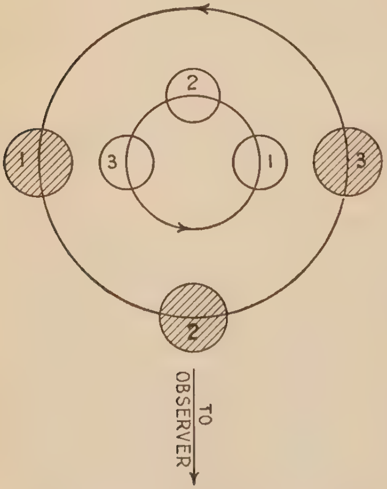

FIG. 168. AN ECLIPSING BINARY

From the light curve of an Algol variable, if it is precisely determined, many details can be learned concerning the two components and their orbits. The theory, begun by Pickering, has been extensively developed by Russell and applied by Shapley and others. The mere existence of an eclipse shows that the inclination of the orbit plane is near 90°. If both primary and secondary minima are observed, the intervals between them determine the eccentricity and the position of periastron—for example, if secondary minima occur halfway between primary minima (the usual case), the orbit is circular.

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[1] Curve *D* might also be produced by a single star, which had the form of a prolate spheroid and which rotated around a minor axis with the Solar System nearly in the plane of its equator.

The depth, form, and duration of the minima make known the inclination, the diameters of the two stars in terms of their distance apart, and their luminosities in terms of their combined light. If the light curve slopes upward after primary eclipse and reaches a maximum just before secondary eclipse (as Stebbins's precise observations have shown to be the case with Algol), the companion is brighter on its hemisphere which faces the bright star, due doubtless to reflection or re-radiation of the latter's light. If the curve reaches a maximum halfway between eclipses, the stars are ellipsoidal in form with their longest axes directed toward one another—at syzygy they are placed endwise toward us and at quadrature sidewise, thus giving us maximum light in the latter position. The extreme example is β Lyræ, which consists of two ellipsoidal stars revolving almost in contact. The ellipsoidal form is not uncommon among Algol stars generally, as might be expected in view of the powerful tidal action of two great stars situated close together.

If velocity curves of both components are available from spectroscopic observations, the combination of photometric and spectroscopic data affords a determination of the diameters of the stars and of their orbits in kilometers, of their masses in terms of the Sun's mass, and hence of their densities. In most Algol stars, according to Shapley, there is evidence that the fainter star is self-luminous, and in no case is it proved to be entirely opaque. Often, the bright star is smaller than its companion. The densities of all the eclipsing binaries of *B* and *A* spectral type are of the order of a tenth that of the Sun; their diameters and masses exceed those of the Sun. The densities of the few redder Algols known range from 0.000001 to 2.6$\odot$.

Some Binaries of Special Interest.—Mizar, ζ Ursæ Majoris, the bright star in the bend of the handle of the Great Dipper, was the first visual double and also the first spectroscopic binary to be discovered. The visual duplicity was discovered by Riccioli of Bologna in 1650. The two components are 14″ apart, of 2.4 and 4.0 magnitude. No orbital motion has been found, but both components of Mizar, and also Alcor, 11′ away, partake of the common proper motion of the Ursa Major Group (page 332). From the group motion, the distance from the Solar System is well determined at 72 light-years, and so the distance from Mizar to Alcor is no less than a quarter of a light-year, while even the components of Mizar are at least 300 astronomic units apart—ten times the distance of Neptune from the Sun. In 1889 E. C. Pickering found with the objective prism that the lines of the spectrum of the brighter component of Mizar were alternately double and single, showing it to consist of a pair of almost equally luminous revolving stars. The period of this pair is about 20.5 days. In 1925 Pease measured the distance and position angle of this close pair with the interferometer at Mount Wilson, finding a separation of from 0″.011 to 0″.013. By combining the spectroscopic and interferometric data, the elements of the real orbit were determined; the eccentricity is 0.53, the inclination to the plane perpendicular to the line of sight is 50°, and the sum of the semi-major axes of the two absolute orbits is 429,000,000 kilometers—somewhat greater than the mean distance of Jupiter. In 1908, both Alcor and the fainter component of Mizar were found, by Frost at the Yerkes Observatory, also to be short-period spectroscopic binaries.

Castor, α Geminorum, was found to be a double star by the English astronomers Bradley and Pound in 1719. The magnitudes are 2.8 and 2.0 and the separation is about 6.″ It is certainly binary, but the period is so long that it cannot yet be accurately determined; the most probable value is about 350 years. Both components are spectroscopic binaries; the orbit of the fainter is nearly circular, with a period of 3 days and a radius which may be as small as 1,300,000 kilometers (about three times the radius of the Moon's orbit), while the period of the brighter is 9 days, its eccentricity 0.50, and the minimum possible value of its semi-major axis about the same as that of the fainter. Both components of the visual pair are of A0 type. More remarkable still, at a distance of 73″ from the bright pair is a ninth-magnitude M-type star which shares the parallax and proper motion of Castor and which, as found by Adams and Joy in 1925, is a spectroscopic, eclipsing binary with alternately double and single lines, of which those of hydrogen and calcium are *bright* (Plate 40). The period of this extraordinary pair is about 0.8 day. The distance of Castor from the Solar System is about 44 light-years; hence, its visual components are about 95 astronomic units apart (three times the distance of Neptune from the Sun), while the bright-line dwarf companion is about 1,000 astronomic units away.

Unusually exact information has been obtained about **Capella**, which was found to be a spectroscopic binary by Campbell in 1899, with a period of 104 days and a nearly circular orbit. In 1921 Anderson and Merrill succeeded in measuring the position angle and separation with the interferometer (page 353). From their work and that of Campbell are derived the elements a = 0″.0536 = 126,630,000 kilometers (between that of the Earth and that of Venus); i = 41°.08; m_1 = 4.2 × Sun, m_2 = 3.3 × Sun; heliocentric parallax

$= 0''.0632$, distance = 52 light-years. Capella has a tenth-magnitude, M-type, proper-motion companion situated at a distance of 12′.

α **Centauri** has the largest apparent orbit of any known binary. Its orbit is highly eccentric ($e = 0.51$) and also highly inclined ($i = 79°$). Its apparent distance varies from 2″ to 22″. The actual distance varies from 11.4 to more than 35 astronomic units.

The visual binary having the shortest known period is *δ* **Equulei,** period 5.7 years. The apparent semi-major axis of its orbit is only $0''.27$. The star having the shortest known period of velocity variation is *γ* **Ursæ Minoris,** period 0.11 day. Its $a \sin i$ is only 35,000 kilometers, and this is outdone by the brighter visual component of *τ* **Cygni,** whose velocity curve indicates a value of $a \sin i$ of only 15,000 kilometers—about a quarter of the distance of the Moon from the Earth. It is probable that these two stars are not really binary, and that the observed velocity variation is due to a pulsation or rhythmic expansion and contraction of a single star, as has been inferred in the case of the Cepheid variable stars (page 381).

The visual binary **42 Comæ Berenicis,** period twenty-five years, is distinguished for having an orbit plane which, according to See, passes directly through the Solar System. If this is precisely true, the star must be an Algol-type variable with an eclipse about every thirteen years; but the event would be very brief, its precise time cannot be predicted, and it has never been observed.

The companion of **Algol** was considered to be perfectly opaque until the work of Stebbins with the selenium photometer in 1910 showed the existence of a minute secondary minimum in the light curve, and that the curve sloped slightly upward preceding the minimum and downward following it. From the spectroscopic elements, the form of the light curve, and a study of the rotational velocity (page 355) made in 1924 by McLaughlin, it appears that the brighter star has a radius of 3.12, a mass of 4.72, a density of 0.16, and a luminosity of 160, all in terms of the Sun; and that the companion, of radius 3.68, mass 0.95, and density 0.02, is brighter on the hemisphere which is turned toward the primary, that hemisphere having a luminosity of 17 and the other 10. The distance between the centers of the two stars is constant at 10,500,000 kilometers. The eclipsing pair revolves around an invisible third body in a period of 1.9 years. The distance of Algol from the Solar System is about 100 light-years.

The binary system of **Sirius,** which is of great interest, is discussed elsewhere (pages 227, 361).

EXERCISES

1. Note the position of Regulus on the star maps in this book. What is the approximate value of the annual part of the reduction of its radial velocity to the Sun on November 23?

Ans. + 30 km./sec.

2. At what sidereal time is the diurnal part of the reduction of the radial velocity of Regulus to the Sun a positive maximum?

Ans. 4 hours

3. Verify the cross motions and space motions of the first five stars in Table 20.

4. Using Map 1 and data from Table 20, show that α and β Centauri will form a striking naked-eye double about 4,000 years from now. (The proper motion of β is so small as to be negligible.)

5. Refer to Fig. 161 and state whether the orbital motion of Krüger 60 causes its position angle to increase or to decrease.

6. Is the orbit of the spectroscopic binary 5 Tauri (Fig. 164) circular?

7. Estimate the velocity of the center of mass of the system of 5 Tauri. (This is the ordinate of a horizontal line drawn across the velocity curve in such a position that equal areas lie between the line and the curve, above and below the line. The computed value of the velocity of the center of mass of 5 Tauri is + 14.2 km./sec.)

8. What is the period of 5 Tauri?

Ans. 960 days

CHAPTER XVI

PHYSICAL PROPERTIES OF THE STARS

Absolute Magnitude and Luminosity.—The apparent brightness of a star depends upon both its intrinsic brightness and its distance. If both the apparent magnitude and the distance are known, it is easy to calculate the intrinsic brightness. This is indicated by the **absolute magnitude,** which is defined as the apparent magnitude which the star would have if removed to a distance of 10 parsecs.

Let l_p be the intensity of light which we receive from a star whose parallax is p seconds, and l_t the intensity of the light which we should receive from the same star if it were situated at a distance of 10 parsecs—*i.e.*, if its parallax were $0''.1$. Since the intensity of a star's light is inversely proportional to the square of its distance, while its parallax is inversely proportional to the distance itself, we have

$$l_p/l_t = p^2 \, (0.1)^2 = 100p^2.$$

But the difference of magnitude of two stars (or of the same star at different distances) is equal to 2.5 times the logarithm of the ratio of their brightness—that is, representing by m the star's apparent magnitude and by M its absolute magnitude and substituting in the equation on page 316,

$$\begin{aligned} M - m &= 2.5 \log 100p^2 \\ &= 2.5 \, (\log 100 + 2 \log p) \\ &= 2.5 \, (2 + 2 \log p); \end{aligned}$$

or, very simply,

$$M = m + 5 + 5 \log p.$$

It is of great interest to compare the intrinsic brightness of the stars about us with that of the Sun. The **luminosity** of a star is defined as the ratio of the amount of light which we should receive from it to the amount which we should receive from the Sun if both star and Sun were removed to the same distance from us, say, ten parsecs. Representing the luminosity by L and the *absolute* magnitudes of Sun and star by S and M, respectively, the fundamental equation (page 316) gives

$$\log L = 0.4\,(S - M).$$

The absolute magnitude, S, of the Sun, is computed as follows: If x be the number of astronomic units in ten parsecs, the Sun would, if removed to that distance, be only $1/x^2$ as bright as now; hence, if s and S be the apparent and absolute magnitudes of the Sun,

$$S - s = 2.5 \log x^2.$$

The value of x is $63{,}000 \times 32.6$, and that of s (page 316) is -26.72. Computing by logarithms from these values, it is easily found that $S = +4.85$; hence, the luminosity of any star of absolute magnitude M is given by

$$\begin{aligned}\log L &= 0.4\,(4.85 - M)\\ &= 1.940 - 0.4M.\end{aligned}$$

For example, the parallax of Vega as given by Schlesinger is $0''.124$, and its apparent visual magnitude according to Harvard is 0.14. Hence, $\log p = 9.093 - 10$, and $M = 0.14 + 5 + 5(9.093 - 10) = 0.60$; and $\log L = 1.940 - (0.4 \times 0.60) = 1.70$; hence, $L = 50$. That is, the candle-power of Vega is fifty times that of the Sun, and if placed at the standard distance of ten parsecs Vega would have an apparent magnitude of 0.60.

The Range of Stellar Luminosities.—In Table 21 are listed a number of stars in the order of their luminosity.

The parallaxes are according to Schlesinger and are all trigonometric except that of Capella, which is derived from the spectrographic and interferometric orbital elements. The numbers following the ± signs in the column of parallaxes are the "probable errors," criteria of the uncertainty of the parallaxes. In computing the absolute magnitudes and luminosities for the last two columns, the parallaxes were used as given, although the last figure is generally very doubtful.

The (intrinsically) faintest known star is Wolf 359, which has only 1/50,000 of the Sun's candle-power; but if situated at a distance of a few hundred light-years this star would be invisible even in our largest telescopes, and there are, without doubt, many stars not yet recorded which are equally faint.

At the other extreme are a few stars, like Rigel and Canopus, which outshine the Sun tens of thousands of times. Still brighter is the ninth-magnitude star S Doradûs, in the large Magellanic cloud, which has, according to Shapley, a luminosity of 600,000, and even this was exceeded for a short time by the seventh-magnitude Nova that appeared in the year 1885 in the great spiral nebula in Andromeda (page 370). The

TABLE 21
INTRINSIC BRIGHTNESS OF STARS

Star	Spect.	Heliocentric Parallax	Dist. in Lt. Yr.	Apparent Vis. Mag.	Absolute Mag.	Luminosity
Wolf 359	M6e	0″.41	8.0	13.5	+16.6	.00002
Procyon's companion	......	.312 ± 6	10.4	13.0	+15.5	.00006
Proxima	M	.783 ± 5	4.2	10.5	+15.0	.00009
Barnard's Star	M5	.555 ± 5	5.9	9.67	+13.39	.00038
Groombridge 34	Ma	.282 ± 6	11.6	8.1	+10.4	.006
The Sun	G0	....	....	−26.72	+ 4.85	1.00000
Altair	A5	.204 ± 6	16.0	0.89	+ 2.44	9.2
Sirius	A0	.371 ± 4	8.8	−1.58	+ 1.27	27
Vega	A0	.124 ± 4	26.3	0.14	+ 0.60	50
Aldebaran	K5	.057 ± 5	57	1.06	− 0.16	101
Arcturus	K0	.080 ± 5	41	0.24	− 0.24	110
Capella	G0	.063	52	0.21	− 0.79	180
α Persei	F5	.015 ± 8	217	1.90	− 2.22	673
Polaris	F8	.007 ± 7	466	2.12	− 3.65	2500
β Draconis	G0	.004 ± 6	815	2.99	− 4.00	3500
Deneb	A2p	.005 ± 8	652	1.33	− 5.17	10200
Rigel	B8p	.006 ± 7	543	0.34	− 5.77	18000
Canopus	F0	.005 ± 10	652	−0.86	− 7.36	77000

ratio of S Doradûs to Wolf 359 is far greater than that of the most powerful searchlight to a candle, and it is emphatically true that "one star differeth from another star in glory."

The Sun is a modest member of the stellar host, exceeded in real brightness by a great majority of the visible stars. If removed to the distance of Aldebaran it would be invisible to the naked eye. Yet it is probable that a majority of all *existing* stars are fainter than the Sun, many being rendered invisible by their faintness and remoteness.

Spectral Criterion of Luminosity.—It was discovered by Adams and Kohlschütter at Mount Wilson in 1914 that the relative intensity of certain lines in star spectra of the F, G, K, and M types differed in the spectra of stars of different luminosity. By a study of these slight but unmistakable differences in the spectra of stars whose absolute magnitude could be determined from their measured parallaxes, curves were formed which correlated the relative intensity of the "magnitude lines" (certain lines of hydrogen, calcium, strontium, and iron) with the stars' absolute magnitude. These curves then give the absolute magnitude of any star whose spectrum can be distinctly photographed; and by reversing the formula on page 347 to read

$$\log p = \tfrac{1}{5}(M - m - 5),$$

it is possible to derive at once the star's parallax. **"Spectroscopic parallaxes"** thus determined have the great advantage over trigonometric parallaxes of being as reliable for distant stars as for near ones. The method has been applied extensively at Mount Wilson and Harvard and in Canada and England, and has more than doubled the number of stars whose distances are known with a reasonable degree of accuracy.

Giants and Dwarfs; the Main Sequence.—Between 1900 and 1910, as knowledge of the distances and luminosities of the stars increased, it became evident that many stars, especially the red ones, fell into two well-defined groups of widely different luminosity, to which Hertzsprung gave the names of **giants** and **dwarfs.** In 1913 the absolute magnitudes and spectral types of about 300 stars were known, and Russell assembled this information in a form now often referred to as a **Russell diagram,** plotting spectral type horizontally and luminosity vertically and representing each star by a dot. The result was an arrangement of dots having roughly the form of a reversed figure 7; the separation of dwarfs and giants was very conspicuous in the K and M types and noticeable in G and even in F, but the A and B stars all appeared to be giants. Figure 169 is such a diagram, compiled more recently by Hubble and containing about 2100 stars. To appreciate the great contrasts shown by the diagram, it must be noted that the vertical scale of luminosities is logarithmic and that stars near the top of the diagram have luminosities thousands of times that of the Sun, while stars at the bottom have luminosities of only 1/100 or less.

Though giants are relatively numerous in the diagram, it is probable that in space they are far outnumbered by the dwarfs, their seeming abundance being due to their visibility at great distances. The majority of all stars may be represented by dots lying along the diagonal from upper left to lower right in the Russell diagram, thus forming a continuous sequence from white giants to red dwarfs. This is called the **main sequence.**

White Dwarfs.—A very few stars are known which are both white and of low luminosity. The most famous of these is the companion of Sirius (pages 227, 361); others are Van Maanen's star, the companion of o^2 Eridani, and probably the companions of Procyon and Mira. These are all too low in luminosity to be shown in Fig. 169; their position is at the left and far below the main sequence. It is probable that white dwarfs are fairly abundant in space, though they are not so in star lists.

The Temperatures of the Stars.—The effective temperature of the radiating surface of a star, like that of the Sun's surface (page 191), may be determined by means of the laws of radiation; but of course, on account of the faintness of the stars, the observations are more difficult. The principal data of observation are in the form of color indices (page 319) and, for a few bright stars, heat indices (page 321) and spectral-energy curves (page 162). The temperature can also be estimated from the degree of ionization shown by the enhanced lines in the spectrum (page 167). The effective temperatures

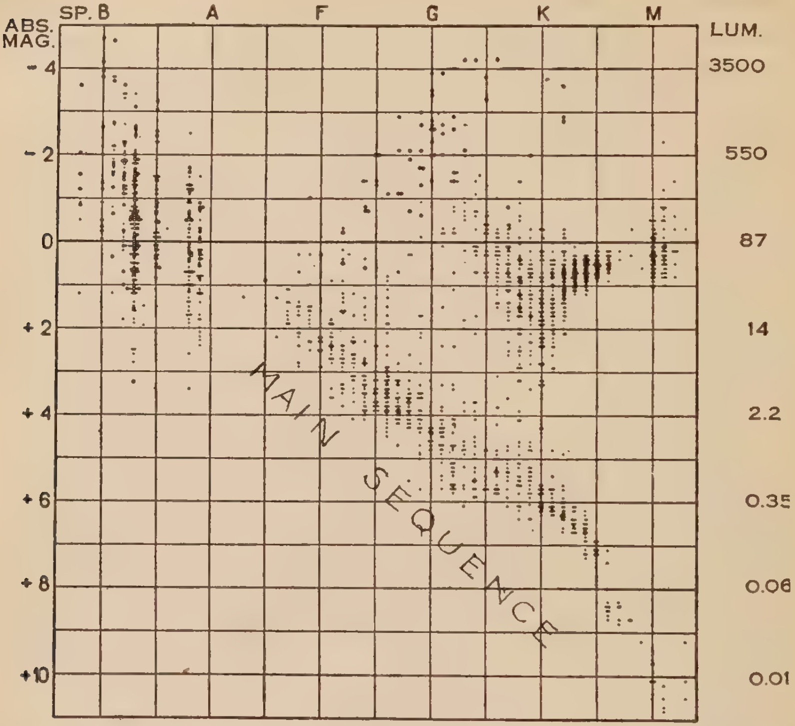

FIG. 169. CORRELATION OF ABSOLUTE MAGNITUDE AND SPECTRAL TYPE

of stars of the principal spectral types are found to be approximately as in Table 22.

TABLE 22[a]

Spectral type	Temperature	Spectral type	Temperature	Spectral type	Temperature
B	23000° K	G dwarf	6000° K	G giant	5600° K
A	11000	K dwarf	5100	K giant	4200
F	7400	M dwarf	3400	M giant	3200

[a] Abridged from Russell, Dugan and Stewart, *Astronomy*, p. 753.

Diameters of Stars Computed from Magnitude and Temperature.—The stars are so distant that, even in the largest telescopes, they appear as mere points, and it is quite impossible to measure their diameters with such an instrument as a micrometer, as is done with the planets. The diameters of many stars, however, have been determined by more or less indirect methods.

A star's rate of radiation in ergs per square centimeter per second may be computed from its effective temperature by Stefan's law (page 160). From its absolute magnitude and color index may be computed the total radiation sent out each second by its entire surface. The total surface of the star in square centimeters is then found by dividing the total radiation by the rate per square centimeter, and the radius is obtained by dividing further by 4π and taking the square root (since the surface of a sphere $= 4\pi r^2$). To obtain the diameter in terms of that of the Sun, this value must of course be doubled and divided by 1.4×10^{11}, the number of centimeters in the Sun's diameter. Since the color index depends on the temperature, the latter may be eliminated from the computation; and the different steps may be combined in a simple formula relating the star's diameter D to its color index I and absolute magnitude M. This formula is derived by Russell, Dugan and Stewart[1] in the form

$$\log D = 0.82I - 0.20M + 0.51.$$

Thus, for Sirius, whose color index is 0.0 and absolute magnitude 1.3, we have $\log D = 0.51 - 0.26 = 0.25$, whence $D = 1.8$ diameters of the Sun. At Sirius's distance of 8.8 light-years, this diameter subtends an angle of only $0''.006$, and there is no wonder that it is imperceptible to direct observation.

Direct Measurement of Star Diameters.—Beginning in 1920, the apparent diameters of a few stars have been directly measured at Mount Wilson with the **interferometer.** The method was described by Michelson as early as 1890 and had been suggested by Fizeau many years earlier. The observations have been made by Pease and Anderson, using a special interferometer in connection with the 100-inch Hooker telescope. A beam of structural steel, attached to the upper end of the tube of the telescope, carries four mirrors, the two outer of which can be moved along the beam so that their distance apart may vary up to twenty feet. The light of the star is reflected by these two mirrors to the two others which are near the middle of the beam and by them to the 100-inch paraboloid, after which it follows the usual course of light in the Cassegrain form of the telescope and finally reaches the eye of the observer (Fig. 170). The interference of the two beams of light thus coming from a point of the star produces, instead of the usual circular diffraction pattern (page 40), a set of exquisitely fine bright and dark **fringes** which appear somewhat like the pickets of a fence. If the star's apparent diameter is great enough, then for a certain separation of the mirrors the bright fringes corresponding to one limb of the star fall upon the dark fringes corresponding to the opposite limb, and the appearance becomes that of a continuous band of light. From the distance between the mirrors at which this disappearance of the fringes occurs, the angular diameter of the star may be accurately computed; and the real diameter in miles or kilometers may then be found with a degree of accuracy about proportional to that of the star's parallax. Until still larger interferometers are built,[1] this method can be successfully applied only to the largest and nearest stars, but enough has already been done to show that the diameters computed from color indices and absolute magnitudes are not seriously wrong.

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[1] *Astronomy*, p. 738.

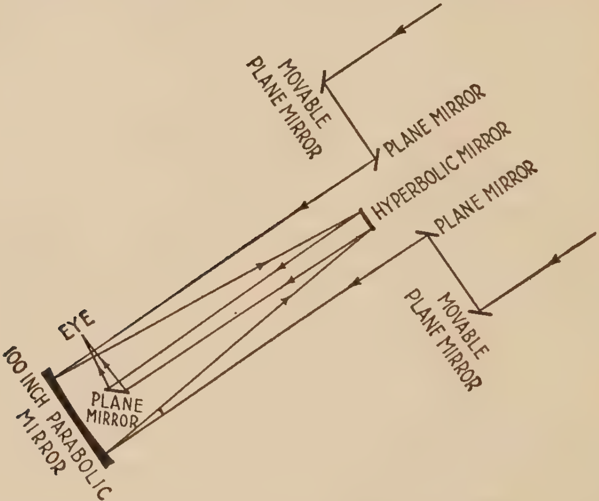

FIG. 170. THE 20-FOOT INTERFEROMETER OF THE MOUNT WILSON OBSERVATORY

The interferometer has been used also for the measurement of position angle and separation of close double stars in which the two components are of equal brightness.

Diameters of Eclipsing Binary Stars.—It has been noted (page 342) that the form of the light curve of an eclipsing binary makes known the diameters of its components in terms of their mean distance apart. Spectroscopic observations, if both spectra are visible, give the projection of this distance in kilometers as one of the elements of the relative orbit ($a \sin i$);

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[1] A fifty-foot interferometer is now (1930) nearing completion on Mount Wilson.

and as sin i (which is always nearly equal to unity in an eclipsing binary) is also known from the light curve, the scale of the system and the diameters of the stars in kilometers can then be readily computed.

Even if only the spectrum of the brighter star is observable, its diameter may sometimes be found by taking advantage of the spectroscopic effect of its rotation. It is safe to assume that both stars of an Algol pair rotate in the same direction and in the same period as they revolve, facing each other as the Moon faces the Earth—the tidal friction in so close a pair would inevitably bring about this result. Soon after the beginning of the principal eclipse, when the light curve is descending to minimum, we receive light mainly from the receding half of the bright star's disk, the other half being hidden by the companion; during the ascent of the light curve after mid-eclipse, we receive light mainly from the approaching half. Hence, the radial velocity departs from a smooth curve just before and just after the light minimum, and the amount of this departure makes known the rotational velocity of the bright star. From this velocity and the known period of rotation, the circumference of the star and hence its diameter are obtained in kilometers, and as the relative diameters of the two stars are known from the light curve, the diameter of the companion may also be found.

It will be noticed that the determination of the diameter of an eclipsing binary does not depend on a knowledge of the star's parallax or distance—an important advantage not possessed by either of the methods which are applicable to single stars.

The Range of Stellar Diameters.—The range in the size of stars, like the range in their luminosities, is enormous. The small red dwarfs like Krüger 60 and Proxima are about the size of the larger planets of the Solar System, while the white dwarfs are smaller still, comparable to the Earth. The red giants, such as Betelgeuse and Antares, have diameters hundreds of times that of the Sun and could readily contain the orbit of the Earth. (It should be borne in mind, however, that the terms giant and dwarf, as applied to stars, refer to luminosity and not necessarily to size.) Most stars of the main sequence have diameters comparable to that of the Sun—ranging from perhaps ten times the Sun's diameter for the average B-type star to one-tenth the Sun's diameter for an M-type dwarf. The diameters of a number of typical stars are illustrated in Figures 171 (giants) and 172 (dwarfs), with Vega included in both diagrams to show the relative scale.

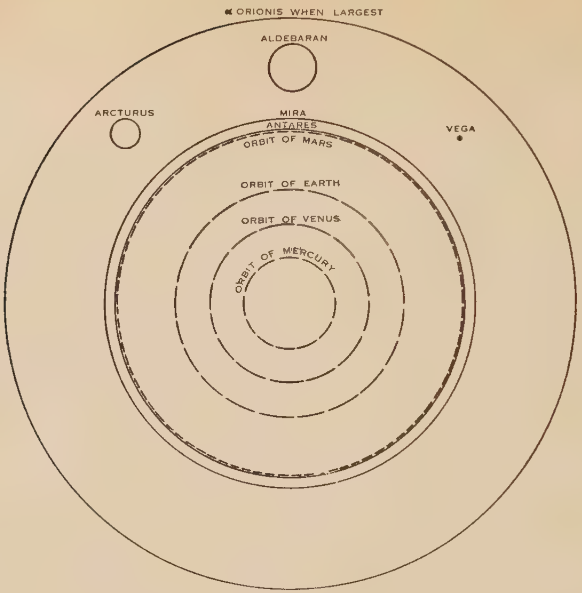
α ORIONIS WHEN LARGEST
ALDEBARAN
ARCTURUS
MIRA
ANTARES
VEGA
ORBIT OF MARS
ORBIT OF EARTH
ORBIT OF VENUS
ORBIT OF MERCURY

FIG. 171. DIMENSIONS OF STARS

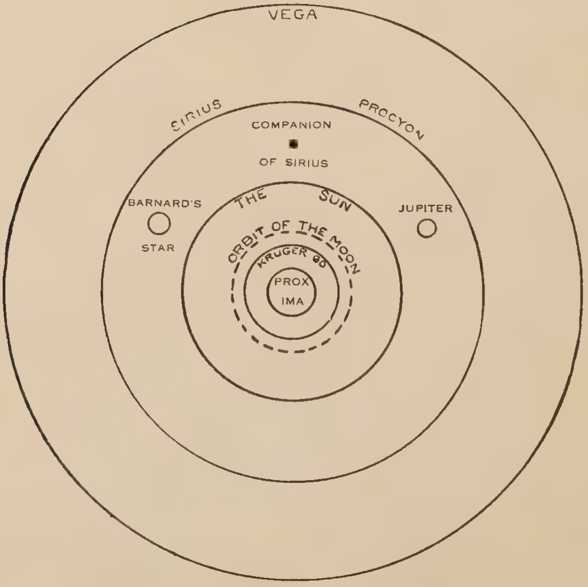
VEGA
SIRIUS
PROCYON
COMPANION OF SIRIUS
BARNARD'S STAR
JUPITER
THE SUN
ORBIT OF THE MOON
KRUGER 60
PROXIMA

FIG. 172. DIMENSIONS OF STARS

The Masses of Stars.—The mass of a star can be directly determined only if the star is a member of a binary system of which the parallax and orbit are known. We have (page 232)

$$m_1 + m_2 = a^3/P^2, \tag{1}$$

where m_1 and m_2 are the masses in terms of the Sun's mass, P the period in years, and a the semi-major axis in astronomical units. For visual binaries, the semi-major axis is determined in seconds of arc. Call this value a''. If p is the heliocentric parallax in seconds, the value of a in astronomical units is given by

$$a = a''/p, \tag{2}$$

whence the sum of the masses

$$m_1 + m_2 = a''^3/P^2p.^3 \tag{3}$$

The separate masses of the two stars can be found in only a few cases, such as that of Sirius (page 227), in which the motion of one or both stars with respect to the center of mass is known.

From observations of spectroscopic binaries, the mass *function* (page 340) is readily determined, and if the value of i is known also from visual or photometric observations the mass itself can be found. For estimating the average mass of a number of spectroscopic binaries, $\sin^3 i$ is assumed to have the value 0.59, which is the mean of all its values from $i = 0°$ to $i = 90°$.

There is no such wide range in stellar masses as we have noted in stellar luminosities and diameters. Certainly very few stars have masses as great as ten times, or as little as one-tenth, the Sun's mass. Stars even three times as massive as the Sun are rare, and Jeans estimates that not one star in 100,000 is ten times as massive as the Sun, although a very few stars of even greater mass than this are known. The star of smallest known mass (the faint component of Krüger 60) is one-fifth as massive as the Sun.

The Interior of a Star.—A star is a body of intensely heated gas. So long as its form and size do not change, its parts must be in equilibrium under the forces which act upon them. These forces are: (1) the mutual gravitation of its parts, the resultant of which is directed toward the center and tends to shrink the star; (2) "gas pressure," due to the motion and collision of the particles, which increases with the temperature and tends to make the star expand; and (3) the pressure of radiation (page 163), which also increases with the temperature and assists the gas pressure. At each point within the star, the two forces which depend on the temperature must together balance the weight of the overlying layers, and it is evident that the temperature and pressure must both increase toward the center. The interior of a gaseous star has been discussed from the standpoint of modern theoretical physics with rare mathematical skill by Eddington, who finds that main-sequence stars, including the Sun, have central temperatures of about 40,000,000° C. In red giants the central temperature is lower, but still to be expressed in millions of degrees, while in white dwarfs it is estimated by Eddington to be as high as 1,000,000,000°.

Referring to Figure 103 (page 162), which gives the wave-length and intensity of radiation at different temperatures according to Planck's formula, the reader will see that at temperatures of millions of degrees the development of energy is stupendous and that the radiation is mostly of very short wave-length (X-rays). The effects of this transcendent temperature as inferred by Eddington are:

(1) The atoms within the star are ionized to a high degree, being in extreme cases completely stripped of their orbital electrons.

(2) As a result the atoms, being much smaller than normal atoms, are packed more closely together and the density is enormous.

(3) The pressure of radiation, though a feeble force at the surface of the Sun or of any star, is enormous inside—17 tons per square inch at a temperature of 1,000,000° and 170,000 tons per square inch at 10,000,000°.

(4) The stellar material, consisting of partly or wholly ionized atoms and free electrons, is surprisingly opaque to the prevailing short-wave radiation, so that the latter leaks out

from the star but slowly, by a process of absorption and re-radiation in greater wave-lengths.

(5) In Eddington's own words,[1] the inside of a star is "a hurly-burly of atoms, electrons, and ether-waves. Dishevelled atoms tear along at 100 miles a second, their normal array of electrons being torn from them in the scrimmage. The lost electrons are speeding 100 times faster to find new resting places. Let us follow the progress of one of them. There is almost a collision as an electron approaches an atomic nucleus, but putting on speed it sweeps round in a sharp curve. . . . After a thousand narrow shaves, all happening in a thousand-millionth of a second, the hectic career is ended by a worse side-slip than usual. The electron is fairly caught, and attached to an atom. But scarcely has it taken its place, when an X-ray bursts into the atom. Sucking up the energy of the ray, the electron darts off again on its next adventure. . . . The stately drama of stellar evolution turns out to be more like the hair-breadth escapades of the films. The music of the spheres has almost a suggestion of—jazz."

Relation of Mass and Luminosity.—Eddington shows that the rate at which radiation escapes from a star is greater, the greater the ratio of radiation pressure to the total pressure inside the star; and that this ratio increases with the mass of the star. In other words, great mass is associated with high luminosity, and *vice versa*, notwithstanding the great disparity in the ranges of stellar masses and stellar luminosities. He has calculated the ratio of radiation pressure to total pressure for gaseous spheres of various masses, and finds that it is near zero for all masses up to 10^{32} grams, and near unity (*i.e.*, the radiation pressure vastly exceeds the gas pressure) for all masses over 10^{36} grams. The mass of most stars is intermediate between these numbers; that of the Sun is 10^{33} grams. Perhaps, therefore, a body of gas having less than a tenth of the Sun's mass would be unable to shine by its own light, while one of a thousand times the Sun's mass would be subject to so great an internal pressure of radiation that it would be liable to burst.

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[1] *Stars and Atoms*, page 26.

Table 23[1] compares masses observed with those computed from Eddington's mass-luminosity curve for stars of different luminosities, the latter being expressed in terms of absolute visual magnitude.

TABLE 23

	Mass			Mass			Mass	
Abs. Mag.	Computed	Observed	Abs. Mag.	Computed	Observed	Abs. Mag.	Computed	Observed
17.5	0.064	....	7.5	0.53	0.55	− 2.5	10	12
15.0	0.11	0.14	5.0	0.92	0.91	− 5.0	35	....
12.5	0.18	0.22	2.5	1.72	1.72	− 7.5	148	....
10.0	0.31	0.34	0.0	3.70	4.00	− 10.0	675	....

Dynamical Parallaxes.—The parallax of a binary star can be computed simply by equation (3), page 357, by solving for p thus:

$$p = \frac{a''}{\sqrt[3]{P^2 (m_1 + m_2)}}.$$

As a first approximation, $m_1 + m_2$ is assumed to be twice the Sun's mass, and the resulting value of p is used for computing the star's absolute magnitude by the relation (page 347)

$$M = m + 5 + 5 \log p.$$

Eddington's mass-luminosity relation then affords a more reliable value of the mass, which is used in the first equation as a second approximation, and the process is repeated until two successive approximations give the same result. Parallaxes determined in this way are called **dynamical parallaxes.** The method, in the hands of Russell and others, has proved a valuable means of determining the distance of stars which are too remote for direct trigonometric measurement.

The Densities of the Stars.—When the mass and size of a star are known, the average density of its material can be obtained immediately by dividing mass by volume. The densities of stars are found to have an amazingly wide range. A red giant such as Betelgeuse is a prodigious bubble of rarefied gas, as tenuous as the air in a fairly good artificial vacuum; a white dwarf such as the companion of Sirius is composed of material thousands of times heavier than lead, of which a pint, if placed at the surface of the Earth, would weigh more than twenty-five tons!

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[1] Abridged from Table XXXIII of Russell, Dugan and Stewart's *Astronomy*.

Observations by Adams have confirmed at the same time the incredibly great density of this particular star and Einstein's prediction of a gravitational shift in the spectral lines of a massive source of light (page 239). In the case of most stars, the Einstein effect is too slight to be easily measured and besides is inextricably combined with the Doppler-Fizeau effect of the star's motion. The motion of the companion of Sirius is accurately given by the orbit of the binary as determined from micrometer observations (page 336), and so the Doppler shift can be allowed for; while its Einstein displacement, due to the enormous value of its surface gravity, is more than 150 times that of the Sun. After allowing for the known Doppler shift, Adams finds a residual redward displacement of 0.32 Angstrom units, in almost perfect agreement with the Einstein displacement calculated by Eddington from the star's diameter of 24,000 miles and density of 53,000 × water. The spectrum is of A type.

Despite its unheard-of density the star must, for Eddington's calculations to be correct, be in the gaseous state—that is, composed of freely moving particles. Eddington's view is that these particles are neither molecules nor atoms in the ordinary sense, but free electrons and **stripped nuclei,** the ionization having proceeded to the extreme point where no revolving electrons are left to the atom. The particles are thus thousands of times smaller than ordinary atoms and so can be packed together thousands of times more closely.

Among stars of the main sequence the range in density is less, though still sufficiently astonishing—from about one-fiftieth that of the Sun (or 25 times that of ordinary air) for the great B-type stars to twice that of platinum for the smallest red dwarfs.

Description of Representative Stars.—In Table 24 are given statistics concerning a few stars which are representative of giants and dwarfs of various spectral classes. Additional facts concerning some of these stars, and some facts concerning other stars of particular interest, are given below.

The first six parallaxes in the table are "spectroscopic" (page 350); the others are trigonometric. The first six diameters were computed from the given parallaxes and the apparent diameters as measured with the interferometer; the others were computed from spectral type and absolute magnitude. The masses of α Centauri, Sirius and its companion, and Krüger 60 are derived from their orbital motion; the others are from Eddington's mass-luminosity relation.

Betelgeuse.—Though Betelgeuse is the twelfth star in the sky in order of visual brightness, it is first in order of radiometric magnitude. It sends to the Earth 7.7×10^{-10} calories per square centimeter per minute. (Compare this number with the solar constant, page 189.) The brightness of Betelgeuse is irregularly variable over a short range. The variation in diameter shown

TABLE 24

REPRESENTATIVE STARS

Name	Spect.	Eff. Temp.	Paral-lax	Diameter in miles	Diam., ⊙ = 1	Abs. Mag.	Luminosity	Mass, ⊙ = 1	Density, ⊙ = 1
Betelgeuse	M2	3100°	0″.011	280,000000 to 460,000000	320 to 540	− 3.8 to − 3.4	2,000 to 2,900	15	0.0000005 to 0.0000001
Mira[1]	M6	2800	0.017	300,000000	350	− 2.1 to + 5.8	0.4 to 600	9	0.0000002
Antares[2]	M1	3100	0.013	290,000000	330	− 3.2	1660	15	0.0000004
β Pegasi	M2	3100	0.014	140,000000	162	− 2.1 to − 1.6	380 to 600	9	0.0000002
Aldebaran	K5	3400	0.055	37,000000	43	− 0.2	105	4	0.00005
Arcturus	K0	4200	0.088	23,000000	27	− 0.1	96	4	0.0002
α Centauri[1]	G0	6000	0.758	1,200000	1.05	+ 4.73	1.12	1.08	1.06
The Sun	G0	6000	....	866500	1.00	+ 4.85	1.00	1.00	1.00
Sirius[1]	A0	11000	0.371	1,500000	1.7	+ 1.27	27	2.5	0.5
Vega	A0	11000	0.124	2,200000	2.5	+ 0.60	50	3	0.2
Rigel[1]	B8	12000	0.006	36,000000	42	− 5.77	18,000	40	0.0005
Krüger 60[1]	M3	3300	0.257	300000	0.34	11.2	0.003	0.26	7
Barnard's star	M5	3200	0.555	140000	0.16	13.4	0.0004	0.16	40
Sirius's companion	A5	7500	0.371	28000	0.034	11.2	0.003	0.96	27,000

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[1] Brighter star of a double.

[2] See page 363.

in the table was discovered by means of interferometer measures (apparent diameter 0″.034 to 0″.054) and implies a pulsation of the star.

Antares.—A great red giant similar to Betelgeuse. The parallax and diameter are doubtful; the parallax determined trigonometrically is twice the spectroscopic parallax which is given in the table and, if accepted, indicates a diameter only half that given. On the other hand, Russell and others consider this star a member of the Scorpius-Centaurus moving group and assign to it a parallax of only 0″.0095, which would give a diameter 480 times that of the Sun—greater than that of any other known star.

V Puppis.—A spectroscopic and eclipsing binary with well-determined orbital elements. Both components are of spectral type B1, with surface temperature (according to Plaskett) of 22,000°. The masses have the unusually high values of 18 and 19 times that of the Sun.

H.D. 1337 (Pearce's Star).—A spectroscopic and eclipsing binary, both stars of type B8, temperature 28,000°. These stars have the greatest accurately known masses—34 and 36 times the Sun's mass. They are ellipsoidal, longest diameters 31 and 48 times the Sun's, and they revolve in a period of 3.5 days with their noses only 350,000 miles apart.

B.D. 6°1309 (Plaskett's Star).—A spectroscopic binary in which the masses are *at least* 63 and 76 times that of the Sun, but as the inclination of the orbit is unknown the actual values of the masses cannot be determined.

Van Maanen's Star (page 328).—A white dwarf, effective temperature 7000°, of luminosity only 1/6000 that of the Sun. Its diameter is about that of the Earth. Its mass is unknown, but its density is believed to be even greater than that of Sirius.

Stellar Evolution.—From the foregoing discussion of the properties of the stars it is evident that their condition cannot remain constant forever. A star, like a human being, must have a life history. Though a stellar lifetime so far transcends a human lifetime and even human history that there is no hope of our detecting evolutionary changes in any one star, it is reasonable to suppose that, among the millions of stars around us, there are representatives of every stellar age and every stage of stellar development. The difficulty lies in recognizing these stages and arranging them in the right order. The subject is in far from a satisfactory state and all that will be attempted here is to outline very briefly the hypotheses of stellar evolution that have been held by astronomers in recent years.

At the end of the nineteenth century and beginning of the twentieth, the general view of astronomers was that a star evolves simply by losing heat, and that the order of stellar evolution is that of the spectral types O-B-A-F-G-K-M—

the hot, white stars being youngest and the red, cooler stars the oldest. The spectral series was at first based merely on the appearance of the spectra, but work in the physical laboratory showed that it could be interpreted in terms of gradually changing temperature and ionization. This view, which has been called the **single-line theory** of evolution, was supported by Campbell's discovery in 1910 (page 327) that the red stars were moving faster than the white; it was considered that their greater age had afforded time to acquire a higher velocity under the gravitational forces of other stars.

At the same time, a **two-branch theory** of evolution was urged by Lockyer, who at first was almost alone in supporting it. According to Lane's law and the Helmholtz contraction theory of the maintenance of radiation (page 192), a star must contract and grow hotter until its density becomes so great and the molecules so crowded that the material no longer behaves like a perfect gas; after which Lane's law no longer applies, and while the star slowly contracts still farther its temperature rapidly falls. A star would begin to shine as a cool, tenuous body of gas, grow denser and hotter until it reached a maximum temperature, and then grow still denser but cooler until it became liquid or even solid and ceased to shine. Lockyer's order of stellar evolution was thus that of the spectral types M-K-G-F-A-B-A-F-G-K-M.[1] This view was adopted and strongly advocated by Russell upon the discovery of giant and dwarf stars, a discovery which showed that while the hot stars were all much alike (giants), the low-temperature stars were divided sharply into two classes (giants and dwarfs) of very different density.

It is now known that the Helmholtz contraction theory is inadequate to account for the radiation of the stars, and even the transmutation of hydrogen into more complex elements could not provide for the degree of longevity contemplated by astronomers.[2] Of known or suggested agencies for maintaining stellar radiation, there remains only the coalescence of protons and electrons (page 193)—the stars are burning themselves out, not by the puny process of combustion but by actual annihilation of their substance. Moreover, as Eddington pointed out in 1924, the ions and electrons inside a star are so much smaller than the normal atoms that the material may be compressed to densities thousands of times that of familiar terrestrial solids without losing its properties of a perfect gas. These considerations, together with Eddington's mass-luminosity relation (page 359), point to an evolution depending on decreasing mass instead of increasing density, and both the single-line and two-branch hypotheses must be acknowledged incorrect or inadequate.

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[1] A small star might, on this hypothesis, pass through its maximum temperature at class A or F or even G without reaching the hottest types at all.

[2] From several lines of evidence, Jeans finds that the present age of the stars is from five to ten millions of millions of years—several thousand times the age of the Earth (page 299).

A remarkable fact which a theory of stellar evolution should explain is that the central temperature of all main-sequence stars appears to be about 40,000,000°, regardless of their position at the top, middle, or bottom of the sequence, while the central temperatures of red giants are lower and those of white dwarfs much higher. Various theories of stellar evolution taking this into account have recently been proposed by Russell, Eddington, and Jeans, which assume the existence within the stars of some form of "active matter" otherwise unknown and subject to being transformed into radiation at certain definite temperatures; but the theories are admitted by their authors to be tentative and likely to be altered in a few years, and it does not seem profitable to undertake their discussion here.

According to any theory of stellar evolution so far proposed, the stars are mortal. After many millions of millions of years, all the matter in the stars which is capable of transformation into energy will be so transformed, and there will be nothing left in the universe but inert material and impalpable radiation —the cold, invisible bodies of stars still pursuing their paths, and light quanta traveling forever at a velocity of 186,000 miles a second.

Cosmic Rays.—Since 1902, it has been known that our air is being traversed by a form of radiation capable of discharging an electroscope and having a far greater penetrating power than X-rays or even γ-rays. While X-rays may pass through a few millimeters of lead and γ-rays through a few inches, this radiation is capable of penetrating a lead block sixteen feet thick. Electroscopes sent up in sounding-balloons show that it is stronger in the upper part of the atmosphere, proving that it comes from outer space, and Millikan finds that its intensity is independent of the solar or sidereal time and of the latitude of the observer, and that therefore it does not come from the Sun or from any particular region among the stars. Millikan calls this radiation **cosmic rays.** Its extremely short wave-length (about 0.00001 Angstrom) shows that in its production an extraordinary amount of energy is liberated. The cosmic rays cannot come from the hot interiors of the stars—that would require even greater penetrating power than theirs. Millikan has made the bold suggestion that they originate in the depths of space by the transformation of radiation back into atoms—the reverse of the process which is believed to go on in the stars. If he is right, there is here an escape from the "heat-death" to which the universe seems otherwise to be condemned.

EXERCISES

1. Verify the absolute magnitudes and luminosities of the first five stars in Table 21 (use the table of logarithms on page 323).

2. Prove that a star's apparent diameter in seconds of arc equals the product of its parallax and its real diameter in astronomical units.

3. Find the apparent diameter of the companion of Sirius.

Ans. 0″.00011

4. Assuming the value of radiation pressure given under (3), page 358, for a temperature of 1,000,000,° verify that given for 10,000,000°.

CHAPTER XVII

VARIABLE STARS

Discovery and Designation of Variable Stars.—A star whose brightness is known to change is called a **variable star.** The study of these stars, though an important branch of astronomy today, is of comparatively recent origin. The ancients regarded the "fixed" stars as symbols of immutability, and seem never to have suspected that any of the known stars might vary in brilliancy. There are in ancient chronicles a number of allusions to such phenomena as the "new star" which was observed for a while between β and ρ Scorpii in 134 B.C., and which is said to have prompted Hipparchus to make the first star catalogue in order to leave a record of the appearance of the heavens in his time for the use of later astronomers in detecting other changes. These allusions, however, are so vague that it is generally uncertain whether they refer to variable stars of the class now called Novæ, or to comets, or to bright meteors.

The first recorded discovery of a recurrently variable star occurred in A. D. 1596, when the Dutch astronomer Fabricius noticed in Cetus a third-magnitude star which had not previously been recorded. It faded in a few weeks and was thought to be a Nova until 1638, when Holwarda, another Dutchman, again observed it and found that after disappearing again it reappeared eleven months later. The star was then found to have been visible also in 1603, when Bayer had mapped it as o (Omicron) Ceti, evidently without suspicion of its identity with the star of Fabricius. The astonishment with which the phenomenon was then regarded is evidenced by the name Mira (wonderful) which was given to o Ceti by Hevelius of Dantzig, and by which it is still known.

The Arabs may have noticed the variability of β Persei, for the name Algol (*al Ghul*, the demon) is somewhat out of keeping with the more complimentary names which they applied to many stars; but there is no other evidence that they did so. Montanari seems to have been the first, in 1669, to announce its variability in Europe, but no careful study was made of Algol until that of Goodricke in 1782. Goodricke also discovered the variability of β Lyræ and δ Cephei.

The discovery of other variable stars at first proceeded slowly; in Argelander's list of 1844 there were but 18 entries, and in Chandler's catalogue of 1888 there were only 225; but since the latter date the number of known variables has increased, chiefly through the application of photography to the study of the stars, to several thousands.

Variable stars not already lettered are designated by capital letters of the Roman alphabet beginning with R, followed by the genitive of the constellation name. They are lettered in the order of their discovery, and after nine variables in a single constellation have thus been designated the letters are repeated in pairs, as RR, RS, etc. Many faint variables, especially in star clusters or nebulæ, have received no designation.

Classification of Variable Stars.—While stellar variability is of every degree and kind and does not fall readily into separate classes, it is convenient to discuss it under the following heads:

1. Algol-type variables or eclipsing binaries
2. Novæ
3. Irregular variables
4. Long-period variables
5. Short-period variables or Cepheids

A star's variation in brightness is best described by the dimensions and form of its **light-curve,** in which are correlated its brightness, expressed by magnitudes as ordinates, and the time from a chosen epoch or phase as abscissae.

Algol-type Variables.—It has been pointedly remarked that variable stars of the Algol type are the only ones for which we have a reliable explanation, and that in reality they are not variable at all; they are binary stars, and it is only because of the location of the Solar System in their planes of motion that their light seems to vary. These stars were discussed in Chapter XV (pages 341 *et seq*).

Novæ.—A "new" star, temporary star, or **nova** is a variable star whose light-curve rises very steeply to a single brilliant maximum and then descends more slowly, usually with many secondary maxima and minima at irregular intervals (Fig. 173). The customary designation for such a star is the word Nova followed by the genitive of the constellation name and the year of principal maximum; for example, Nova Persei 1901.

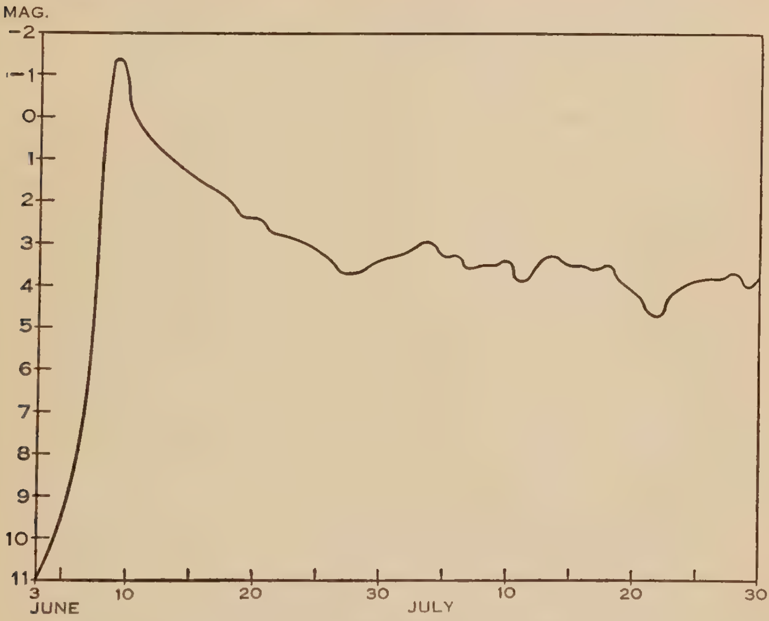


Fig. 173. Light-curve of Nova Aquilæ 1918

It was formerly believed that novæ were new creations which, after a brief lifetime, went out of existence; but such a view is wholly at variance with what we know of other stars and of the material universe generally, and besides a number of recent novæ have been identified, on photographs and otherwise, with previously known stars and are still observed as faint stars many years after their outburst.

Up to A.D. 1670, thirteen novæ were recorded, all of which were in or near the Milky Way. No more were discovered until 1848, but since that year, and especially since the beginnings of astronomic photography, discoveries of novæ have been frequent. Altogether, about fifty novæ have been observed near the Milky Way, most of which were bright enough at maximum to be seen by the unaided eye.

Undoubtedly many fainter novæ have escaped detection. From the number found on Harvard photographs, Bailey has estimated that an average of ten or more reach a maximum of the ninth magnitude or brighter each year. At that rate, in the course of a few hundred million years the total number of old novæ in the sky would equal the present number of visible stars, and it is possible that most stars experience such an adventure at some time during their careers.

In addition to all these galactic novæ, about a hundred have been found in the great nebula of Andromeda (Messier 31), and a few in other spiral nebulæ. Only one nova—Nova Coronæ Borealis 1866—is known to have appeared in any part of the sky except in or near the Milky Way or Magellanic clouds or in the spiral nebulæ. With one exception, nebular novæ have all been very faint—mostly about the eighteenth magnitude—and have been discovered photographically with large reflecting telescopes. It is probable, however, that their actual luminosity is of the same order as that of galactic novæ, their apparent faintness being due to their enormous distance. The exception was Nova Andromedæ 1885, which appeared about a quarter of a minute of arc from the nucleus of Messier 31, attained a magnitude of 7.5, and disappeared even to the largest telescopes in less than a year.

The most famous novæ of the past were Nova Cassiopeiæ 1572, known as the star of Tycho Brahe, and Nova Ophiuchi 1604, called Kepler's star. When Tycho first saw the nova of 1572 it was already brighter than Jupiter. and it grew brighter still during the next few days, so that it rivaled Venus and was visible in broad daylight; it then faded slowly, and disappeared in about eighteen months. Tycho's determination of its position was not sufficiently accurate to make its identification at the present day certain; it may be either of two stars of about the twelfth magnitude which are near the place given by Tycho, or possibly it has faded to a point below the reach of even modern instruments. Kepler's star became brighter than jupiter faded from naked-eye vision in a little more than a year, and left no identifiable trace.

The twentieth century has so far witnessed the outburst of a number of faint galactic novæ and of five bright ones, from which, with the great telescopes and spectrographs of today, much information about these unusual stars has been gathered. The five bright novæ are: Nova Persei 1901, maximum magnitude 0.0, discovered by Anderson of Edinburgh; Nova Geminorum 1912, maximum 3.4, discovered by Enebo in Norway; Nova Aquilæ 1918, maximum −1.0, discovered independently by many observers, principally in America; Nova Cygni 1920, maximum 1.8, discovered by Denning in England; and Nova Pictoris 1925, maximum 1.1, discovered by Watson in South Africa.

The Spectra of Novæ.—The spectra of novæ undergo rapid changes which, in view of the stability of most stellar spectra, are most astounding. If observed during its rapid rise in brilliancy, the typical nova exhibits a spectrum similar to that of a star of B or A type, except that the dark lines are greatly displaced toward the violet, as if by a velocity of approach of several hundred kilometers per second. This displacement finds a reasonable explanation on the hypothesis of a shell of gas expanding around the star with explosive velocity. The intensity of the lines increases with the brightness of the star, and at maximum the spectrum is likely to be of a modified A type having very broad absorption lines. As the star starts on the downward portion of its light-curve, its color changes from white to yellow, and *bright* lines, particularly of hydrogen and ionized iron, appear in about their normal positions, on the redward side of the absorption lines. These bright lines soon broaden enormously into wide bands of complicated structure. A few days later, a new set of dark lines appears, even more displaced toward the violet than the first set (in the case of Nova Aquilæ 1918, the displacement was such as to indicate a velocity of −1800 km./sec.), and then the bright lines increase further in intensity, while the continuous background fades. The color of the star then rapidly changes to a deep red, due to the fading of the continuous spectrum and to the predominance of the bright Hα. After a few weeks the bright lines of the nebular spectrum appear and the former spectrum gradually vanishes, leaving the spectrum like that of a gaseous nebula except that the lines are broad. The star then assumes a green color due to the intense nebular line at

5007 Angstroms. Later still, as the star subsides to inconspicuousness, the continuous spectrum reappears, and with it emerge the bands characteristic of the O-type or Wolf-Rayet stars. In this final stage the color of the star is again white.

Nebular Phenomena of Novæ.—Not only do novæ show affinity with nebulæ by their spectra, but in some cases nebulæ have developed around them which have been seen and photographed. The most striking case is that of Nova Persei 1901, which, for a few days in February of that year, was the brightest star in the northern heavens. In August, when the light of the nova had diminished to less than one per cent of its maximum, Wolf of Heidelberg discovered by photography a faint shell of nebulosity centered upon the star. Successive photographs of this shell, obtained by Ritchey at Yerkes and Perrine at Lick during the following autumn and winter, showed that it was expanding at the prodigious rate of between two and three seconds of arc a day (Plate 41). The parallax of the nova proved to be about 0″.01, corresponding to a distance of about 300 light-years, and so the outburst which was observed in 1901 actually occurred about the year 1600, before the landing of the Pilgrims. At that distance, the observed rate of expansion of the nebulous shell indicated an actual velocity equal to that of light, and it became evident, as Seeliger pointed out, that Ritchey and Perrine were witnessing the successive illumination of more and more distant parts of a nebula already existing, as the light of the star traveled farther and farther out.

Fifteen years later, long after the great light-echo had spent itself, Barnard discovered visually at the Yerkes Observatory a much smaller nebulous envelope, which also spread outward but much more slowly, the rate being about ½″ a year. The expansion of this little nebula is attributed to an actual motion of the gas at a velocity of some hundreds of kilometers a second. The nova at the time of Barnard's discovery had descended to the thirteenth magnitude, having lost all but about 1/160,000 of its maximum light.

A similar development of small planetary nebulæ was observed to take place around Nova Aquilæ 1918 and Nova Cygni 1920.

In the case of the former, the images formed by the nebular lines in the slitless spectrograph used by Moore and Shane at the Lick Observatory grew about twice as rapidly as the $H\beta$ image, indicating a more rapid expansion of "nebulium" than of hydrogen, and the lines corresponding to different parts of the little nebula were so displaced as to indicate an intricate internal motion. In 1926–27 the nebula was still visible though very faint on photographs made with the 100-inch telescope at Mount Wilson, and had attained an apparent diameter of about 18″. The angular rate of expansion combined with the radial velocity indicate a distance of the star of about 1200 light-years.

Conjectures as to the Cause of Novæ.—There can be no doubt that the sudden enormous increase in the brightness of a nova is due to some tremendous cataclysm, of a magnitude far exceeding that of any other which man has ever observed. For example, Nova Aquilæ 1918 rose in four days from magnitude 10.5 to magnitude −1.0—a 40,000-fold increase in luminosity. So far, no really satisfactory theory as to the nature of the cataclysm has been proposed. Among the principal conjectures are the following:

1. A grazing collision of two stars. This theory was propounded as early as 1878 by Bickerton of New Zealand, who believed the nova to be a third body formed of matter torn off the two stars, rapidly expanding due to the enormous evolution of heat. He predicted successfully many of the features since observed in the spectra of novæ, but his theory has met with little acceptance because of the vastness of the distances between the stars as compared to their diameters, which should render collisions very rare indeed, while novæ are rather frequent.

2. A close approach of two stars, resulting in vast tidal upheavals, as in the origin of the Solar System pictured in the Planetesimal Hypothesis. This occurrence, though a little less improbable than an actual collision, still would happen very infrequently and also would be less effective.

3. The passage of a star through a nebula, the temperature of the star being raised as that of a meteor is raised by passing

through the air. While this hypothesis receives support from the dark nebula which evidently had existed around Nova Persei previously to its flaring up and which was illuminated by the star, it is difficult to conceive of so much energy being liberated by the star's passage through a medium of such extreme tenuity as the diffuse nebulæ are known to be. Moreover, the hypothesis is ruled out by the brevity of the phenomenon and the enormous size of known nebulæ.

4. The sudden release of energy, probably of a sub-atomic origin, stored up within the star. This release might be brought about, as See and W. H. Pickering have suggested, by a body of planetary or smaller dimensions colliding with the star, and the probability of such a collision is much higher than that of a collision of two stars. Russell points out that such a body would strike the star with terrific velocity and, plunging deep beneath the photosphere, be stopped by the denser layers below. Its kinetic energy would thus be transformed into heat, and a pocket formed in which the temperature would be millions of degrees. Russell conjectures that this temperature might act as a trigger to release energy from an unknown source within the star, the heated gas would become still hotter and, expanding, would blow off the overlying layers of the star with catastrophic force. With the relief of pressure the temperature would drop again so that the brightness of the star would subside, while the expanding gas would form a nebula of increasing size.

Irregular Variables.—The most famous of irregularly variable stars is probably η Argûs (or η Carinæ), situated in the "keyhole" nebula in the southern circumpolar region (Plate 45). It was first noticed in 1677 by Halley, and from that year until 1825 it varied irregularly from the second to the fourth magnitude. In 1827 it rose to the first, but fell again to the second by 1830. In 1843 it reached magnitude -1.0, and so was the brightest star in the sky except Sirius; but during the following years its brightness greatly diminished, and since 1886 it has been about constant at magnitude 7.5. The spectrum is one of a very rare type, consisting almost wholly of bright lines in which hydrogen and ionized iron predominate. No other bright irregular variable is known which has a range of brightness approaching that of η Carinæ. α Herculis, α Orionis, and β Pegasi are all examples of irregular variables of small range. A number of variables which are sometimes classed with Mira have shorter periods which are much less regular. These stars have spectra which, although of the M type, contain no bright lines.

In the great nebula of Orion, and also in the similar diffuse nebula Messier 8 Sagittarii, there are many faint red stars which vary quite capriciously over a range of several magnitudes. It has been suggested that these are "friction variables," a part of their light being due to friction with portions of the nebula which vary in density; but in view of the probable extremely low density of all known nebulosity, a better explanation is perhaps that the light of these stars is obscured to varying degrees by passing wisps of dark nebula. Either explanation must be regarded as a mere guess, and probably not the right guess.

Stars of the Types of R Coronæ and SS Cygni.—Somewhat resembling the long-period variables, but much less regular, are a few stars which fall into two classes. Of one of these, R Coronæ Borealis is the model. It is ordinarily of the 5.5 magnitude, but is subject to sudden drops of several magnitudes, even down to the twelfth, whose date and duration are unpredictable. Stars of this type lie near the Milky Way.

The other class is represented by SS Cygni, which, usually of the eleventh magnitude, suddenly *rises* to the ninth or eighth. It often shows two alternating types of maximum, one of longer duration than the other, but occurring at irregular intervals.

The Periods of Variable Stars.—Figure 174 shows graphically the numbers of known variable stars (excluding eclipsing variables) having various periods. The scale of periods (abscissæ) is made logarithmic in order to give room on the diagram for both long and short periods. About 750 stars have periods which cluster around 300 days. These are called long-period variables. About 150 have periods of from three days to two months, and about 100 have periods less than a day. Stars of these last two groups differ greatly from the long-period variables but resemble each other, and so are classified together as short-period variables.

Long-period Variables.—The range of light variation of the long-period variables is from four to nine magnitudes. More than half of these stars have periods between 250 and 400 days. Their light-curves do not repeat themselves exactly, either in the range of brightness or in the interval between maxima. In most cases, the increase of brightness is more rapid than the decrease. All are red stars, mostly of spectral type M, while a few are of types N, R, and S. At and near the time of maximum light, bright lines, particularly those of hydrogen, are superposed upon their spectra. None are known to be connected with nebulæ except R Aquarii, in the spectrum of which Merrill found bright lines of "nebulium," and around which Lampland discovered a small nebula of peculiar form.

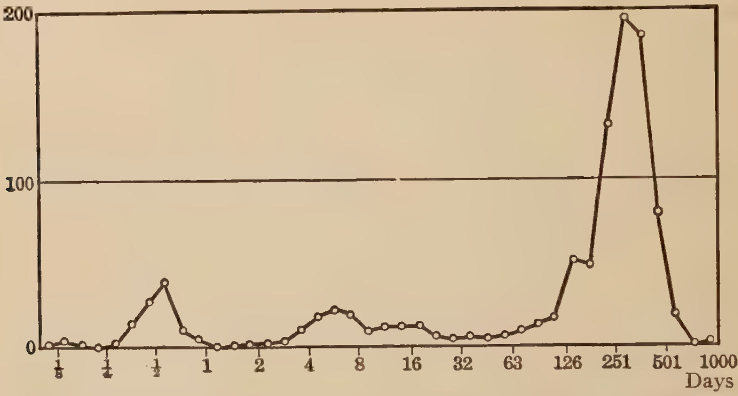


FIG. 174. DISTRIBUTION OF PERIODS OF VARIABLE STARS. (FROM RUSSELL DUGAN AND STEWART'S *Astronomy*, BY PERMISSION OF THE PUBLISHERS, GINN AND COMPANY)

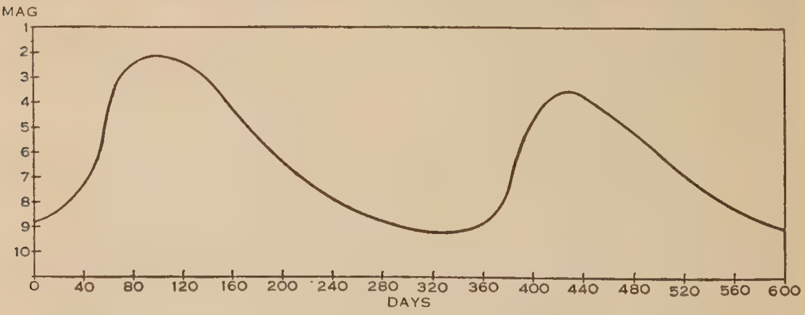


FIG. 175. LIGHT-CURVE OF MIRA

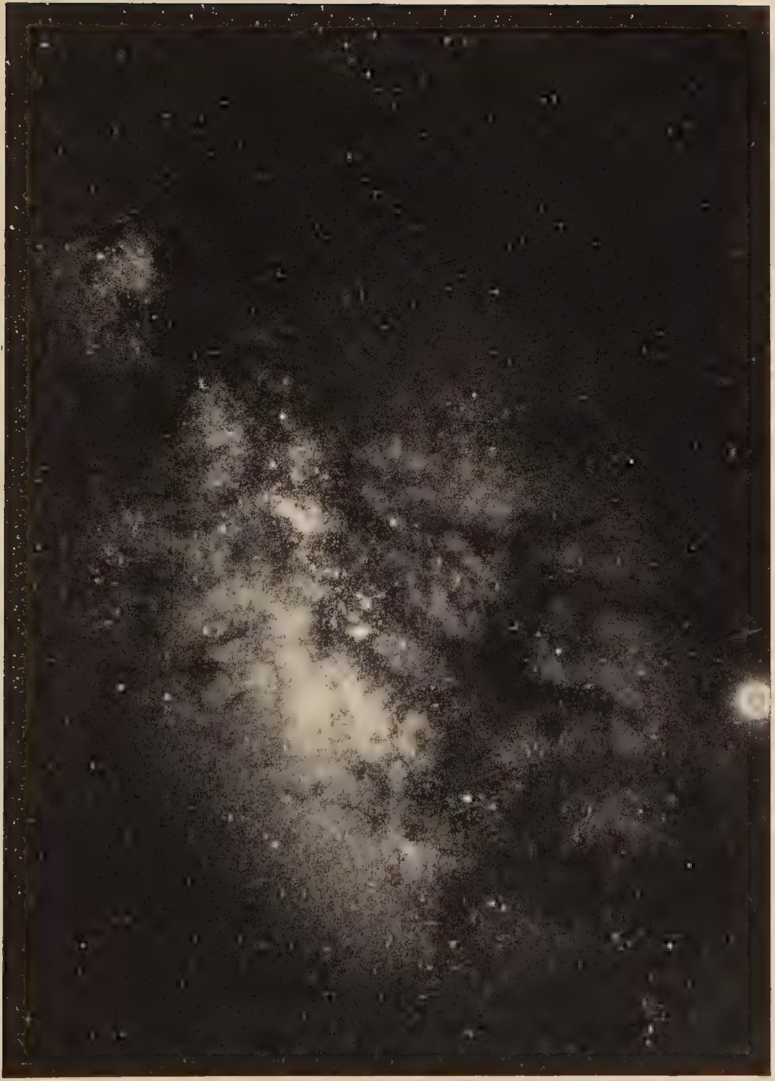


PLATE 42. THE MILKY WAY IN SAGITTARIUS, photographed by Duncan with 2.2-inch lens at Mount Wilson, 1922 July 21. Exposure 3 hours 45 minutes

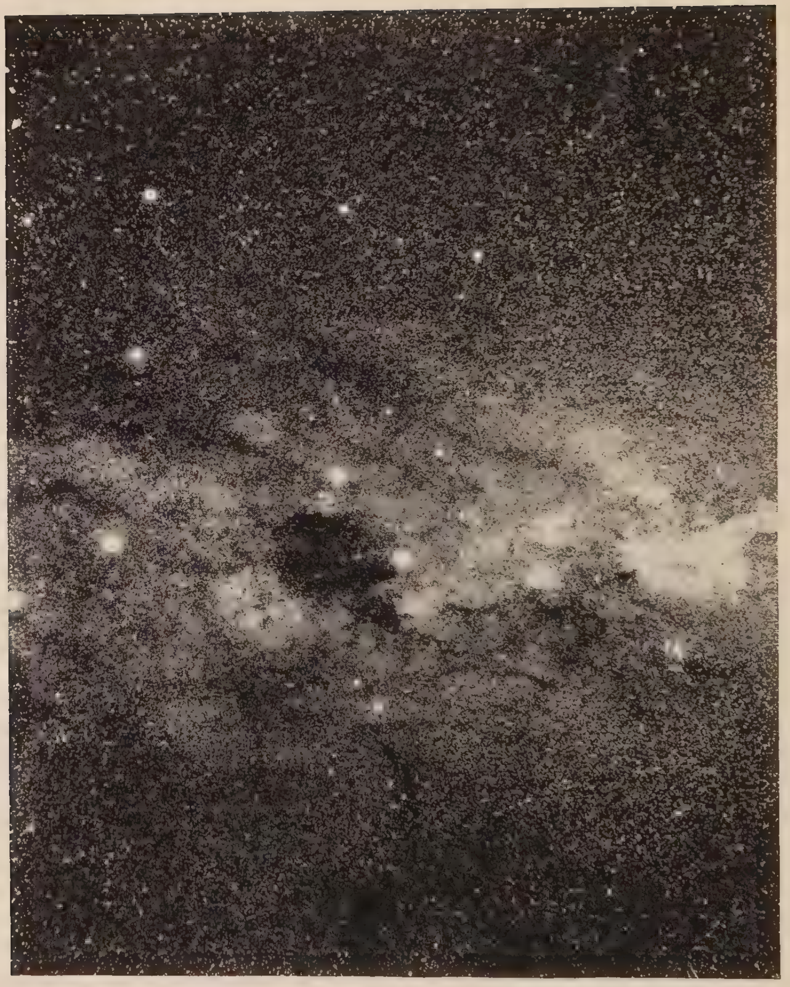


Plate 43. The Milky Way in the Region of the Southern Cross, photographed by Margaret Harwood at Arequipa with 1-inch lens, 1923 May 12–13–18. Exposure 11 hours 39 minutes. The Cross is upright near the center of the plate and below it is the Coal Sack. The Stars γ and δ Crucis appear relatively faint because of their yellow color. The bright stars near the left edge of the plate are α and β Centauri

Variable stars of the long-period class show no marked preference for the Milky Way.

The type star of the class is Mira Ceti, which has probably been studied more fully than any other star. Its period averages 330 days, but varies from this at times by as much as 15 days either way. At maximum light, the star is sometimes as bright as magnitude 1.5 and sometimes as faint as 5.6, but usually is about 3.5; while at minimum it is usually about 9.2, but has been as faint as 10.0 and sometimes has sunk only to 8.0. Fig. 175 shows the light curve during two periods.

The spectrum of Mira varies from M6 at maximum to M9 at minimum. According to a recent study by Joy at Mount Wilson, no emission lines are present at minimum, but bright hydrogen lines appear soon after minimum and increase in intensity until about maximum. Low-temperature emission lines of iron, magnesium, and silicon appear some time after maximum. The dark bands of titanium oxide are especially strong at minimum.

The effective temperature of the star varies from about 1,500° C. at minimum light to about 2,000° C. at maximum. Measures o the total energy made with the thermocouple (page 320) by Nicholson and Pettit indicate a "radiometric" brightness about eight magnitudes higher than the visual and a variation in total radiation of only about one magnitude. The great difference between the visual and the radiometric range is easily explained by the displacement with changing temperature of the point of maximum energy in the spectrum according to Wien's law (page 161); at minimum, the greater part of the energy is contained in the infra-red radiation, but at maximum the peak of the energy curve is shifted into the visible spectrum, and the visual light increases more rapidly than the total radiation. This effect is accentuated by the intensification of the dark titanium oxide bands at minimum, since they lie entirely in the visible part of the spectrum.

The radial velocity indicated by the absorption lines varies through a range of about 12 km./sec. in the period of the light-variation. The emission lines, on the other hand, show a variation of 19 km./sec. At minimum light, their velocity is the same as that of the absorption lines, and at all other times they show a relative motion toward the observer.

Interferometer measures by Pease show that the star is of enormous size, its diameter being about 500,000,000 kilometers. The variation of radial velocity cannot be due to orbital motion, for the semi-axis of the orbit computed on this interpretation is only 26,000,000 kilometers, about 11 per cent of the radius; if the velocity curve is due to the influence of a companion, the companion must be inclosed far within the visible star. The most natural explanation of the oscillation of the spectral lines is that the star *pulsates*, dilating and contracting in the period of its light-changes. The shell of gas which gives rise to the emission lines evidently pulsates through a wider range than the lower-lying reversing layer.

Although not a spectroscopic binary, Mira has a visible companion whose existence was inferred by Joy in 1922 from peculiarities in the bright-line spectrum. At Joy's suggestion, the companion was sought for visually by Aitken with the Lick refractor, and easily found by that skillful observer at a distance of 0".9, although it had eluded previous scrutiny with large telescopes by several observers, including its discoverer. It is a white star of about the tenth magnitude, but appears to be slowly growing fainter. Its spectrum exhibits the bright lines of hydrogen, helium, ionized iron, and calcium, but no dark lines except those of hydrogen.

Like the novæ, the long-period variables are awaiting an adequate explanation. It is certainly to be expected that a pulsating star would vary in brightness; but it should be hottest and brightest when smallest, and this is not found to be the case with Mira, which is brightest when at about the middle of the contracting phase. An older suggestion postulates a variation of internal activity similar to that which produces the eleven-year cycle in the Sun (whatever the cause of that may be), but the Sun's variation is of so much longer period and so much smaller amplitude than that of the red stars that it would seem to be a different kind of phenomenon.

Cepheids, or Variable Stars of Short Period.—Though the variable stars of short period are separated naturally into two groups (Fig. 174), they all resemble one another in many ways and are often called **Cepheids** from their type-star δ Cephei. When it is necessary to distinguish between the groups, those of periods less than a day are called **cluster variables** because, though found sparingly in all parts of the sky, they are very numerous in star clusters.

In contrast with variable stars of the long-period or Mira type, the Cepheids are punctual and regular in their variation, the period and form of light-curve changing but little; the range of variation is small, usually less than half a magnitude and seldom exceeding 1.5 magnitudes visually; and the spectral types are A, F, or G.

Most Cepheids have light curves which rise more rapidly than they fall. Every Cepheid which is bright enough to have been studied spectroscopically shows a variable radial velocity in a period identical with the period of the light curve. The variation, however, cannot be due to eclipses, for the maximum light invariably occurs near the time of maximum velocity of approach (Fig. 176), and minimum light at maximum velocity of recession, whereas in the Algol stars the minimum necessarily occurs on the rising branch of the velocity curve. The spectra of different Cepheids are related to the periods—stars with periods less than a day are of class A, those of about 4 days of class F, and those over 8 days of class G. The spectral type of a given Cepheid changes slightly with the brightness of the star, being of a cooler and redder type at minimum, so that the range of photographic magnitude is greater than that of the visual magnitude. Cepheids having periods greater than three days are found mostly in the Milky Way, and have small peculiar motions (averaging about 12 km./sec.). The cluster variables which are not in star clusters are scattered in all parts of the sky, and are moving much more rapidly than the galactic Cepheids. Stars of both groups are giants; those of

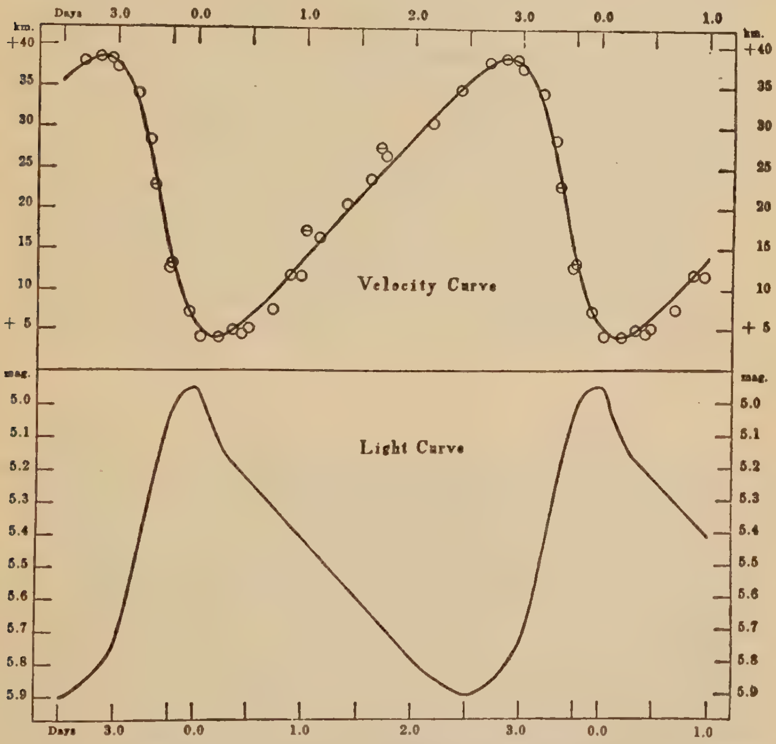


Fig. 176. Light- and Velocity-curves of the Cepheid RT Aurigæ (Duncan)

longer period are several hundred times, and the cluster variables scores of times, as luminous as the Sun.

The Period-Luminosity Relation in Cepheid Variable Stars.—The Cepheids possess a remarkable property which gives an indication of their distance, however great it may be. This property was discovered by Miss Leavitt at the Harvard Observatory in 1908 by a study of the small Magellanic cloud. She found many faint Cepheids in this cloud, and upon determining their periods of variation in brightness, found that *the longer the period the greater was the apparent brightness of the star.* Since all the stars in the cloud are at practically the same distance from the Earth, their apparent magnitude must differ by a constant from their absolute magnitude which therefore is also correlated with their periods. Bailey of Harvard discovered in several of the globular star-clusters many Cepheid variables displaying the period-luminosity relationship, and in 1917 Shapley pointed out that, assuming the relationship to hold for all Cepheids, if the distance and absolute magnitude of one or more Cepheids could be determined by independent means, those of all the others and of the star-clusters of which they are members could be immediately obtained from their periods. He accordingly formed the curve shown in Fig. 177, eleven points of which he fixed with some precision from the absolute magnitudes of relatively near Cepheids. The curve gives at once the absolute magnitude of any Cepheid whose period is known by observation; and

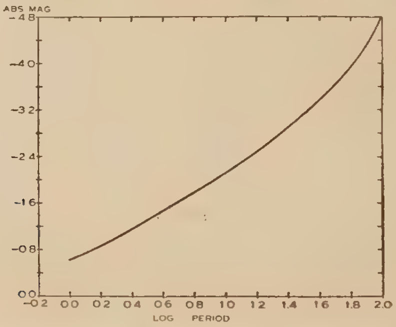
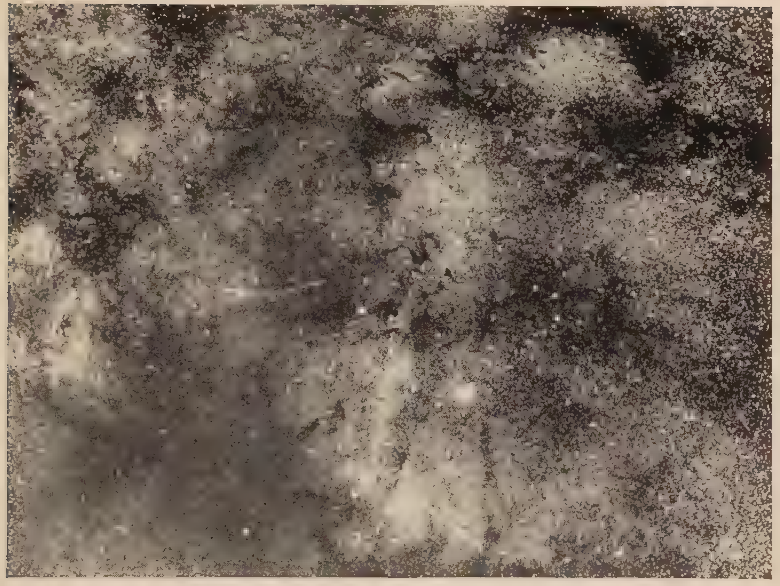
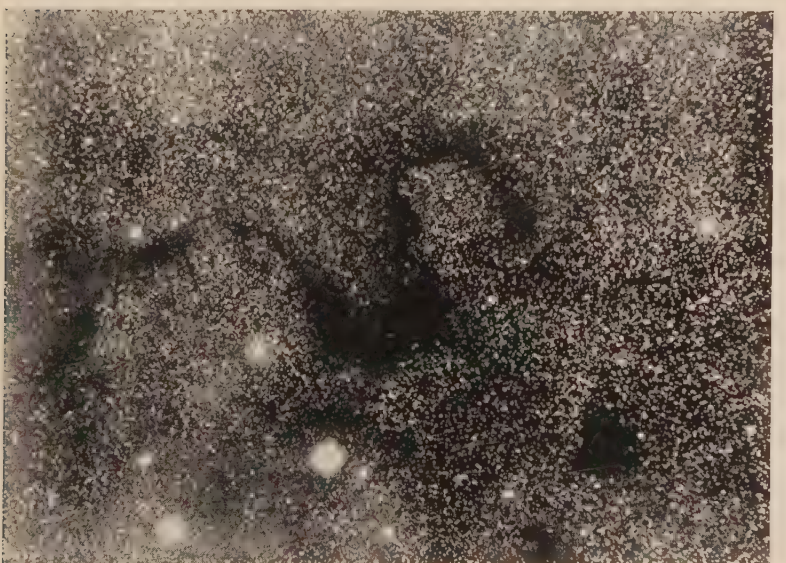


Fig. 177. Correlation of Period and Luminosity of Cepheids

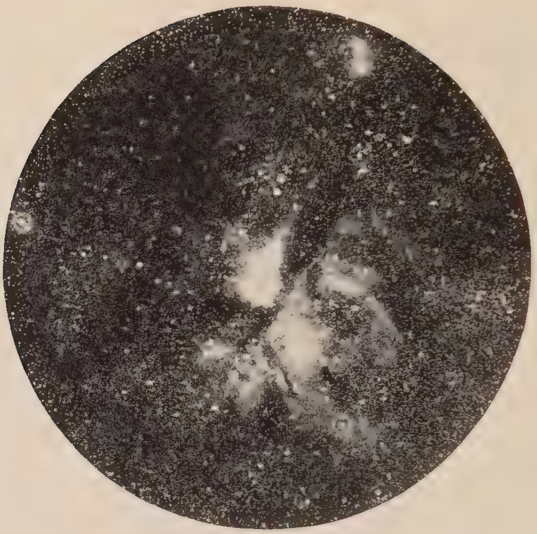
PLATE 44. THE MILKY WAY IN OPHIUCHUS



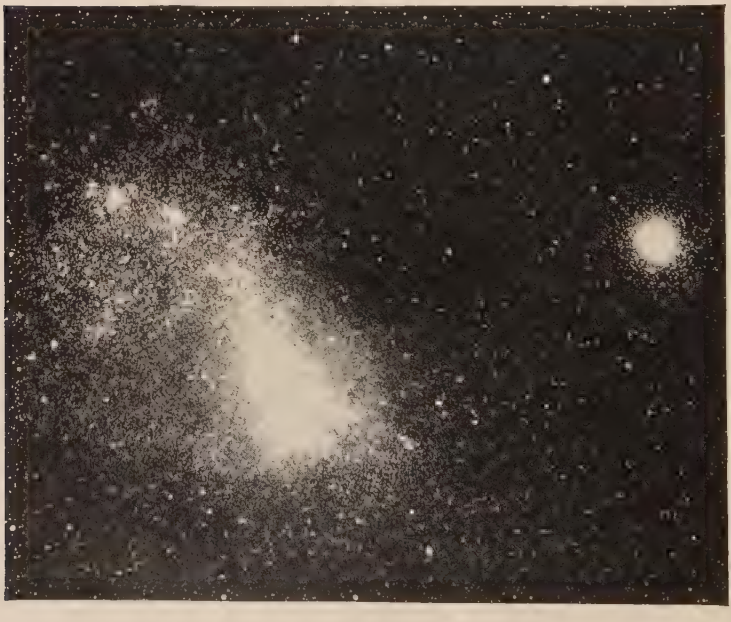
The Milky Way near θ Ophiuchi, photographed by Barnard with 10-inch lens at Yerkes Observatory



The dark S-shaped nebula near the centre of the upper plate, photographed by Duncan with 100-inch Hooker Reflector



The Keyhole Nebula in Carina, photographed by Bailey at Arequipa with 13-inch telescope, 1894 May 25–28. Exposure 13 hours 44 minutes



The Small Magellanic Cloud and the Globular Star Cluster 47 Tucanæ, photographed by Bailey with 24-inch telescope at Arequipa, 1898 November 10. Exposure 5 hours from this and the observed apparent magnitude the heliocentric parallax can be obtained by the equation (page 350)

$$\log p = \tfrac{1}{5}(M - m - 5).$$

From the value of p thus obtained, the distance of the star is of course known at once by the relations on page 312. There is little doubt of the correctness of the relative distances obtained by this method, although their absolute values may be considerably in error. Shapley finds in this way that the scattered Cepheids are situated at distances ranging from about 200 to about 20,000 light-years. The distances of the clusters come out much greater, and will be discussed in Chapter XVIII.

Theories of Cepheid Variation.—Two lines of explanation have been offered for the phenomena of Cepheid variation and, as in the case of other variable star theories, neither is satisfactory. They are referred to as the binary and the pulsation hypotheses. According to the first, a Cepheid is a close binary, surrounded by a tenuous envelope through which the two stars move in their orbits. The brighter star, which alone impresses our instruments, is either heated by contact with this medium more on its preceding hemisphere than on the following, or its own slightly opaque atmosphere is brushed back from the preceding side. In either case the star would be, as observed, brightest when at the lowest point of its velocity curve, due in the first case to greater emission and in the second to lesser absorption of its light. The binary hypothesis accounts for all the facts of observation, but fails in one respect: the lack of measurable parallax in any of the Cepheids and the spectroscopically determined great luminosities of many indicate that they are giant stars, while the orbits obtained on the hypothesis of their binary character are all so small that, if the bright star really is a giant in size, the companion must revolve inside it. Moreover, no Cepheids are known to exhibit periodically double lines, as some might be expected to do if they were binaries.

According to the pulsation hypothesis, each Cepheid is a single giant star, and the variation of its velocity is to be explained by a pulsation, the star expanding and contracting periodically as Mira appears to do (page 377). This explains the forms of the velocity curves fairly satisfactorily, fits in with the correlation of period and luminosity, and gives a reasonable explanation of the existence of a variation in light since a body of gas is cooler when in an expanded condition than when compressed; but the theory fails completely to account for the peculiar relation between light curve and velocity curve and is somewhat strained to account for the lack of symmetry in the light curve.

Jeans has recently overcome some of the difficulties of the pulsation hypothesis in a mathematical treatment of a pulsating star which is at the same time rotating, as most stars probably are, and in which the pulsations combined with the rotation are in the act—an act which is likely to be protracted some 100,000 years—of producing a fission of the star into two bodies which will ultimately develop by tidal action (page 244) into an ordinary binary.

EXERCISES

1. According to observations made by members of the American Association of Variable Star Observers, a maximum of Mira occurred on July 19, 1929 (Julian Day 2425812), and the mean period between maxima is 331.6 days. Predict the date of maximum for the present year.

2. Find Mira's place in the sky with the help of Maps 6 and 7. On every convenient night during several weeks before and after maximum, determine the magnitude of Mira by the method of Argelander (page 318). Note the star's color. Convenient comparison stars for naked-eye study of Mira are listed below:

Star	*Mag.*	*Star*	*Mag.*
α Arietis	2.2	γ Ceti	3.6
Deneb Kaitos	2.2	α Piscium	3.9
α Ceti	2.8	ξ Piscium	4.8

3. Identify β Lyræ by means of Map 2. Observe it by the method of Argelander on every available night for a month, and form a light-curve from your own observations. Convenient comparison stars for β Lyræ are:

Star	*Mag.*
γ Lyræ	3.3
μ Herculis	3.5
θ Herculis	4.0

4. The following dates and Greenwich times of minima of Algol were computed from observations by Stebbins (the fraction of the Julian Day is counted from Greenwich mean noon):

J. D. 2425980.2634 = 1930 January 3, 6:19 P.M.
6344.4106 = 1931 January 2, 9:51 P.M.
6708.5578 = 1932 January 2, 1:23 A.M.

J.D. 2427075.5724 = 1933 January 3, 1:44 A.M.
7439.7196 = 1934 January 2, 5:16 A.M.
7806.7341 = 1935 January 4, 5:37 A.M.
8170.8813 = 1936 January 3, 9:09 A.M.
8535.0286 = 1937 January 1, 0:41 P.M.
8902.0431 = 1938 January 3, 1:02 P.M.
9266.1903 = 1939 January 2, 4:34 P.M.
9630.3375 = 1940 January 1, 8:06 P.M.

The period determined by Stebbins is $2^{\text{d}}.867301 = 2^{\text{d}}\ 20^{\text{h}}\ 48^{\text{m}}\ 55^{\text{s}}$. From the above data, predict a number of minima for the present year.

5. Identify Algol among neighboring stars with the help of Maps 6 and 7. Determine its usual magnitude by the method of Argelander. On a night when the minimum occurs at a convenient time, watch the changes of the star's brightness by comparison with stars of the following list:

Star	*Mag.*
γ Andromedæ	2.2
β Andromedæ	2.4
ζ Persei	2.9
δ Persei	3.1
α Trianguli	3.6

CHAPTER XVIII

STAR CLUSTERS AND NEBULÆ

The Milky Way.—Almost imperceptible to city dwellers through the illumination of the sky by artificial light, in country places and at sea the Milky Way or **Galaxy** merits Milton's description of it as

> A broad and ample road whose dust is gold
> And pavement stars.

To the naked eye it appears as a band of soft, misty light encircling the sky; but even a small telescope, as Galileo discovered, shows it to be made up of myriads of stars. Larger telescopes, and especially photographs, show it to contain also many large diffuse nebulæ, both luminous and dark.

The central line of the Milky Way is almost exactly a great circle of the celestial sphere (the **galactic circle**) which crosses the celestial equator in right ascension $6^h\ 44^m$ and $18^h\ 44^m$ at an angle of about 62°, the galactic poles being situated in the constellations of Coma Berenices and Cetus. In the latitudes of the United States, at about 12^h of sidereal time the Galaxy lies so near the horizon that it is scarcely visible, while at 5^h and 21^h it passes near the zenith. It is most conspicuous in the evening during autumn.

The Galaxy reaches its greatest north declination in Cassiopeia and its greatest south declination in the Southern Cross. Its boundaries are very irregular and indefinite; its width varies from about 5° to 30° or more. In Auriga, Orion, Monoceros, Canis Major, and Argo (galactic longitude 140° to 240°), it is broad and faint. It is bifurcated in Cygnus, a branch extending southward on the west side into Ophiuchus. The brightest part lies in Sagittarius, Scorpius, and Scutum (galactic longitude 310° to 0°).

In many parts of the Milky Way, particularly in Sagittarius, Scorpius, Scutum, and Cygnus, there are great bright condensations or **star clouds** which in a clear sky are very conspicuous to the naked eye, and in which photographs by Barnard, Bailey, and others have revealed hundreds of thousands of stars in masses and streams interspersed with dark lanes and occasional luminous nebulæ (Plates 42, 44). In many parts also occur dark rifts which are visible to the naked eye; the most famous of these is the **coal sack** in the Southern Cross.

It will be shown in the next chapter that the appearance of the Milky Way is one of perspective and is due to the fact that we live in the midst of a vast aggregation of stars, star clusters, and nebulæ (the **Galactic System**) which has roughly the form of a disk or of two saucers placed rim to rim and bottom outwards. Looking toward the rim, we see vast numbers of stars (the Milky Way), most of which are exceedingly faint because of their great distance. Looking into other parts of the sky, we see fewer stars and especially fewer very faint stars because the stars which we do see in these directions are relatively near.

The Belt of Bright Stars.—It was pointed out by Sir John Herschel and emphasized by Gould that a belt of bright stars, containing many of the brightest in the sky, intersects the Galaxy in Cassiopeia and Crux at an angle of about 20° and traverses Orion, Canis Major, Argo, Centaurus, Crux, Lupus, Scorpius, Taurus, Perseus, Cassiopeia, Cepheus, Cygnus, Lyra, Hercules, and Ophiuchus. This belt contains practically all of the bright B-type stars. The numerous first-magnitude stars contained in that portion extending from Perseus to Canis Major are probably responsible for the often-expressed opinion that the skies of winter are clearer than those of summer; it is not, however, the clearness of the winter sky but the brightness of the winter stars that gives this impression.

The Magellanic Clouds.—About 20° from the South Pole, and hence always invisible in the latitudes of the United States, lie the **nubeculæ** or **Clouds of Magellan,** named for the great Portuguese navigator. To the naked eye they appear like detached portions of the Milky Way, and the telescope shows them to be composed of stars and nebulæ. The Nubecula Major or Greater Cloud is about 7° in diameter, and is situated near the border of Dorado and Mensa. The Lesser Cloud (Plate 45), which is less than 4° in diameter, lies between Hydrus and Tucana. Their brightness, according to

Sir John Herschel,[1] "may be judged of from the effect of strong moonlight, which totally obliterates the lesser, but not quite the greater." Displacements of lines in the spectra of included nebulæ show that both clouds are receding, the larger at a velocity of 276 km./sec., and the smaller at 170 km./sec.

Globular Star Clusters.—Among the most magnificent objects in the sky, as seen with a large telescope, are the crowded swarms of stars which are known as globular star clusters. Each contains many thousands of faint stars which at the center are crowded so closely together (Plates 45, 46) that they are resolved with difficulty even by the largest instruments. Photographs made with great reflectors add more and fainter stars with each increase in length of exposure, and it is probable that in none of the globular clusters have all the stars yet been perceived. Away from the center, the congestion grows gradually less, and the precise boundary of the cluster is difficult to determine; but in general the boundary is approximately circular, and the impression made upon the beholder is that of looking into a vast globe filled with stars.

Fewer than a hundred of these great clusters are known, and these lie mostly in the constellations Argo, Centaurus, Sagittarius, and Ophiuchus, between galactic longitudes 235° and 5°. All are near the Milky Way, but none are in the midst of it; they are almost completely absent in galactic latitudes $-10°$ to $+10°$. Their spectra reveal large radial velocities (from -410 to $+225$ km./sec.), but their proper motions are too small to be detected with certainty, showing that the clusters are vastly remote.

The globular cluster having the greatest apparent brightness and size is ω Centauri, which appears to the naked eye as a hazy, fourth-magnitude star, while in a large telescope it is seen as a mass of stars of the twelfth magnitude and fainter, occupying a circular area in the sky of about two-thirds the diameter of the full Moon. It is too far south for observation in the northern half of the United States. The second-largest cluster, 47 Tucanæ, is still farther south, only about 17° from the south pole, and near the lesser Magellanic cloud. The finest clusters visible in our latitudes are M 13 Herculis and M 22 Sagittarii, each barely visible to the unaided eye but containing more than 60,000 stars of the thirteenth magnitude and fainter.

[1] *Outlines of Astronomy*, 1857, p. 515.

The faintest known globular cluster is N. G. C. 7006 Delphini, which contains no stars brighter than the seventeenth magnitude.

Open Star Clusters.—The open clusters, of which about 600 were catalogued by the Herschels, are much less densely populated with stars than the globular clusters, their shape is more irregular, and their apparent size usually greater. Many of them contain nebulosity, which none of the globular clusters do. They differ also in their distribution in the sky, all but a few of the open clusters being situated in the Milky Way. Most of them contain a few hundred stars each and are easily resolved by a small telescope. Many are visible to the naked eye as faintly luminous spots or as condensations in the Milky Way; among these are Præsepe Cancri, which is resolved into eighth-magnitude stars by a good opera glass, and the beautiful pair of clusters h and χ Persei (Plate 47). The Pleiades form a fine cluster of stars in which seven are easily visible to the naked eye; four more may be seen without optical aid on an exceptionally clear, moonless night, and several hundred stars and some great masses of nebulosity are revealed by the telescope and on photographs (Plate 47). Other examples of a clustering of naked-eye stars are the Hyades, Coma Berenices, and the region of the Belt of Orion. Many groups in which the stars are fewer and more scattered are known, and in fact there seem to be clusters of all degrees of density down through multiple stars like θ Orionis to the double stars.

Distances and Dimensions of Open Clusters.—The stars in a few of the nearer open clusters have perceptible common proper motions, and the distances of these are readily determined by the method discussed on page 332. Estimates of the distances of more remote clusters are made by a less direct and less rigorous method: it is assumed that the mean absolute magnitude, M, of the stars of a given spectral type or color index in the cluster is the same as that of known stars of the same type; their mean apparent magnitude, m, then gives the heliocentric parallax of the cluster by the relation (page 350).

$$\log p = \tfrac{1}{5}(M - m - 5).$$

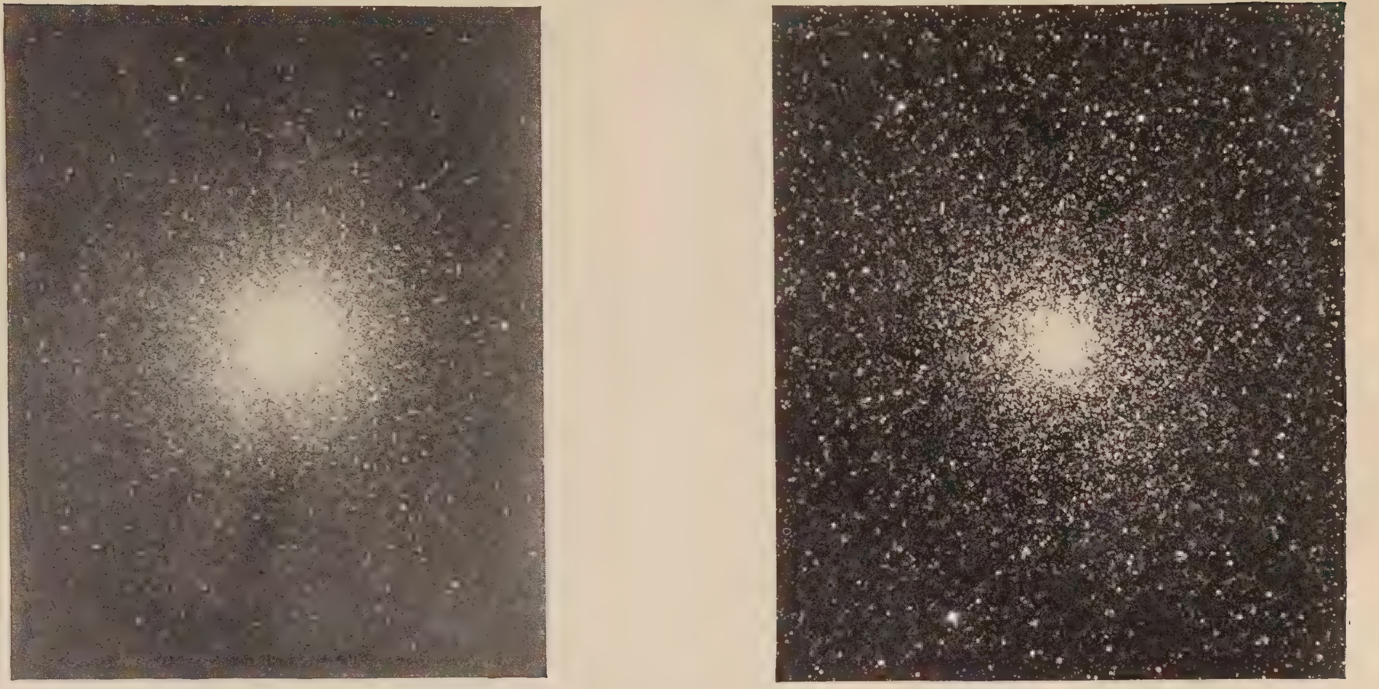
By these methods Trumpler finds that the Hyades are about

120 light-years, the stars of Coma Berenices 270 light-years, and the Pleiades 500 light-years away, while the majority of telescopic open clusters range in distance from 1,500 to 15,000 light-years. From the distances and apparent sizes he finds that the diameters of open clusters range from 10 to 80 light-years. As within these great spheres several hundred stars are contained, their distances apart, though a matter of light-years, are much less than the average spacing of stars in the neighborhood of the Solar System. Many stars in open clusters are giants, and an inhabitant of a planet near the center of such a cluster would have a view of the starry heavens splendid beyond our conception, with scores of stars far outshining Sirius.

Distances and Dimensions of Globular Clusters and Star Clouds.—Many of the globular clusters are rich in variable stars of the Cepheid type, and Shapley has determined their distances by the method outlined on page 380. From the distances he computed the linear diameters of the clusters and the absolute magnitudes of their stars, and found that the globular clusters thus studied were all of about the same size and that the brightest stars in them (which are all red giants) were of a remarkable uniformity of brightness, of about -1.5 absolute photographic magnitude. By assuming these uniformities to extend to all globular clusters, he then found the distances of those which do not contain known variable stars from their apparent diameters and the apparent brightness of their brightest stars. His results are astonishing, to say the least. The nearest globular clusters, as ω Centauri, Messier 13 Herculis, and Messier 22 Sagittarii are, according to Shapley, about 20,000 light-years away, while the distance of the most remote, N. G. C. 7006, is 220,000. At such distances, stars fainter than the Sun would be quite invisible, and the many thousands of stars that appear on long-exposure photographs of these clusters must be only the giants, while still greater numbers of lesser stars doubtless escape our detection.

On the distance of N. G. C. 7006 Sir James Jeans comments as follows: "The mere figure of 220,000 light-years can convey but little conception of the distance of this remotest of star clusters from us. We may apprehend

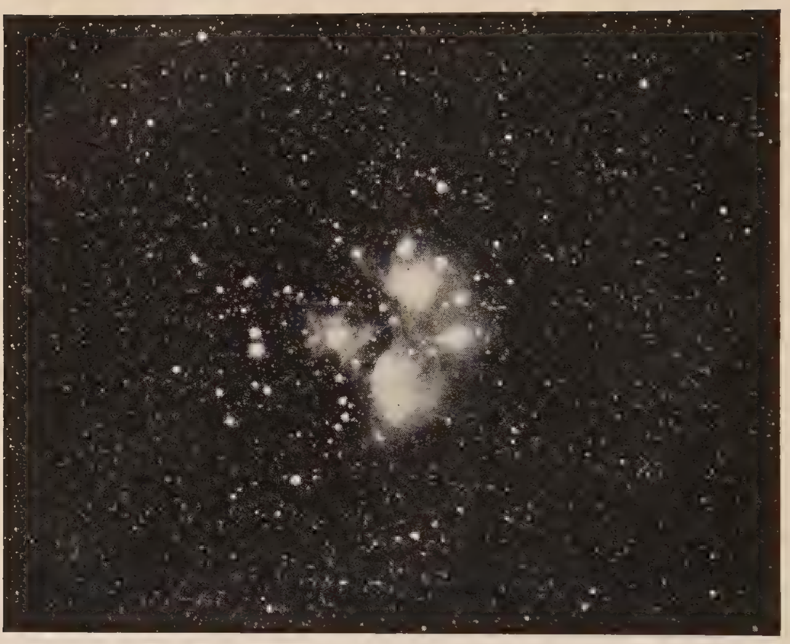
PLATE 46. GLOBULAR STAR CLUSTERS PHOTOGRAPHED WITH THE 60-INCH MOUNT WILSON REFLECTOR



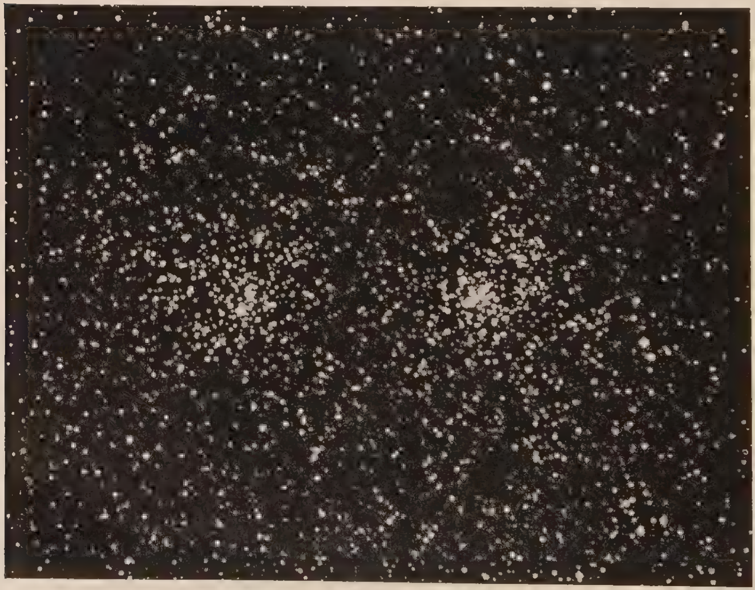
M 13, 1910 June 6–7–8. Exposure 11 hours
(Ritchey)

M 22, 1918 August 6. Exposure 3½ hours
(Duncan)

PLATE 47. OPEN STAR CLUSTERS



The Pleiades, photographed by Barnard with 10-inch lens at Yerkes Observatory



CLUSTERS h AND χ PERSEI. Photographed by Isaac Roberts with 20-inch Reflector, 1890 January 13. Exposure, three hours it better if we reflect that the light by which we see the cluster started on its long journey from it to us somewhere about the time when primæval man first appeared on earth. Through the childhood, youth and age of countless generations of men, through the long prehistoric ages, through the slow dawn of civilization and through the whole span of time which history records, through the rise and fall of dynasties and empires, this light has travelled steadily on its course, covering 186,000 miles every second, and is only just reaching us now. And yet this enormous stretch of space does not carry us to the confines of the universe. . . ."

In diameter, the denser central part of each globular cluster is about 15 light-years, and of the outlying regions fully 100 light-years. In the central portion of each there are about 1500 times as many stars as in an equal volume of the space near the Solar System; but, even so, the average distance between the stars must be some 50,000 astronomic units.

For each of the Magellanic clouds Shapley finds a distance of about 100,000 light-years. The smaller cloud is about 6500 light-years in diameter. The larger one has a diameter of about 14,000 light-years and contains many great nebulæ and star clusters. One of these nebulæ, of small angular extent, covers about the same volume of space as the whole constellation of Orion. The bright star-clouds of the Milky Way are placed by Shapley at a distance of 50,000 light-years, or more.

THE NEBULÆ

Observations of Nebulæ Prior to the Use of Photography.— The word **nebula** is used in astronomy to denote any object outside the Solar System which occupies a perceptible area in the sky and which cannot be resolved by large telescopes into stars.

The term *stella nebulosa* was applied by the early astronomers to any hazy, luminous spot which seemed fixed among the stars. In his Almagest, Ptolemy listed six, all of which have proved to be star clusters or loose groups of stars. They include Præsepe, the double cluster in Perseus, a bright cluster (Messier 7) in the tail of the Scorpion, and three loose groups such as the one composed of φ_1, φ_2 and λ Orionis in the head of Orion, which, with attention, are easily resolvable by the naked eye. Some of the *stellæ nebulosæ*, as *h* and χ Persei, ω Centauri, and 47 Tucanæ, were given stellar designations on the maps of Bayer and Flamsteed.

Very few of the nebulæ which are still recognized as such are visible to the unaided eye; among these few the only one easily seen is the great nebula in Andromeda (Messier 31), which was recorded by Al Sufi in the tenth century. Remarkably enough, Galileo makes no mention of true nebulæ, although he made a careful examination of the region of Orion which contains one of the brightest. The honor of the first telescopic discovery of a nebula seems to belong to an admirer of Galileo's, Peiresc of Provence, who found the great nebula of Orion (Messier 42) with a telescope in December, 1610. Two years later, the Andromeda nebula was first observed telescopically by Simon Marius in Germany, who compared it to a candle shining through a plate of horn.

The first catalogue of nebulæ and star clusters was made by the French astronomer Messier in 1781. Messier's chief interest lay in the discovery of comets and, as many of the nebulæ resemble faint comets when seen in a small telescope, he found it expedient to list the brighter ones with their right ascensions and declinations and a brief description in order that they might be readily recognized. His list contains 103 objects, more than half of which are star clusters, although Messier described many of these as "*nébuleuse sans étoiles*." For many of these nebulæ and clusters, their numbers in Messier's list are the most common designation.

No great interest seems to have been taken in the nebulæ until 1783, when Sir William Herschel began his systematic study of the heavens with an eighteen-inch reflector (page 412), during the course of which, with the aid of his sister Caroline, he found more than 2,500 new nebulæ and clusters which he classified and described. This work was extended to the southern hemisphere by his son Sir John Herschel, who took the eighteen-inch telescope on an expedition to the Cape of Good Hope in 1834. In 1864 Sir John published a *General Catalogue* describing over 5,000 nebulæ and clusters, only 450 of which were discovered by other observers than the Herschels. This catalogue is now superseded by the *New General Catalogue* published in England by J. L. E. Dreyer in 1888. With its two later additions, the *Index Catalogues*, Dreyer's compilation contains more than 13,000 objects. Most nebulæ and star clusters are commonly known by their numbers in Dreyer's catalogue, the name of which is abbreviated by N. G. C. Thus, the great nebula of Andromeda is referred to as Messier 31 or as N. G. C. 224.

About the middle of the nineteenth century a six-foot reflector was built in Ireland by Lord Rosse, and with this the detailed structure of many nebulæ was revealed, the most important discovery being that of a spiral or whirlpool form. Lord Rosse compared many nebulæ rather fancifully with familiar objects, applying to them names by which they are still conveniently known, although the resemblance is not always striking. Examples are the Owl nebula (Messier 97), the Crab nebula (Messier 1), and the Dumb-bell nebula (Messier 27). Other valuable observations of the forms of nebulæ were made by the English astronomer Lassell, who transported a twenty-four-inch reflector for this purpose to the island of Malta.

It was commonly believed in Herschel's time that all nebulæ were "resolvable" if only a sufficiently large telescope were used, and that many were "island universes" or sidereal systems comparable in size with the Galactic System; but Sir William Herschel later expressed the view that some of the nebulæ were composed of a shining fluid which was not of a starry nature. This view was strongly confirmed by Sir William Huggins, when in 1864 he examined the spectrum of the little bright nebula (N. G. C. 6543, Plate 49) which is very near the north pole of the ecliptic, and saw but a single bright line in the green (many fainter lines have since been observed), showing that the nebula was "not an aggregation of stars, but a luminous gas." Many nebulæ have a spectrum similar to this, while many others have spectra like those of the stars (Plate 48).

Photographic Studies of Nebulæ.—In the study of nebulæ, photography possesses two great advantages over visual observation: first, a well-made photograph is a permanent and accurate record of the appearance of the object observed, a record which is free from personal bias or misjudgment on the part of the observer. Second, by prolonging the exposure of the plate, often to a duration of many hours, details are shown which are much too faint to be perceived by the eye.

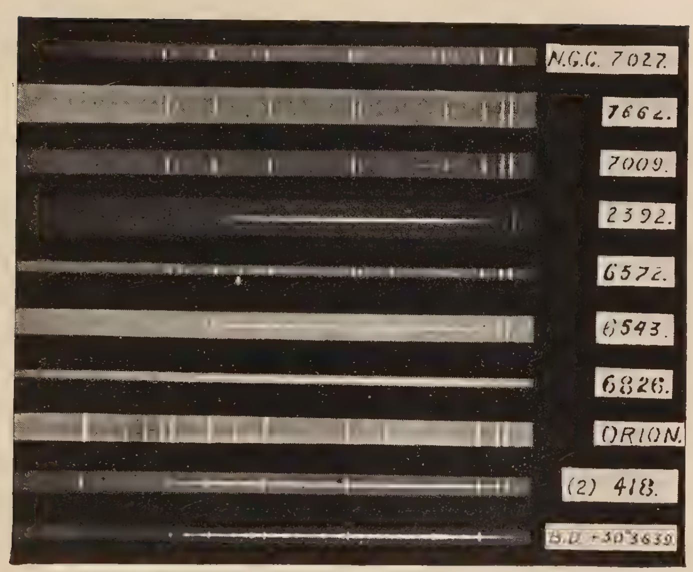
The first successful nebular photograph was made by Henry Draper of New York in 1880, when he photographed the great nebula of Orion with an eleven-inch refractor, exposing the plate fifty-one minutes. This was soon surpassed by plates taken with a three-foot reflector by Common in England. In 1886 Isaac Roberts began in England an extensive program with a twenty-inch reflector, and by the end of the century he had published two fine volumes of photographs of nebulæ and star clusters. His three-hour exposure on the Andromeda nebula in 1888 was a great revelation, for it showed that certain vague dark lanes previously noted by visual observers were really a part of a convoluted structure and that this great nebula is one of the spirals. Roberts's photographs were improved upon by Keeler at the Lick Observatory, who used a three-foot reflector from 1899 to 1901; while during the last decade of the nineteenth century and the first two of the twentieth, wonderful photographs of extended nebulosities and of Milky Way structure were made with large lenses of the portrait type by Barnard at the Lick and Yerkes Observatories, Wolf at Heidelberg, and Bailey at the Harvard Observatories in Massachusetts and Peru.

Probably the most perfect nebular photographs yet obtained are those made by Ritchey with superior reflectors of his own construction, first a twenty-four-inch which he used at the Yerkes Observatory from 1900 to 1904, and later the sixty-inch Mount Wilson reflector, which he used principally from 1908 to 1910. The one hundred-inch mirror of the Hooker telescope at Mount Wilson was also constructed under Ritchey's supervision, and has been used in nebular photography with excellent results since its completion in 1918.

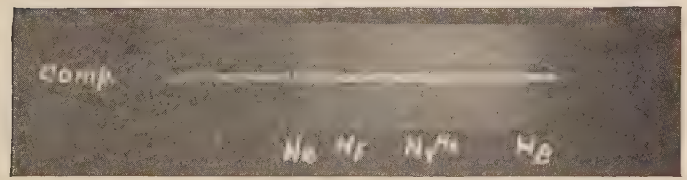
For photographing objects whose diameters are less than a degree, or for the detailed study of larger objects, no instrument is better than a reflecting telescope; its mirror is perfectly achromatic, and if constructed with a large ratio of aperture to focal length (in most modern reflectors this ratio is 1 : 5), it is very rapid. For the general study of more extended bodies, such as the galactic clouds and the largest nebulæ, the field of good definition in the reflector is too small, and better results are obtained with large lenses of the type used commercially in the making of portraits.

Method of Reproduction of Photographs of Nebulæ in this Book.—Most nebulæ are extremely faint, even as they appear on the photographic plate. In the illustrations reproduced in this book, the contrast of the nebula with the sky background was usually heightened by successive copying on slow, fine-grained plates. A glass positive was made from the original negative, a second negative from this, and a paper print from this second negative,

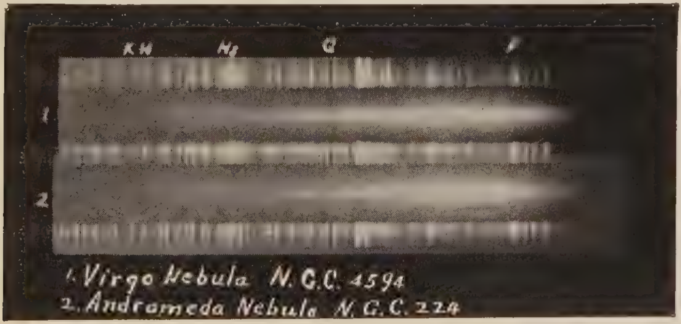
PLATE 48. SPECTRA OF NEBULÆ


N.G.C. 7027.
7662.
7009.
2392.
6572.
6543.
6826.
ORION.
(2) 418.
B.D. +30°3639.

SPECTRA OF GASEOUS NEBULÆ (Lick Observatory)


Comp.
Hδ
Hγ
Hβ

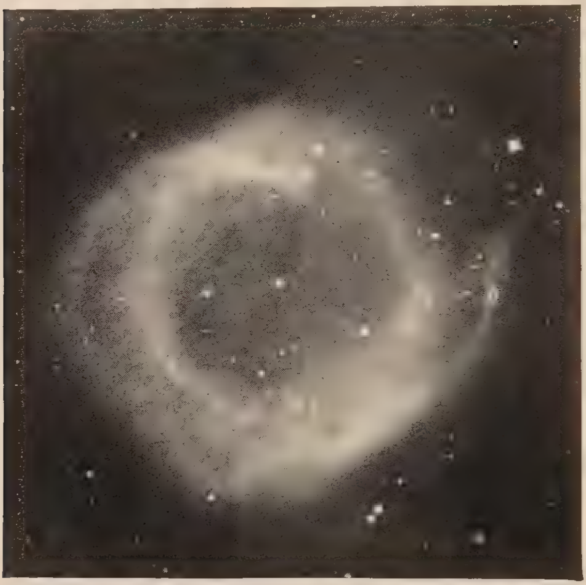
SPECTRUM OF THE MEROPE NEBULA, photographed by V. M. Slipher in 1912 with an exposure of 21 hours. It is similar to the spectrum of Merope and affords evidence that nebulæ derive their light from stars.


K H
Hδ
G
F
1
2
1. Virgo Nebula N.G.C. 4594
2. Andromeda Nebula N.G.C. 224

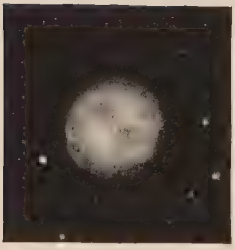
SPECTRA OF SPIRAL NEBULÆ, photographed by V. M. Slipher. N.G.C. 4594 shows high positive velocity and rapid rotation. The velocity of N.G.C. 224 is large and negative.

PLATE 49. PLANETARY NEBULÆ

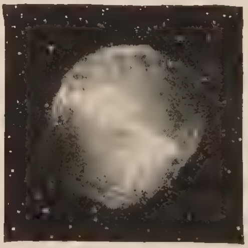
Photographed with 60-inch and 100-inch reflectors at the Mount Wilson Observatory.
Scale for bottom row, 1 inch = 50″; for others, 1 inch = 6.′5



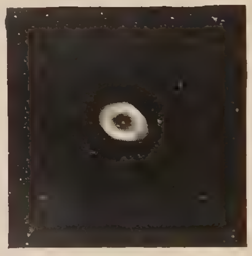
N.G.C. 7293 (Hubble, 100-inch)



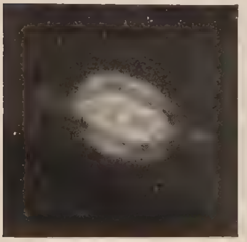
M 97 (Ritchey, 60-inch)



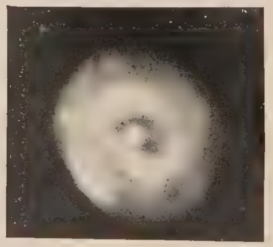
M 27 (Duncan, 100-inch)



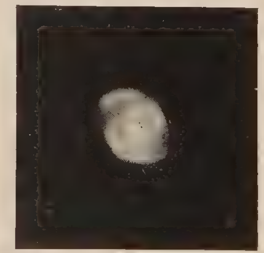
M 57 (Duncan, 100-inch)



N.G.C. 7009 (Pease, 60-inch)



N.G.C. 7662 (Pease, 60-inch)



N.G.C. 6543 (Pease, 60-inch)

and the copper-plate used in printing the half-tone illustration was made by a photographic process from the paper print. This procedure is necessary in order that the faint details of the original negative be not lost in reproduction. The scale of the photograph was in many cases enlarged or reduced to suit the available space. The long rays which appear attached to the bright stars in the photographs made with reflecting telescopes are caused by diffraction of the star's light by the thin steel webs which support the secondary mirror.

Classification of Nebulæ.—There is probably an even greater diversity among the nebulæ than among the stars. The classification of Sir William Herschel has been revised from time to time as additional information has been gathered. The form given below, which is due to Hubble, is in harmony with the results of recent researches.

The division of nebulæ into two great classes is based partly upon their apparent distribution and partly upon their probable nature. The planetary and diffuse nebulæ are all situated apparently in or near the Milky Way, and there is good reason to believe that they belong to the Galactic System and have distances of only a few hundred or at most a few thousand light-years. The elliptic and spiral nebulæ are found only in moderate and high galactic latitudes, and are probably entirely extraneous to the sidereal system to which the Sun belongs. Many of them may be "island universes" in the sense in which the term was used by Herschel, and the distances of the majority of them must be many millions of light-years.

CLASSIFICATION OF NEBULÆ

- Galactic
  - Planetary
  - Diffuse
    - Luminous
      - Having bright-line spectra
      - Having dark-line spectra
    - Obscure
- Extra-galactic
  - Spiral
    - Normal
    - Barred
  - Elliptic
  - Irregular

The Spectra of Gaseous Nebulæ.—All the planetaries, many diffuse nebulæ, and small parts of some of the spirals have spectra characterized by bright lines, and these nebulæ are often classed together under the term **gaseous nebulæ.** The brightest visible line is usually one at wave-length 5007 Angstroms, due to doubly ionized oxygen. Near it are another oxygen line (λ4959) and the Hβ of hydrogen, and the dominance of these lines gives to many such nebulæ a vivid green color. The strongest photographic line is usually one at λ3727, due to ionized nitrogen. The Balmer series of hydrogen is always conspicuous but, unlike the case of the solar photosphere, the red Hα is relatively weak. Normal helium is often present, and so is the line or band at λ4686, due to ionized helium. Other lines which are often found are due to ionized carbon and nitrogen. As might be expected, none of the metals or other heavy elements are present.

The lines at 5007, 4959, and 3727 were long unidentified, and were attributed to a hypothetical light element which Sir William Huggins named "nebulium." It was expected that this element would eventually be discovered in the chemical laboratory—an expectation strengthened by the chemical discovery of helium in 1895; but it has now long been known that there is no room in the periodic table (page 166) for any unknown light elements. In 1927 Bowen of Pasadena cleared up the mystery by showing, on the basis of modern atomic theory, that emissions of the wave-lengths of the "nebulium" lines would be expected from ionized oxygen and nitrogen under nebular conditions. The "nebulium" lines have never yet (1930) been produced in the laboratory.

Planetary Nebulæ.—The term **planetary nebulæ** was applied by Herschel to objects having small, well-defined, circular or elliptic disks which, as seen in moderate-sized telescopes, often resemble those of planets. They range in apparent size from mere points, which can be distinguished from stars only by their spectra, to the helical nebula N. G. C. 7293 Aquarii (Plate 49), which is about 15′ in diameter. The parallaxes of a few have been measured, but with considerable uncertainty; probably none are nearer than 300 light-years,

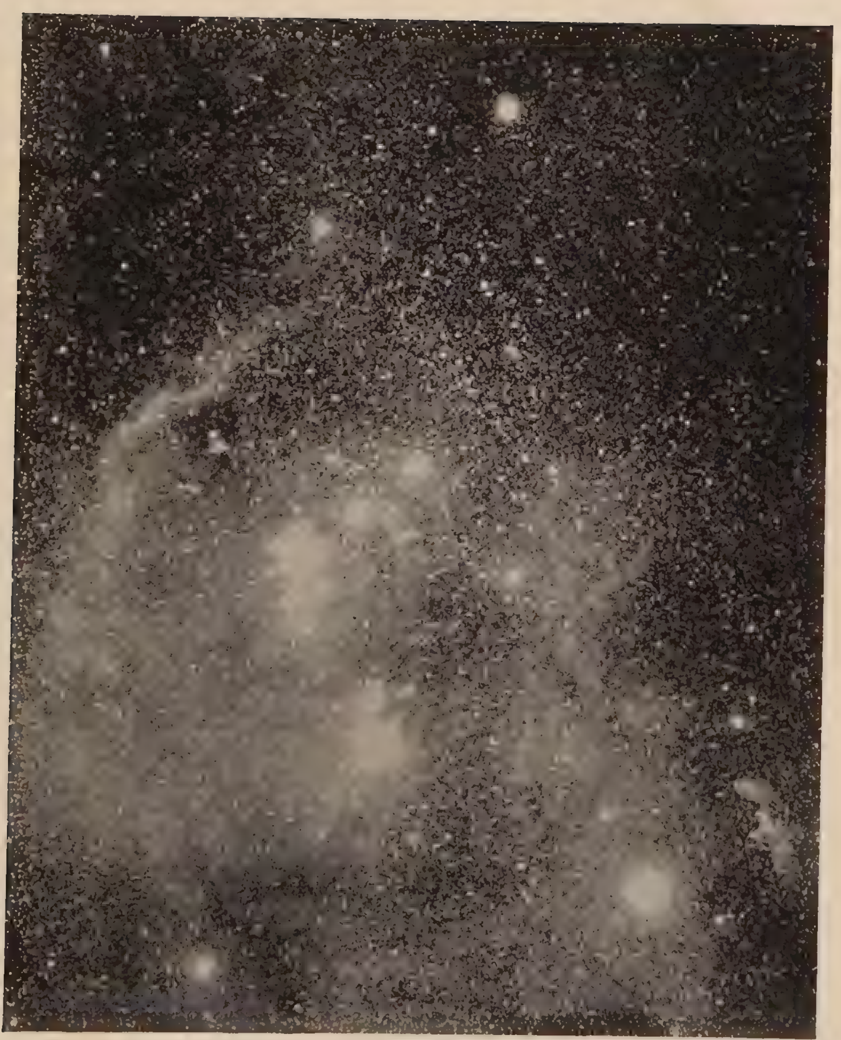
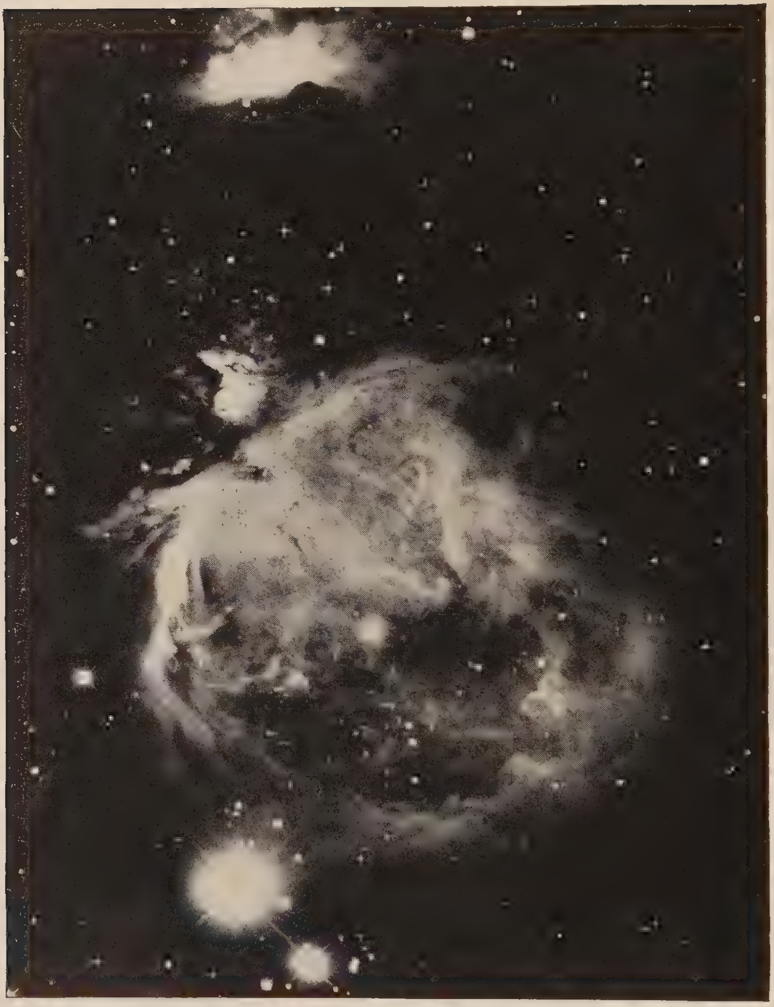


PLATE 50. ORION AND ITS NEBULOSITIES, photographed by F. E. Ross, 1927 January 6, with 3-inch Ross lens. Exposure, 5 hours

Plate 51. The Sword of Orion



Showing the Great Nebula (M 42) around the multiple star θ Orionis and, at the top of the plate, the Nebula N.G.C. 1977 around c Orionis. The wide double star ι Orionis appears at the bottom of the plate. Photographed by Ritchey, 1901 October 19, with 24-inch Reflector. Exposure 1 hour while some may be thousands of light-years away. Their real diameters must be several hundreds or even thousands of times that of Neptune's orbit.

The large planetaries, of which the Owl and the Dumbbell (Plate 49) are additional examples, are very few in number and rather faint; the small ones, mostly a few seconds in diameter, number about 130, and many of them are so bright that they can be photographed with present-day reflectors with only a few seconds' exposure. Most of the known planetary nebulæ contain centrally located stars which, in the few cases that are bright enough for spectrographic observation, are of the B type or hotter. Where no central star appears, there is usually evidence of dark material by which it may be concealed. Sometimes the central star is much brighter than the nebula, producing what Herschel called a **nebulous star;** more frequently it is fainter than the nebula and its light is often so concentrated in the violet end of the spectrum that the star is imperceptible visually in all but the largest telescopes, although showing conspicuously on photographs.

As photographed with large telescopes, the disks of the planetary nebulæ show numerous details. Many are brighter at the circumference, giving the appearance of a ring, as in the famous nebula in Lyra (N. G. C. 6720; Plate 49); some, like N. G. C. 7009 and 7662 (Plate 49), have two or more concentric rings; and a few appear to be spirals, evidently not of the flat or watch-spring variety, but helical like a corkscrew. All are fairly symmetrical with respect to a central point or star. It is practically certain that few of these nebulæ are really flat rings, for only one or two present an appearance that can be due to a thin ring turned edgewise; they must be translucent shells of gas, the appearance of a ring being due to the greater thickness of gas in the line of sight at the apparent edge of the shell.

The spectra of planetary nebulæ consist of bright lines only or of bright lines on a continuous background (Plate 48). Photographed with a slitless spectrograph, the bright lines become monochromatic images of the nebula, and it is thus often found that the form of the nebula appears different in light of different wave-lengths, due probably to a variety of distribution of the constituent gases.

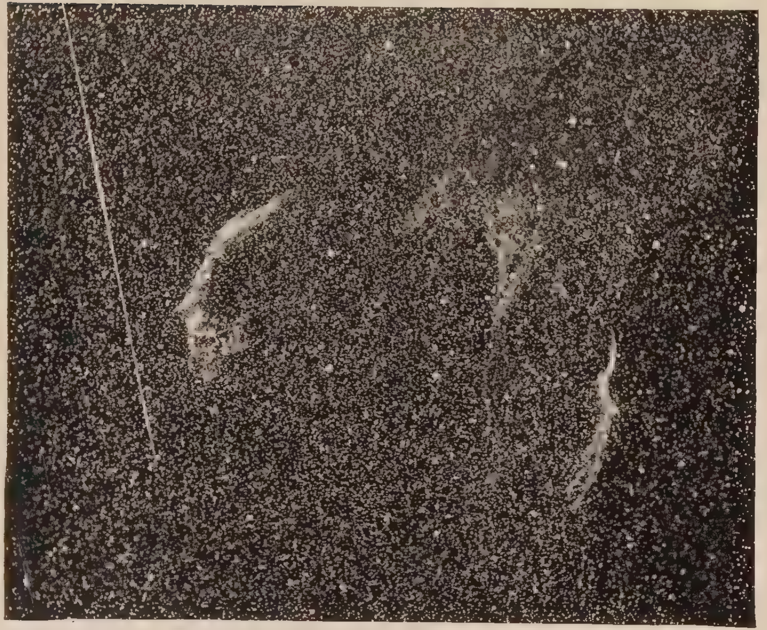
The spectral lines of most planetary nebulæ are considerably displaced, indicating velocities which average about 30 km./sec. —considerably higher than the average velocities of the stars. The lines, when the slit is placed centrally, are often inclined in such a way as to indicate a rotation around the shortest axis of the nebula, and in some cases they are doubled and in some distorted in a manner difficult to interpret. A simple doubling of the spectral lines where the slit of the spectrograph crosses the center of the nebula is probably due to an expansion of the nebulous shell; for these nebulæ seem to be nearly transparent so that the light comes to us from both the front and the rear; and an outward motion would shift the lines toward the violet for the particles on the nearer side and toward the red for those on the farther.

The rotational velocities found by Campbell and Moore at the Lick Observatory signify, on reasonable assumptions of distance, periods of thousands of years and masses several times the mass of the Sun. Their densities must be exceedingly low, even in comparison with those of giant stars.

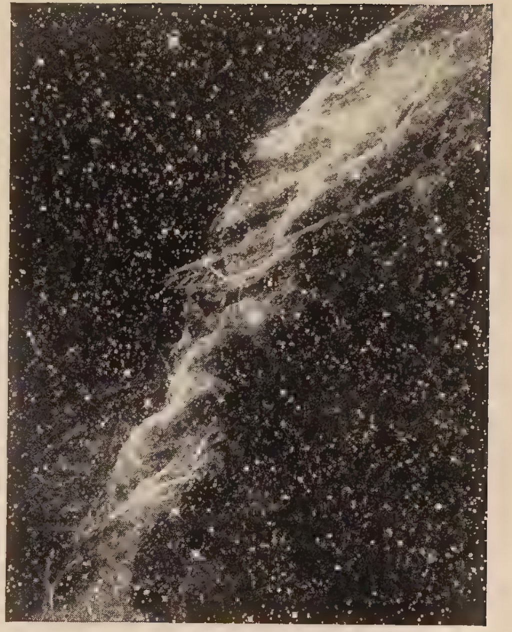
The central stars of planetary nebulæ are mostly of the O type of spectrum, a state to which, as we have seen, novæ are eventually reduced. This fact and the further one that such nebulæ have actually been observed to form around novæ (page 372) have led to a belief in such a catastrophic origin of all planetary nebulæ. It is probable that they are the shells of gas which, perhaps after many tens of thousands of years, are still expanding around the sites of ancient explosions.

The Colors of Planetary Nebulæ.—As observed visually with the one-hundred-inch telescope, many of the planetary nebulæ present beautiful colors. The brightest ones are mostly of a greenish blue, while others are predominantly red. The two bright planetaries N. G. C. 7662 and N. G. C. 3242 have very bright greenish-blue rings inclosed in fainter and larger shells of a rosy hue. The central stars, which are probably white, appear yellow in contrast with the blue ring. The "Saturn" nebula (N. G. C. 7009) consists of a bright greenish-blue ring or "ball" with reddish ansæ resembling in form the ring of Saturn, and with a yellow-appearing central star. The celebrated O-type star BD + 30° 3639, which was found spectroscopically by Campbell

PLATE 52. THE VEIL NEBULÆ, N.G.C. 6960 AND 6992 CYGNI



Photograph by Barnard with 10-inch lens.
Bright Meteor Trail at Left



N.G.C. 6992, photographed by Ritchey with
60-inch reflector

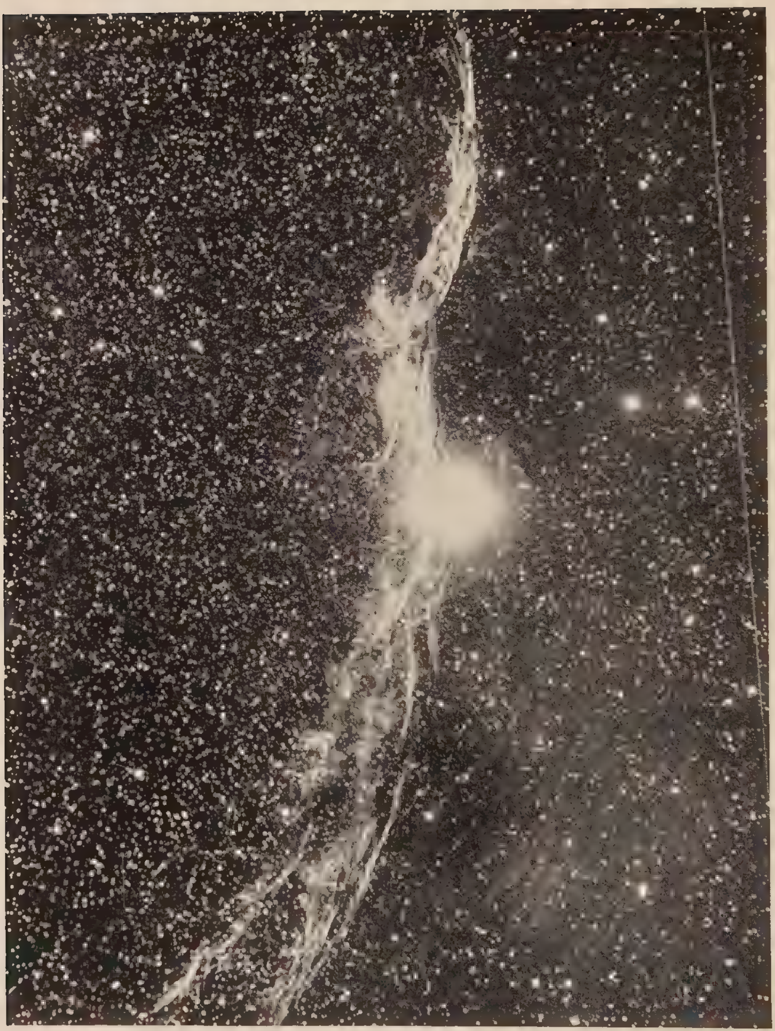


PLATE 53. THE VEIL NEBULA N.G.C. 6960 CYGNI
Photographed by Duncan, 1921 August 3, with the 100-inch Hooker telescope. Exposure 7 hours. A meteor trail appears at the right

to be surrounded by a small gaseous nebula, appears in the one-hundred-inch telescope in the center of a beautiful rosy envelope about 4″ in diameter.

Diffuse Nebulæ.—Galactic nebulæ of the second class display a great variety of form, size, and brightness. Few show any of the sharpness of outline that characterizes the planetary nebulæ, and most of them fade imperceptibly into the background of the sky; hence the term **diffuse.** Some, like the great faint wreath in the constellation Orion (Plate 50), extend over many degrees on the celestial sphere, while some are only tiny wisps. The brightest of the diffuse nebulæ is the central portion of the great nebula in Orion (M 42, Plate 51), which is very dimly visible to the unaided eye as a slight mist around the middle star of Orion's sword. Some are so faint as to require many hours' exposure to record them on photographic plates at the focus of a modern reflector; and some, the dark nebulæ, are manifest only as they are silhouetted against a brighter background of stars or luminous nebulosity.

The spectra of most luminous diffuse nebulæ are characterized by bright lines and are similar to those of planetary nebulæ; many appear to consist of bright lines only, while others have a continuous background. Some have dark-line spectra. The radial velocities shown are usually small.

The parallaxes of such ill-defined objects could not be directly measured even if they were large. From the *motus parallacticus* of the stars involved in the Orion nebula, Kapteyn has estimated its distance at 600 light-years.

The density of these objects must be exceedingly small. It may be shown that, if the mass of the Orion nebula were as great as 100,000 times the Sun's mass, its gravitational attraction would affect perceptibly the proper motions of the neighboring stars, which it does not; and yet even the bright portion near the center is about a third of a degree in diameter, corresponding to a real diameter of about 3 light-years, or 20,000,000 times the diameter of the Sun. The mean density is therefore at most of the order of 10^{-17}, or about a millionth that of the best vacuum yet produced artificially on the Earth.

The Source of the Light of Galactic Nebulæ.—That such extraordinarily tenuous objects as the galactic nebulæ could shine by virtue of their own high temperature is difficult to believe; in fact, the word temperature would have no meaning as applied to such material. It has often been suggested that the source of the nebular light is the stars; and Hubble, from a study of many galactic nebulæ, finds strong evidence in favor of this view, as follows:

1. Every luminous *diffuse* nebula (except one—N. G. C. 6960–6992, Plate 52) surrounds or lies near a conspicuous star or stars, usually of the hot spectral types. At the center of each bright *planetary* nebula is a star, of the O or B type in each case for which there is evidence, while the planetaries in which no star is perceptible are faint and may contain stars too faint to be observed.

2. A definite relation exists between the spectrum of the nebulosity and that of the associated stars: if the stellar spectrum is of the Bo or hotter type, the nebular spectrum consists of bright lines only or of bright lines on an extremely faint continuous spectrum; if the star is of type B1 or cooler, the continuous background is conspicuous, and in some cases the nebula has a dark-line spectrum.

3. For a given exposure of the plate (corresponding to given brightness of the nebula) the extent of the illumination is the greater, the greater the brightness of the star; the relation being that which would be expected if the intensity of the illumination varies inversely as the square of the distance.

The manner in which the stars cause the nebulæ to shine is not known. V. M. Slipher, who discovered that the nebulous brush around Merope in the Pleiades (Plate 48) had a dark-line spectrum identical with that of the star, considered the star's light to be simply reflected by the nebula; and this view was made practically certain by the photometric work of Hertzsprung, which showed that the nebula was only about 5 per cent as bright as a white surface, reflecting starlight from a similar position, would appear to be. Slipher's view may be generally true of nebulæ connected with B- and A-type stars. It cannot be true of most bright-line nebulæ, since their spectra differ radically from those of their dominating stars; these nebulæ must in some way be excited to self-luminosity by the action of the stellar radiation.

It is probable that the apparent form of a luminous diffuse nebula seldom corresponds to the actual distribution of matter in space, since we perceive only the portion of it which is

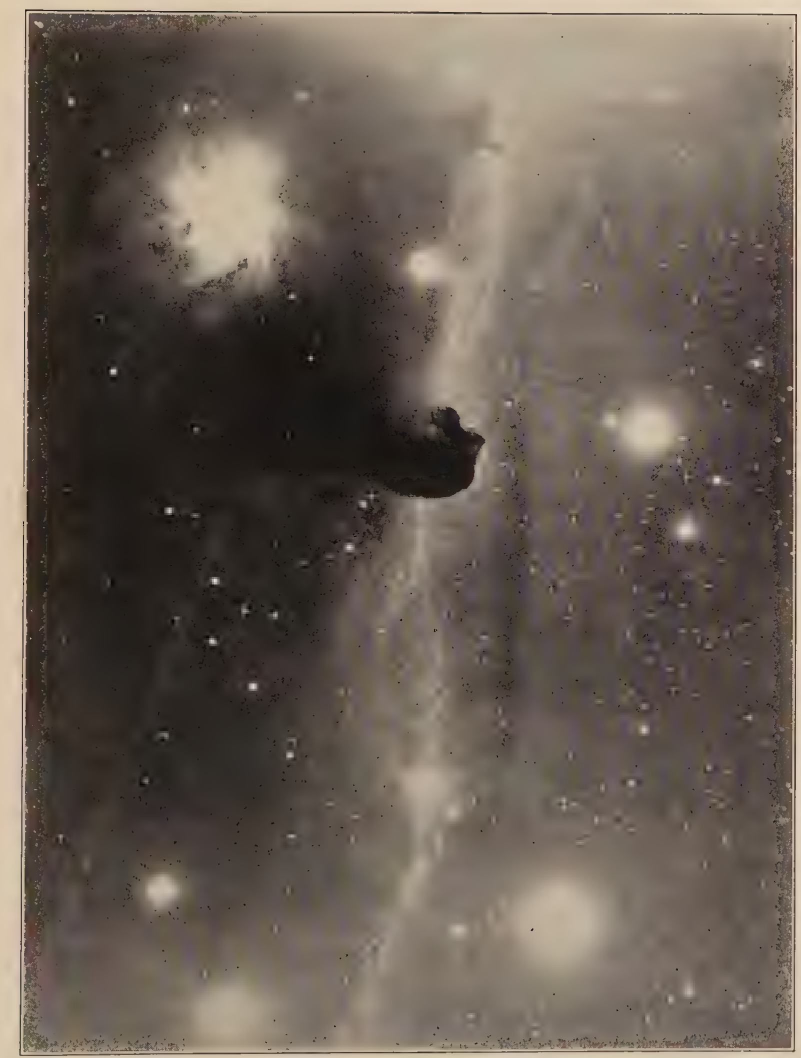
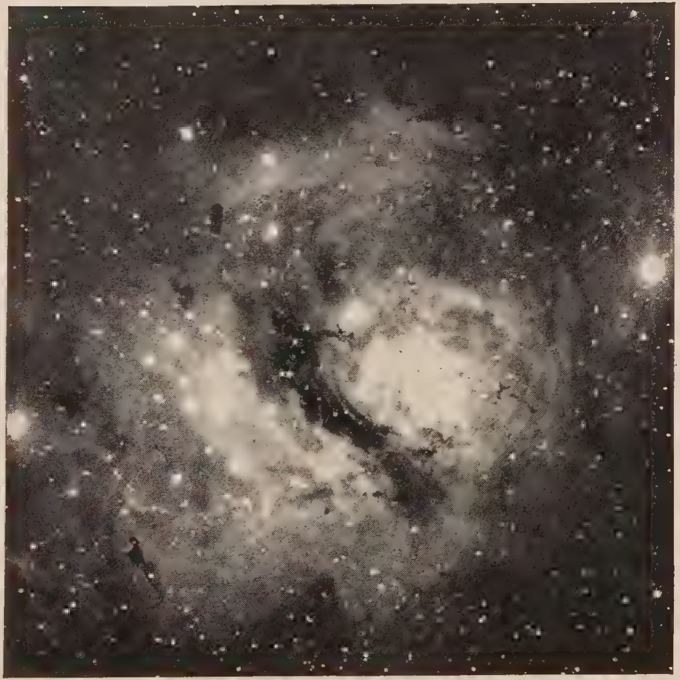


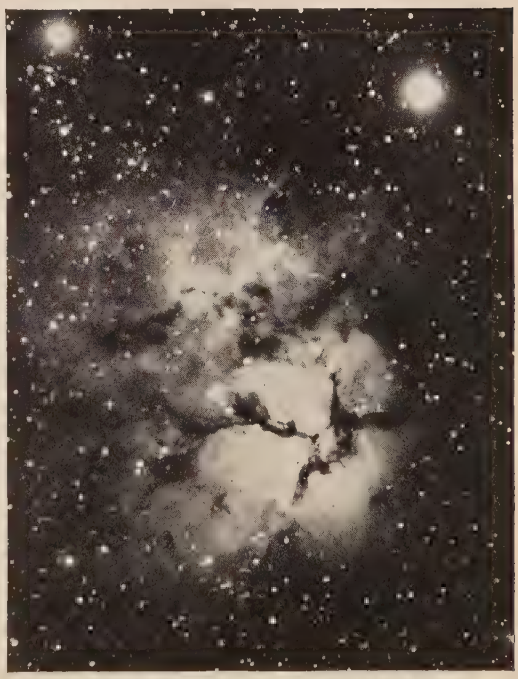
PLATE 54. NEBULOSITIES SOUTH OF ζ ORIONIS

The dark "Horse-head" nebula, Barnard 33, is near the centre. Photographed by Duncan, 1920 November 13, with 100-inch reflector. Exposure 3 hours

PLATE 55. DIFFUSE NEBULÆ IN SAGITTARIUS, photographed by Duncan at Mount Wilson Observatory



Messier 8, 1919 June 27. Exposure 3 hours. 60-inch Reflector



Trifid Nebula (M 20), 1921 June 30. Exposure 2 hours 30 minutes. 100-inch Reflector brought to luminescence by the influence of the associated stars. The visible portion may usually be surrounded by dark nebulosity which is of the same nature as the bright. Abundant evidence of this dark nebulosity is found in certain regions, as in the neighborhood of ρ Ophiuchi, where a luminous nebula with a spectrum like that of the star which it surrounds is itself surrounded by a border of comparatively starless sky.

Obscure Nebulæ.—In many parts of the Milky Way, even in the midst of the densest star clouds, there are areas almost totally devoid of stars. Sir William Herschel is recorded to have exclaimed, when he encountered one of these in a star cloud in Scorpius during his systematic sweep of the sky, "*Hier ist wahrhaftig ein Loch im Himmel!*" As holes in the sky, or at least as vacancies among the stars, they were long regarded, notwithstanding the fact that, to appear starless, they must be tunnels extending through vast depths of starry space and pointed directly toward the Solar System—a highly improbable circumstance even if the stars were motionless, and practically impossible after they have moved about at random for a few millions of millions of years.

During the present century, the conviction has grown in the minds of astronomers that these dark areas are not vacancies, but obscuring masses—**dark nebulæ** of vast dimensions which obscure the stars beyond them. Barnard, who discovered many of them, was at first doubtful of this interpretation, but later became its chief advocate and convinced its opponents. Some of these objects, like the dark "horsehead" south of ζ Orionis (Plate 54) and the dark S in Ophiuchus (Plate 44), when photographed with large telescopes, leave no doubt as to their character of opaque clouds. Many of them are feebly luminous, as Barnard first pointed out, and Bailey has shown from photographs made at Arequipa that faint luminosity pervades them almost universally. It is probable, as Bailey remarks, that obscuring nebulosities are the principal cause of all the apparent irregularities of the Milky Way, including the Coal Sacks in Crux and Cygnus and the Great Rift which bifurcates the Galaxy from Cygnus to Centaurus, and that, if the dark nebulæ were absent so as to reveal the Galaxy as it really is, "we should behold a zone of faint stars, little if at all broken by rifts or holes, and without definite limits in width."

Russell has offered the theory that the diffuse nebulæ are clouds of fine dust, with particles widely separated and accompanied, in many regions, by molecules of gas, which have been repelled to remote regions of space by the radiation pressure of the stars. Those particles which pass near hot stars are excited to luminosity and are visible as luminous nebulæ, while those that lie at a distance from exciting stars are evident to us mainly by their obscuring effect.

Evidence of an All-Pervading Interstellar Nebula.—There are many spectroscopic binaries of type B in whose spectra the H and K lines of ionized calcium do not partake of the periodic shift of the other lines due to the star's orbital velocity, and therefore cannot originate in the star's atmosphere. In some cases, possibly in all, the D lines of sodium behave as do H and K. The stationary lines are always weak and sharply defined instead of strong, broad, and diffuse as in the solar spectrum, and it is therefore not surprising that they are not found in the spectra of cooler stars where they would be concealed by the corresponding lines of the star's own spectrum. The velocity of the calcium (and sodium), when freed from the motion of the Solar System, is in all cases small and in general differs from that of the center of mass of the binary system. The generally accepted explanation is that the light of the stars reaches us through a tenuous cloud or nebula containing sodium and ionized calcium. Eddington points out that, as the diffuse nebulæ have no definite boundaries, the outer portions of one nebula may merge with those of another, "so that there is always some residual density in interstellar space." He holds the view that the ionized state of the calcium is due to starlight which, even in the depths of space, is strong enough to ionize an atom occasionally, while the atoms are so far apart that their chance of picking up an electron and so restoring the normal state is small. He calculates that the density is so low that there is but one atom to each cubic inch. A tube of one square inch cross-section, extending from here to one hundred times the distance of α Centauri, would then contain about the same number of atoms as a cubic inch of ordinary air. Eddington remarks, "It depends upon our point of view whether we regard this as an amazing fullness or an amazing emptiness of space."

Diffuse Nebulæ of Special Interest. The Orion Region.—Pre-eminent among all nebulæ is the great nebula of Orion, Messier 42, N.G.C. 1976 (Plate 51), which by many is considered the most beautiful object in all the sky. It surrounds the star θ Orionis, described by Tennyson as

> "A single misty star
> Which is the second in a line of stars
> That seem a sword beneath a belt of three."

A small telescope shows this star to consist of several, four of which, of magnitudes five to eight, form a Trapezium (Fig. 28) about 20″ in length, while about 2′ southeast of this group extends a row of three which beginners, forgetting the magnification of the telescope, often mistake for Orion's belt. All are B-type stars and glitter with intense blue-white light. Surrounding the Trapezium is a right triangle of nebulosity whose hypotenuse, about 2′ long, extends about parallel to the celestial equator on the north of the Trapezium. This nebulosity, which is the brightest part of the whole formation, presents a curdled appearance. Toward the southeast, beyond the row of three stars, extends a curved arm of nebulosity sometimes called the Waterspout, while to the northwest extends a brighter, broader curved portion like a wing. East of the Trapezium is a dark rift often called the Fishmouth or Sinus Magnus, which is easily seen on photographs to be slightly luminous. North of the rift is a circular wisp of nebulosity (M 43) surrounding a star of the eighth magnitude. Long-exposure photographs show that the space between M 42 and M 43 is filled with faint nebulosity, and that the main nebula extends more than half a degree toward the south, while on the north it joins the nebulosity around c Orionis, the most northerly star of the Sword. The surrounding region contains many luminous and also many obscure nebulæ, and there is little doubt that the whole constellation is enveloped in nebulous matter, most of which is invisible.

The color of the great nebula is greenish, and is easily noticed in a small telescope. In larger telescopes, especially reflectors, this color is intense. Along the eastern edge of the bright triangle, however, and also on the concave edge of the Waterspout, a rosy tinge is perceptible, as Barnard noted years ago. In the one-hundred-inch reflector this is very evident, and so is the red color of many of the fainter stars (irregular variables) which contrasts strongly with the green of the main nebula and the blue-white of the bright Trapezium. The stars down to about the twelfth magnitude are so numerous that, were the nebulosity removed, the region would still be remarkable as a star cluster.

The spectrum of the great nebula consists of bright lines on a very faint continuous background (Plate 48). The radial velocity ranges in different parts of the nebula from about + 10 to about + 25 km./sec.

South of ζ, the eastern star of Orion's belt (Plate 54), extends an irregular line of luminous nebula, very faintly visible in the telescope and distinct on long-exposure photographs. East of this line the sky is largely devoid of faint stars, while west of it they are numerous. Evidently the eastern side is the location of a dark nebula, for a great dark cloud (Barnard 33), about 5′ wide and shaped somewhat like a horse's head, protrudes from it over the bright nebulosity. This cloud has a "silver lining," as if a bright star or stars were concealed behind it. At the upper left corner of Plate 54 may be seen the bright nebula N.G.C. 2023 which surrounds an eighth-magnitude star and which is probably due to excitation of the cosmic dust of the otherwise dark nebulosity by the influence of this star. North of this, above the region shown on Plate 54, is a curious combination of bright and dark nebulæ. None of these nebulæ are conspicuous visually.

The Region of Sagittarius and Scutum.—The Milky Way in Sagittarius and Scutum abounds in wonderful nebulous forms, both bright and dark.

The brightest is Messier 8 (Plate 55), which is faintly visible to the naked eye. It surrounds a cluster of stars of the eighth to tenth magnitude and, as shown on photographs, is an extraordinary combination of bright and dark nebulosities. In color and spectrum it resembles the great nebula in Orion. About a degree north of Messier 8, and faintly connected with it, is the beautiful **Trifid** nebula, M 20 (Plate 55). A little farther north are M 16 and M 17, both remarkable combinations of bright and dark nebulosities.

In a dense star cloud situated between M 17 and the Trifid nebula are the two dark nebulæ Barnard 92 and 93 (Plate 56), the former of which is noticeable in a small telescope; and in another dense cloud south of Messier 8 is Barnard 86 (Plate 56), which is probably the most conspicuous visually of all the dark nebulæ. Its eastern border is very sharply defined. West of this dark nebula is a fine open cluster, and on its northeastern edge is an orange-colored star of the seventh magnitude.

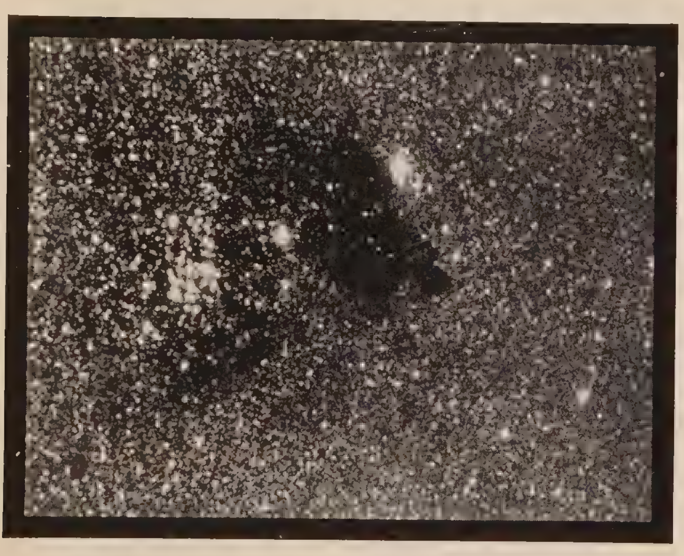
The Cygnus Region.—The constellation Cygnus contains the vast but faint **America** nebula, N.G.C. 7000 (Plate 57), which is about three degrees long and lies about three degrees east of the giant star Deneb from which, on Hubble's theory, it derives its light. It is bordered on either side by dark lanes, presumably nebulous, while the "Gulf of Mexico" and "Hudson Bay" are marked by dark nebulosities. West of the America nebula is a formation which, from its appearance on the Hooker telescope photograph (Plate 57), has been called the **Pelican** nebula.

Surrounding γ Cygni, the giant star at the intersection of the arms of the Northern Cross, are a number of faint nebulosities, while south of ϵ are the two beautiful **Veil** nebulæ, N.G.C. 6960 and 6992 (Plate 53, frontispiece), which are faintly connected to form a wreath. By comparing plates recently made with the sixty-inch reflector with those made by Ritchey fifteen years earlier, Hubble has detected outward motion in parts of this wreath which he interprets as due to an explosion which probably occurred thousands of years ago, when perhaps a nova was formed at the center of the wreath.

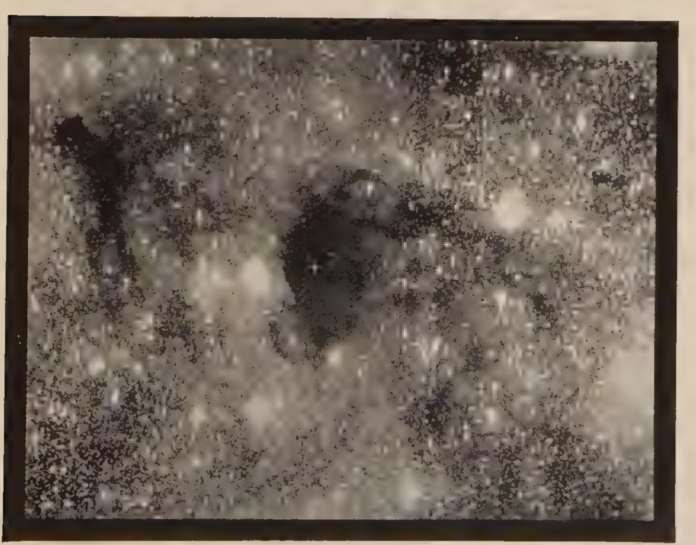
Region of η Carinæ.—Surrounding the irregular variable star η Carinæ (page 374) in the south circumpolar regions and covering about a square degree of the sky is the great mass of luminous and obscure nebulosities known as the **Keyhole** nebula (Plate 45). It derives its name from the dark nebula which is situated in the brightest mass. The famous variable star is near the eastern edge of this "keyhole," which is not very conspicuous on long-exposure photographs.

The Crab Nebula.—The nebula Messier 1 Tauri, called the **Crab** nebula by Lord Rosse, is classed as a planetary by Curtis in his extensive study of these forms, but in appearance resembles more the diffuse nebulæ. It is quite small, about $4' \times 6'$, but bright enough to be an easy object in a small telescope. Its structure is filamentous, somewhat like that of the Veil nebulæ in Cygnus. Its spectrum, photographed with some difficulty by Slipher and by Sanford, is continuous with superposed bright lines which are widely double and also inclined, which may be interpreted as evidence of rapid motions of expansion and rotation. Lampland at Flagstaff detected changes in this nebula which were confirmed at Mount Wilson; a comparison

PLATE 56. DARK NEBULÆ IN THE MILKY WAY IN SAGITTARIUS, photographed by Duncan with 100-inch Reflector



Barnard 86, 1925 July 19. Exposure 2 hours 30 minutes



Barnard 92 and 93, 1922 June 24. Exposure 4 hours

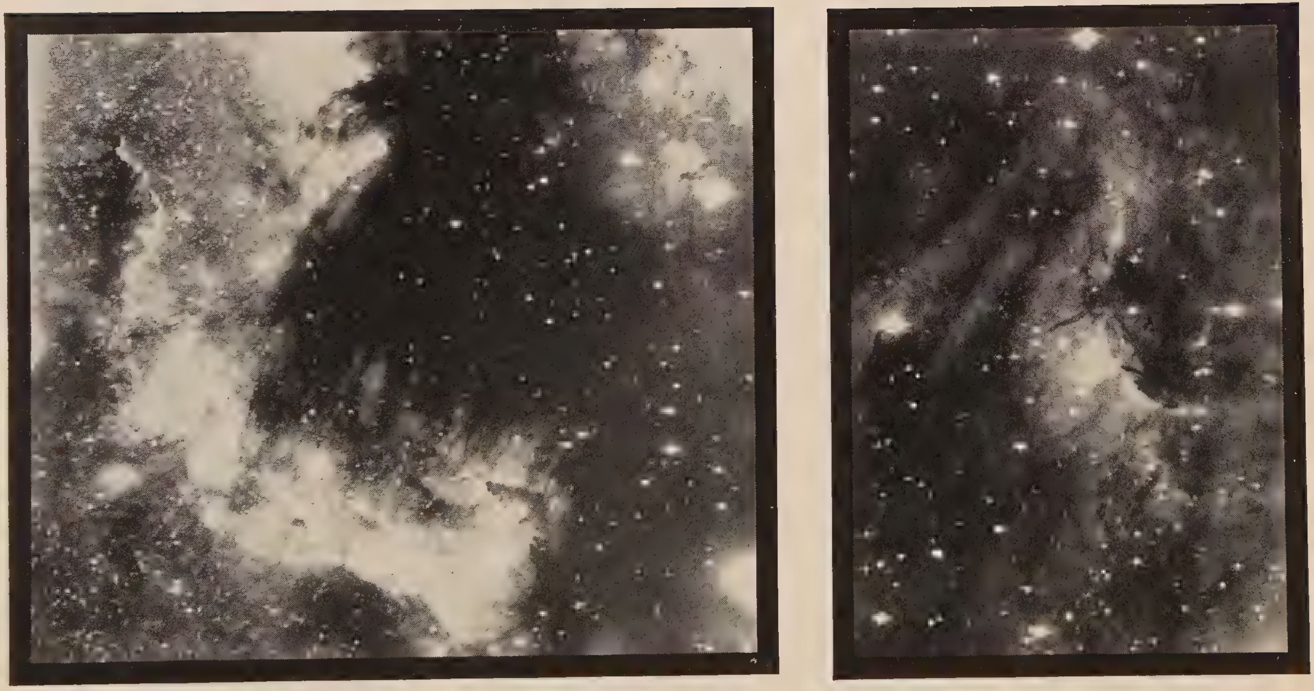


Plate 57. Nebulæ East of α Cygni
Photographed by Duncan with 100-inch Reflector
Left—Region of the "Gulf of Mexico" in the America Nebula, N.G.C. 7000, 1922 July 26. Exposure 5 hours
Right—The Pelican Nebula, 1925 July 20. Exposure 4 hours 45 minutes

of two photographs made with the sixty-inch telescope at an interval of eleven years shows an outward motion of certain details of the nebula at a rate which, if constant, must have carried them from the center to their present position in the short time of about a thousand years. At the center of the nebula is a close double star of about the fifteenth magnitude.

Variable Nebulæ.—The few nebulæ that are known to vary in brightness are associated with variable stars. The most notable ones are connected with T Tauri, R Coronæ Australis, and R Monocerotis. There is little doubt that the variation of the nebula is in each case due to the variation of the exciting influence of the star. The nebula surrounding R Coronæ Australis is inclosed in a comparatively starless border probably due to dark nebulosity. Observations at Flagstaff, Mount Wilson, and Helwan, Egypt, have shown that, soon after the star sinks to a minimum, the nebula fades almost to imperceptibility; and that after the star brightens, a wave of luminosity sweeps out over the nebula. The natural interpretation is that the illumination is due to a light-echo similar to the one which swept out over Nova Persei 1901 (page 372), and this view is supported by the fact that the spectrum of the nebula is very similar to that of the star, both including bright hydrogen lines. The form of the nebula is not the same at successive brightenings, due perhaps to changes in the distribution of the nebulous material and to relative motion of the star and the nebula. Similar phenomena have been observed by Lampland, Slipher, and Hubble in the comet-shaped nebula attached to R Monocerotis.

Extra-galactic Nebulæ.—The nebulæ outside the Milky Way are very numerous; while estimates of their number differ greatly, it is safe to say that at least a half million are within the reach of modern photographic telescopes, and it is probable that the more distant ones are still more numerous.

The spiral of greatest apparent size, the great nebula of Andromeda (Plates 58, 59, 60), is at least $2^{\circ}.5$ long and a quarter as wide. The next largest, Messier 33 Trianguli (Plate 60), is about a degree in diameter; but there are no others of dimensions exceeding $30'$. The great majority of extra-galactic nebulæ are only a few seconds or less in diameter.

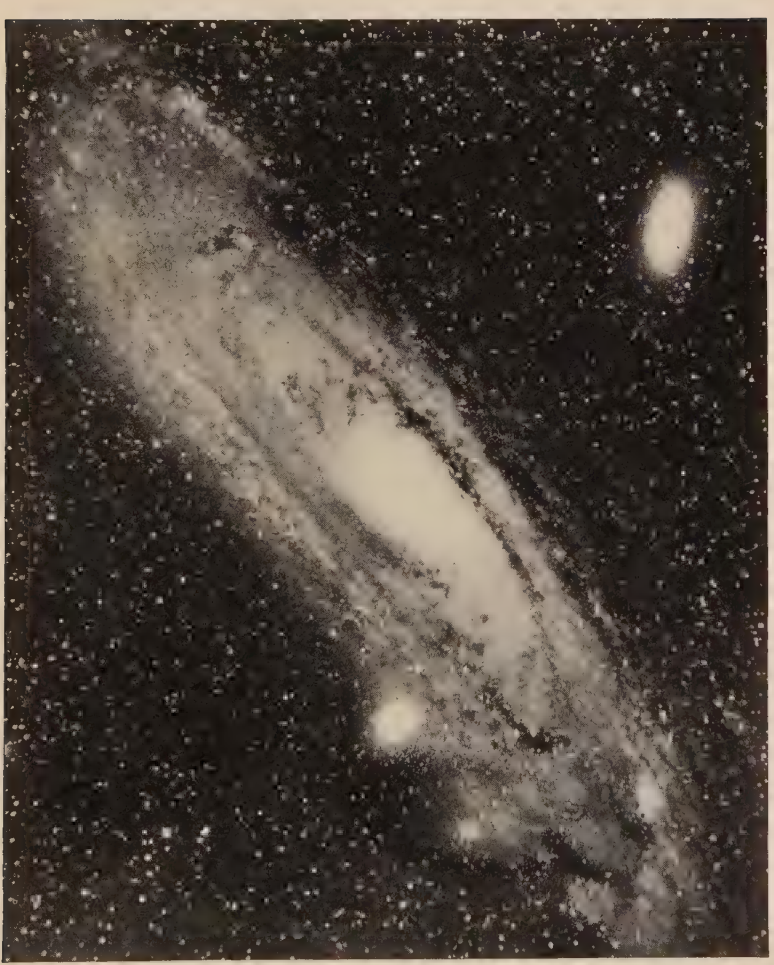
These nebulæ not only avoid the Milky Way, but have a distribution in the two galactic hemispheres that is far from uniform. In a region extending from Virgo to Ursa Major small nebulæ are very numerous; near the north galactic pole in Coma Berenices, which is included within this region, is a compact cluster of them, and in Virgo is a somewhat larger cluster of larger nebulæ. A photograph taken with the one-hundred-inch reflector in the midst of the Coma cluster (Plate 61) shows more than three hundred small nebulæ in a circle no larger than the full Moon. Most of the large spirals lie south of the Milky Way, but the very small nebulæ are not so numerous in the southern galactic hemisphere as in the northern.

The spectra of about fifty of the brightest extra-galactic nebulæ of all three classes have been photographed, principally by V. M. Slipher. With the exception of localized condensations within their structure, which have spectra containing bright lines, all have dark-line spectra resembling the spectra of stars, the types ranging from F to K. The radial velocities, with a few exceptions, are positive and very large—hundreds and even thousands of kilometers a second. In a number of cases, where the nebula is placed edgewise to the Solar System, the lines are inclined, giving evidence of rotation around the shortest axis. Examples of the spectra of spiral nebulæ are shown in Plate 48.

Spiral Nebulæ.—The **normal** spiral nebula, of which Messier 51 (Plate 60) is a good example, consists of two whorls which in many cases are branched and which, departing on opposite sides of a central nucleus, wind around it in the same plane and in the same direction. Most such nebulæ have approximately the form of an equiangular or logarithmic spiral—that is, a flat spiral in which the tangent to the curve makes a constant angle with the line joining the center to the point of tangency. Some are presented to us edgewise and some in plan, and others at every angle between. In many of those which we see edge-on (Plate 60), dark lanes extend lengthwise through the center, betraying the existence of a dark extension which would otherwise be invisible.

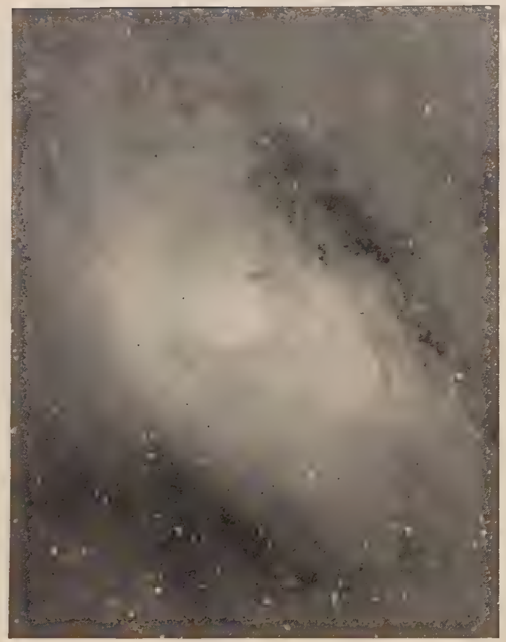
The **barred** spirals are characterized by a straight bar of

PLATE 58. GREAT NEBULA OF ANDROMEDA (M 31)

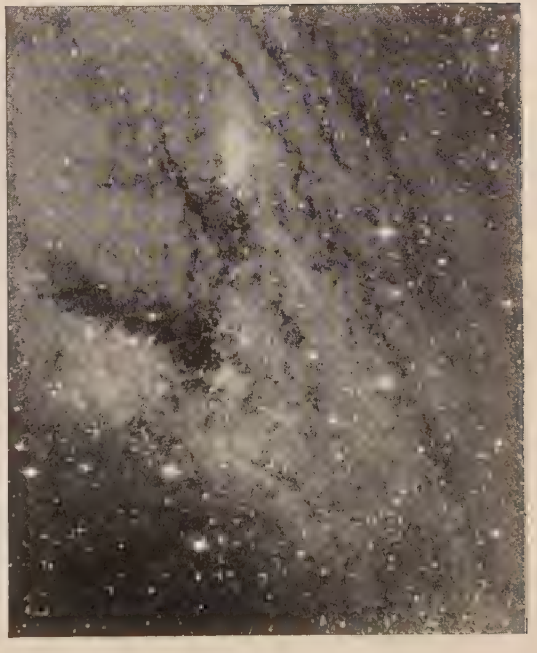


Photographed by G. W. Ritchey, 1901 September 18, with 24-inch Reflector. Exposure 4 hours

Plate 59. The Great Nebula of Andromeda (Messier 31)



Central part, photographed by Ritchey with 60-inch Reflector, 1909 October 13. Exposure, 2 hours



South-preceding end, photographed by Duncan with 100-inch Reflector, 1925 July 24. Exposure, 2 hours nebulosity passing through the nucleus; whorls similar to those of the normal spirals spring from the ends of the bar. Barred spirals, while fairly common, are not so numerous as normal spirals.

Spiral nebulæ of either type may be arranged in a series, at one end of which are nebulæ which glow with soft, misty light nearly free from knots and condensations, while at the other end are those in which the whorls are composed almost entirely of knots, many of which are so small and definite as to differ in no way from ordinary stars. This progression of nebular types is shown in Plate 62, in which three normal spirals are shown on the left and three barred spirals on the right. In some spiral nebulæ, of which the great nebula in Andromeda and Messier 81 (Plates 58-60) are fine examples, the central portion is of the soft, amorphous type of nebulosity, while the outer whorls contain numerous condensations and stars.

Elliptic Nebulæ.—Many extra-galactic nebulæ show no structural features whatever, even when observed with the largest telescopes. In apparent form they are ellipses or lenticular figures in which the ellipticity (ratio of the difference of longest and shortest axes to the longest) ranges from 0.0 to 0.7. The circular form occurs too frequently to be attributed to accidental tilt of a disk or spheroid; many of them must be actually globular. Plate 63 shows a progression of four elliptic nebulæ from the circular form to the lenticular. The largest known elliptic nebula (M 32) has a major axis of about 4′. The vast majority of extra-galactic nebulæ are so small as to show no details of structure; they may as well be elliptic as spiral.

Irregular Extra-Galactic Nebulæ.—About 2.5 per cent of the extra-galactic nebulæ which have been classified show no evidence of rotational symmetry and are classified as **irregular.** The distribution of their parts is chaotic. More than any other known objects, they resemble the Magellanic clouds. The largest example is N. G. C. 6822 Sagittarii, which covers an area of the sky about 10′ × 20′ and has a bright core about 3′ × 8′ in which stars are densely crowded. The object contains five diffuse nebulæ which resemble galactic nebulæ. Other irregular or "Magellanic" non-galactic nebulæ include Messier 82 and N. G. C. 4449, illustrated in Plate 63.

The Distances and Dimensions of Extra-Galactic Nebulæ.—It is certain that these objects are at enormous distances and that their dimensions are correspondingly immense. The "island universe" theory, according to which they are galaxies or vast aggregations of stars and nebulæ similar to the Galactic System and entirely external to it, has been suggested at different times and has been especially strongly advocated by H. D. Curtis since 1915, at which time the following facts were known to support it:

1. Their spectra are about what would be yielded by a vast congeries of stars.

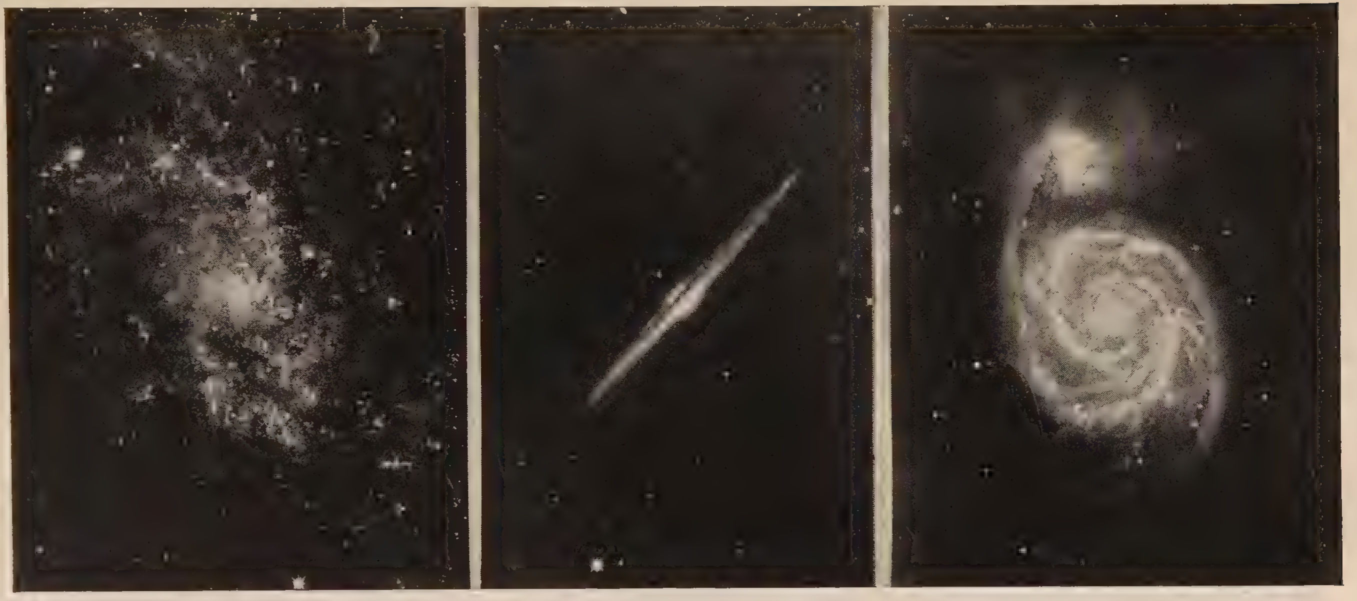
2. Their apparent avoidance of the Milky Way finds a reasonable explanation on the assumption that external bodies in that plane are hidden by the nebulæ that are known to exist in our system, or by dark material in the peripheral regions of the galactic disk similar to that seen in many spiral and elliptic nebulæ which are turned edgewise to our view.

3. Many novæ have been found in spiral nebulæ, but they are rare in other parts of the sky except the Milky Way. The nebular novæ are very faint, and if comparable with galactic novæ in actual luminosity are necessarily very distant.

4. The high velocities of extra-galactic nebulæ, detected spectrographically, place them in a class apart from galactic objects.

5. Not only are measurable parallaxes lacking, but, notwithstanding the high spectrographic velocities, no motion at right angles to the line of sight has been certainly detected.

This last argument was seemingly refuted in 1921 by Van Maanen, who measured pairs of photographs of a number of the great spirals, made with the sixty-inch Mount Wilson reflector at intervals of about ten years, and found minute apparent displacements of the nebulous condensations outward along the whorls. On the assumption that the actual velocity of these particles was not much greater than the rotational velocities observed spectrographically in edge-on spirals, the distance comes out only about 25,000 light-years. In view of the strong evidence in favor of much greater distances, however, it would appear unsafe to regard these motions as proved until observations can be obtained after a much longer interval.

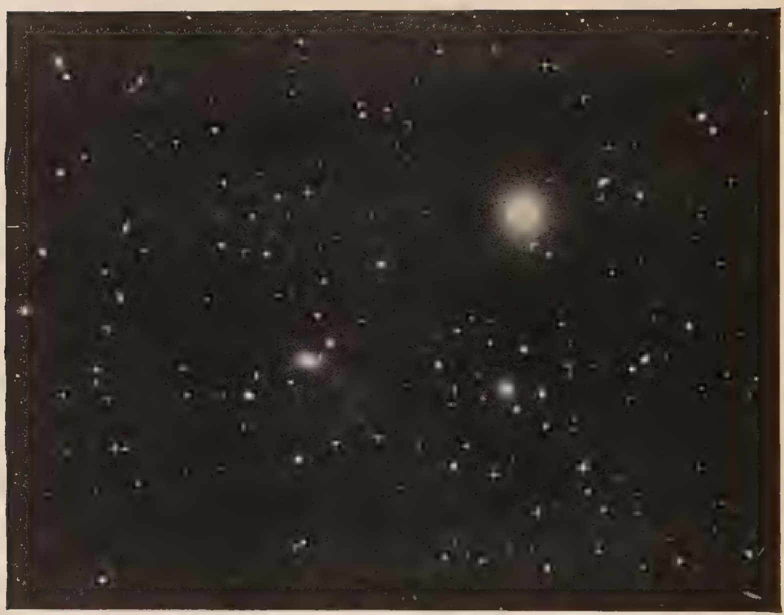


Messier 33 Trianguli, 1910 August 5, 6, and 7. Exposure, 8h. 30m.

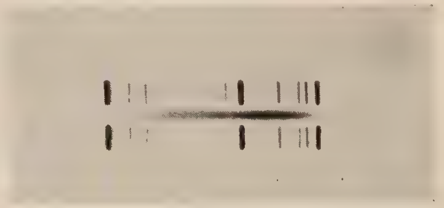
N. G. C. 4565 Comæ, 1910 August 6 and 7. Exposure 5h.

M 51 Canum Venaticorum, 1910 April 7–8. Exposure 10h. 45m.

PLATE 60. THREE GREAT SPIRAL NEBULÆ photographed by Ritchey with 60-inch Reflector. The photograph of Messier 33 (left) is reproduced on a smaller scale than the others; its diameter is really about four times that of either of the other nebulæ



CLUSTER OF SEVERAL HUNDRED EXTRA-GALACTIC NEBULÆ IN COMA BERENICES, photographed by Duncan, 1921 May 8, with 100-inch telescope. Exposure 4 hours. Most of the nebular images are so small that, in the engraving, they cannot be distinguished from stars



SPECTRUM OF N.G.C. 4860, one of the nebulæ near the center of the upper plate, photographed by Humason in 1929 with a small spectrograph attached to the 100-inch telescope. Exposure, 42 hours. Helium comparison spectrum. The nebular spectrum is displaced toward the red by an amount which corresponds to the enormous velocity of 7800 km/sec. The engraving is enlarged ten times from the original negative. This nebula is so faint and so remote that to obtain even so small a bit of evidence required the greatest of existing telescopes and the untiring vigilance of a patient observer

Strong additional evidence in favor of the island universe theory was found in 1924 by Hubble through his study of many faint, long-period Cepheid variables in the irregular nebula N. G. C. 6822, in the great spiral Messier 33, and in the great Andromeda nebula, the outer portion of which is resolved by the one-hundred-inch telescope into myriads of faint stars (Plate 59). The position of these Cepheids on Shapley's period-luminosity curve (page 380) indicates for the three objects the following distances and dimensions:

	Distance	Longest diameter
N.G.C. 6822	700,000 light-years	4,000 light-years
Messier 33	900,000 "	16,000 "
Andromeda nebula	900,000 "	47,000 "

From statistics concerning several hundred extra-galactic nebulæ, Hubble finds that the relation between apparent total brightness and apparent diameter is about that which would be expected if their real diameters were all of the same order—that is, while one might be ten times as big as another, it would not be consistent with the statistics for one to exceed another a thousand times. This result means that their apparent diameter is largely a matter of distance and leads to distances greater than 100,000,000 light-years for the host of small nebulæ which have apparent diameters less than 10″.

Gaseous Nebulæ Contained in Extra-galactic Nebulæ.—Many of the condensations in extra-galactic nebulæ exhibit bright-line spectra similar to those of many diffuse galactic nebulæ. These have been noted especially in the great spirals Messier 33 and 101, in N. G. C. 6822, and in the Magellanic clouds. The velocities shown by their line-shifts correspond as far as known to those of the great nebulæ of which they form parts, not to those of gaseous nebulæ in our own system. Presumably they bear the same relation to the stellar members of their systems that the diffuse galactic nebulæ bear to galactic stars. Some of these nebulæ appear to be of enormous dimensions: the largest one (30 Doradûs) in the Nubecula Major, accepting Shapley's distance, is 260 light-years in diameter, and one in Messier 33 has a diameter of 200 light-years. In comparison with these, our own Orion nebula, which is within

600 light-years of the Solar System and only about 10 light-years wide, seems modest.

The Little and the Great—Astronomy deals with the smallest and the largest of known dimensions, and with many which are intermediate. Table 25 exhibits the approximate values of a few astronomical dimensions for convenient comparison.

TABLE 25

APPROXIMATE DIMENSIONS

Diameter of proton (conjectural)	10^{-13}	centimeters or less	
Diameter of hydrogen atom	10^{-8}	cm.,	100,000 × diameter of proton
Wave-length of red light	7×10^{-5}	cm.,	7,000 × diameter of H atom
Aperture of largest refractor (Yerkes)	10^{2}	cm.,	1,400,000 × wave-length of red light
Radius of Earth	6.4×10^{8}	cm.,	6,400,000 × aperture of Yerkes telescope
Distance of Earth from Sun	1.5×10^{13}	cm.,	23,000 × radius of Earth
Distance, Sun to α Centauri	4×10^{18}	cm.,	270,000 astronomical units
Distance, Sun to M 31	9×10^{23}	cm.,	225,000 × distance to α Centauri
Farthest recorded nebula (conjectural)	10^{27}	cm.,	1,000 × distance to M 31

The greatest distance recorded in the table is 10^{40}, or

10,000,000,000,000,000,000,000,000,000,000,000,000,000,000,

times the least.

CHAPTER XIX

COSMOGRAPHY

The Number of Stars of Different Magnitudes.—To the casual observer of a clear, moonless sky it often seems that the stars visible to the unaided eye are innumerable; but if the attention is concentrated on a limited area, such as the square of Pegasus or the bowl of the Dipper, the stars in that area are found to be easily counted. In fact, only a few thousand stars can be seen in the whole sky without optical aid even on the best nights, and a little moonlight or haze conceals the majority of these, proving that the faint stars outnumber the bright. This increase in number with faintness is continued among the telescopic stars, as is illustrated vividly in Plate 64.

The stars brighter than the ninth magnitude have been catalogued and counted. Table 26 gives the total number of stars brighter than each visual magnitude as determined at Harvard (the numbers cannot be given with perfect precision because of the variability of certain stars and the fact that many lie near the border line between two magnitude-classes):

TABLE 26

NUMBER OF STARS BRIGHTER THAN GIVEN MAGNITUDES, WITH RATIO OF EACH NUMBER TO ITS PREDECESSOR

Vis. mag.	Number	Ratio	Vis. mag.	Number	Ratio
2	40	...	6	4,800	3.2
3	135	3.4	7	15,000	3.1
4	450	3.3	8	46,000	3.1
5	1,500	3.3	9	134,000	2.9

By a laborious investigation of the number of stars in selected regions of the sky, photographed principally with the sixty-inch Mount Wilson reflector, Seares and van Rhijn have formed curves and tables giving the numbers of stars down to the twenty-first photographic magnitude in different galactic latitudes and to the twentieth visual magnitude for the whole sky. From their curves are derived the numbers in Table 27, which extends the table given above:

TABLE 27

NUMBER OF STARS BRIGHTER THAN GIVEN MAGNITUDES (*Continued*)

Vis. mag.	Number	Ratio	Vis. mag.	Number	Ratio
7........	14,300	...	14.......	13,800,000	2.4
8........	41,300	2.9	15.......	32,000,000	2.3
9........	117,000	2.8	16.......	70,800,000	2.2
10........	324,000	2.8	17.......	148,700,000	2.1
11........	868,000	2.7	18.......	296,000,000	2.0
12........	2,260,000	2.6	19.......	560,000,000	1.9
13........	5,700,000	2.5	20.......	1,000,000,000	1.7

The numbers for the last three or four magnitudes are of course extrapolations.

From the first of these tables we find that the number of stars visible to the naked eye is about five or six thousand; but since only half the celestial sphere is visible at one time and the faint stars are extinguished by atmospheric absorption when several degrees above the horizon, two thousand is a liberal estimate of the number that are visible at one time even under the best conditions. The total number down to the twentieth magnitude, which is about a magnitude fainter than can be perceived by the eye at the one-hundred-inch telescope, appears to be about 1,000,000,000.

The Total Number of Stars.—A significant thing about the numbers in the above tables is the ratio of each to its predecessor, which is printed in the third column. This ratio diminishes as fainter stars are added, and apparently would become unity before reaching the thirtieth magnitude. This would mean that the total number of stars brighter than the thirtieth magnitude is the same as the total number brighter than the twenty-ninth—that is, that there are no stars so faint as the thirtieth magnitude. The total number of stars (exclusive of globular clusters and extra-galactic nebulæ) is estimated from the numbers in Seares's table to be of the order of thirty or forty thousand million.

If the stars were all of the same actual brightness, their differences of magnitude being due to difference of distance only, their apparent brightness would be inversely as the square of their distance; that is, each star of magnitude n would be $\sqrt{\rho}$ times as far away as one of magnitude $n - 1$ where ρ is the light-ratio (page 315). All stars brighter than the nth magnitude would be contained in a sphere of radius $\sqrt{\rho}$ times the radius of the sphere containing stars brighter than the $(n - 1)$st magnitude, and the volume of the larger sphere would be $\sqrt{\rho^3}$ times the volume of the smaller. If, further, these stars of uniform intrinsic brightness were distributed uniformly through space, the number of stars brighter than the nth magnitude would be to the number of those brighter than the $(n - 1)$st as the volume of the larger sphere is to that of the smaller. Now, ρ being equal to $2.512\ldots$, $\sqrt{\rho^3}$ is very nearly equal to 4. Therefore, the fact that the ratio of successive numbers in Tables 26 and 27 is less than four shows that the stars are not uniformly bright bodies distributed uniformly throughout infinite transparent space.

Although there is known to be a vast range in the intrinsic brightness of stars, yet in dealing with great multitudes the numbers in different magnitude-classes should still depend upon their distribution. The observed correlation between number and brightness might be due to an absorbing medium which occults the more distant stars. It is known that absorbing material—dark nebulæ, dark stars, planets and meteors—does exist, but its quantity is not generally considered great enough to account for the diminishing ratio of the fainter stars.

The generally accepted explanation of the diminishing ratio is that we are in the midst of a vast but finite aggregation of stars—the **Galactic System**—which is separated from all other bodies or systems by distances which are great compared with its diameter. Many of these stars must be concealed by dark nebulæ, and Shapley would increase Seares's estimate to 100,000,000,000, while Eddington, on dynamical grounds, estimates the total mass of the Galactic System at 270,000,000,000 times that of the Sun.

General Distribution of Stars with Reference to the Milky Way.—Counts of stars, both bright and faint, in different parts of the sky show that the **star density,** or average number of stars per square degree, is greatest near the galactic circle and, in general, diminishes with increasing galactic latitude. Systematic study of this kind was begun in the latter part of the eighteenth century by Sir William Herschel by the method called by him "star gauging," which consisted in counting the stars visible in an eighteen-inch reflector having a field 15′ in diameter. He made 3,400 "gauges," and his son Sir John made 2,300 more with the same telescope in the southern hemisphere. Their work has been confirmed and extended by others, the most recent results being those of Seares and van Rhijn, which are summed up in Table 28. This table gives, for certain galactic latitudes, the average number, per square degree, of stars brighter than photographic magnitude *m*. In the last column is given the ratio of the star density on the galactic circle to that at the galactic poles. Table 28 plainly

TABLE 28

AVERAGE NUMBER OF STARS PER SQUARE DEGREE, BRIGHTER THAN PHOTOGRAPHIC MAGNITUDE *m*, IN DIFFERENT GALACTIC LATITUDES

m	Galactic Latitude				Galactic Concentration
	0°	30°	60°	90°	
4.0	0.0156	0.0074	0.0051	0.0045	3.5
5.0	0.0449	0.0214	0.0148	0.0130	3.4
6.0	0.128	0.0614	0.0421	0.0372	3.4
7.0	0.361	0.173	0.118	0.103	3.6
8.0	1.01	0.482	0.325	0.278	3.6
9.0	2.81	1.31	0.871	0.723	3.9
10.0	7.71	3.49	2.23	1.81	4.3
11.0	20.8	9.06	5.47	4.33	4.8
12.0	55.6	22.7	12.8	9.89	5.6
13.0	146	54.4	28.6	21.4	6.8
14.0	371	125	61.0	44.3	8.4
15.0	910	272	123	87.1	10.4
16.0	2140	561	236	163	13.2
17.0	4780	1090	428	288	16.6
18.0	10200	1990	733	482	21
19.0	20800	3440	1190	769	27
20.0	40100	5620	1820	1160	34
21.0	73600	8690	2650	1670	44

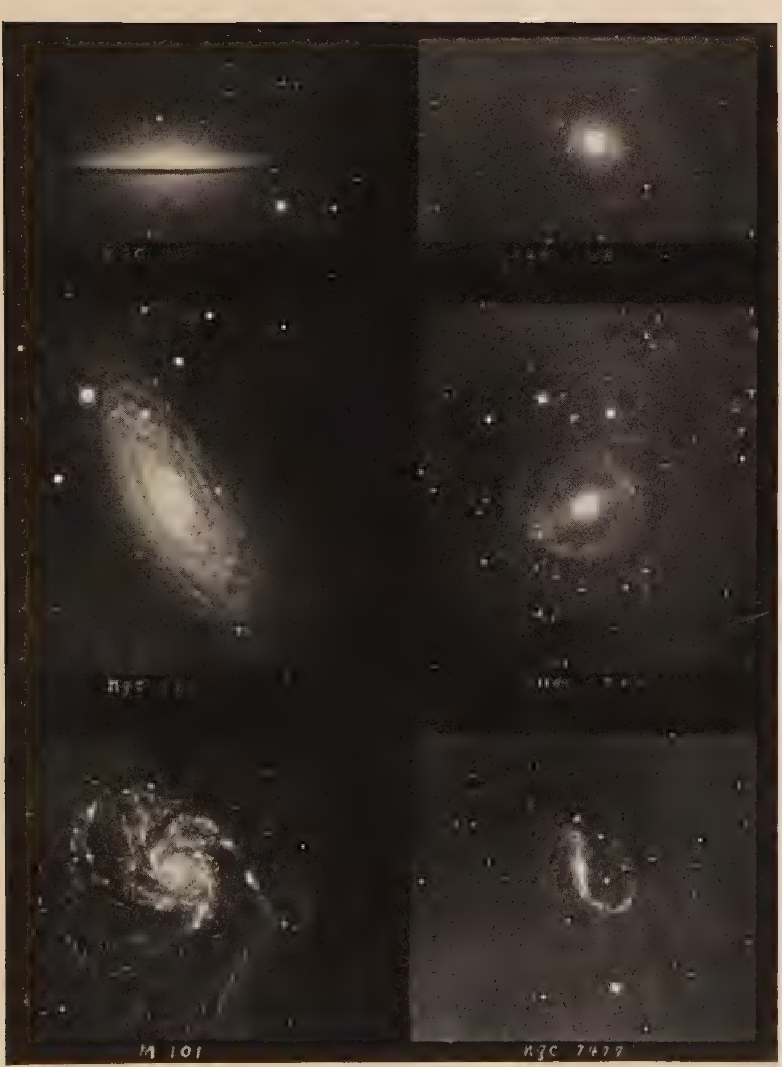

M 101
NGC 7479

PLATE 62. EXAMPLES OF SPIRAL NEBULÆ (Hubble)

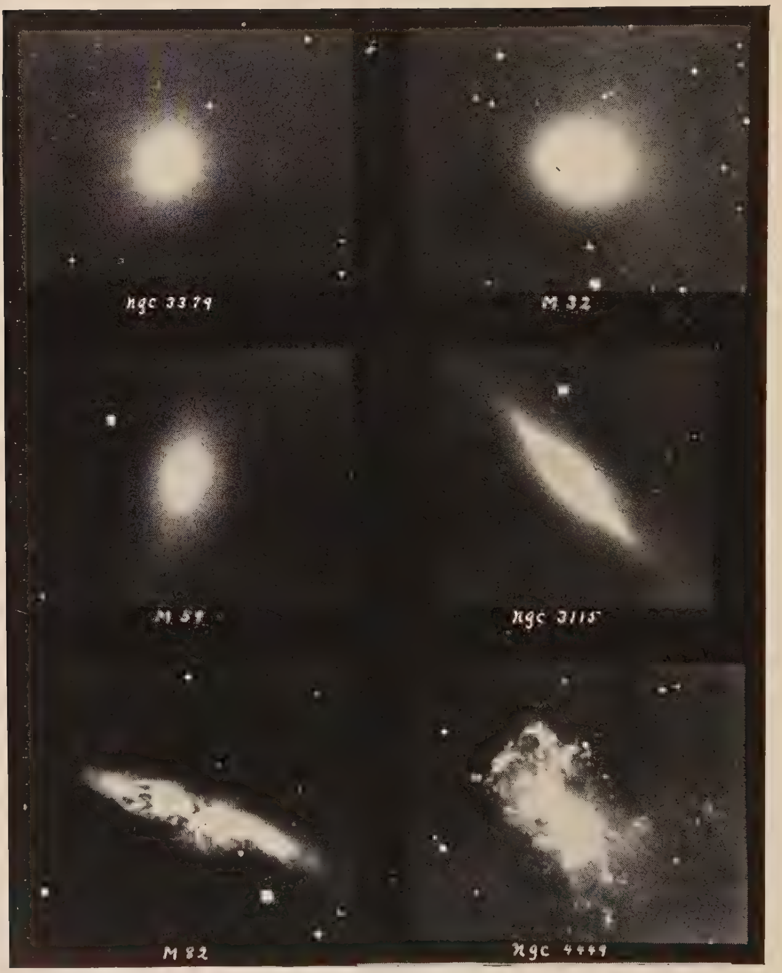

ngc 3379
M 32
M 59
ngc 3115
M 82
ngc 4449

Plate 63. Examples of Elliptic and Irregular Extra-Galactic Nebulæ (Hubble)

establishes two noteworthy facts: first, that there is a marked crowding of stars of each magnitude-class toward the Galactic Circle (Plate 65), and second, that this **galactic concentration** is greater for faint stars than for bright stars.

Galactic Concentration and Spectral Type.—Studies in the distribution of stars of different spectral type, especially those made at Harvard from the data of the *Henry Draper Catalogue*, show marked differences in galactic concentration. It is found that, for stars brighter than magnitude 8.25, there is little or no galactic concentration of F and G stars, that the K stars are concentrated only moderately toward the Galaxy, and that the concentration of A and especially of B stars is very marked indeed. Different degrees of concentration are shown by the M stars of different magnitudes and different sub-types. While the brighter M stars show little or no galactic concentration, that of the faint Ma and Mb stars is strongly marked.

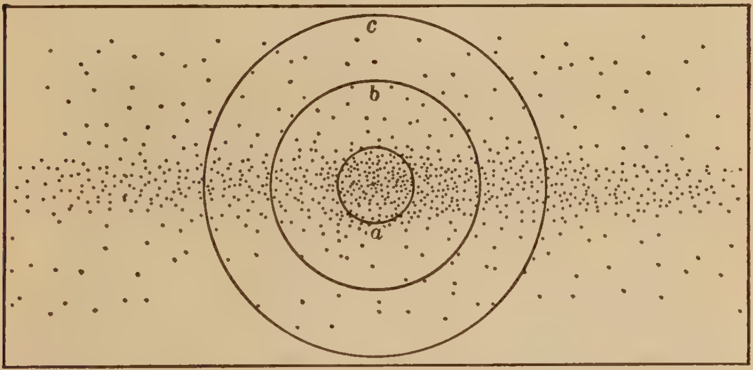


FIG. 178. GALACTIC CONCENTRATION. (FROM RUSSELL, DUGAN AND STEWART'S *Astronomy*, BY PERMISSION OF THE PUBLISHERS GINN AND COMPANY)

The Form of the Galactic System.—The first noteworthy speculations on the structure of the universe appear to have been published in 1750 by Thomas Wright, a scholarly English amateur. His idea was advocated and developed by Herschel and is strongly supported by Herschel's "star gauges" and by the counts of stars made by subsequent observers. It is evident, as Wright suggested, that the region of space which contains the majority of stars visible as such in our telescopes—that is, the **galactic system**—is shaped generally like a disk with a diameter several times its thickness, or like two saucers placed rim to rim and bottom outwards; and that the Solar

System is far inside this disk. An edge-on view of such an aggregation of stars is suggested in Fig. 178. The brightest stars are in general the nearest, and for an observer at the center these would be the stars included in the circle *a*, which would have a fairly uniform distribution in the sky. Fainter and more distant stars, such as those in circle *b*, would be pronouncedly concentrated toward the plane of the disk, and the galactic concentration would increase with increasing faintness as it is actually observed to do.

The bifurcation of the Milky Way and the irregularities of its structure are due, according to Herschel, to irregularities in the form of the disk or in the distribution of the stars; and he suggested the cross-section of the Galactic System shown in Fig. 179. According to a theory proposed by Easton of Rotterdam in 1900, the Galactic System has the form of a flat spiral similar to that of many spiral nebulæ. Fig. 180 is Easton's chart of the Milky Way, in which the central figure indicates in a general way how the stellar accumulations might be arranged in spiral form so as to produce the observed appearance, the Solar System being at the point *S*. Neither Herschel nor Easton recognized the important part which is now known to be played by obscuring nebulosities (page 399).

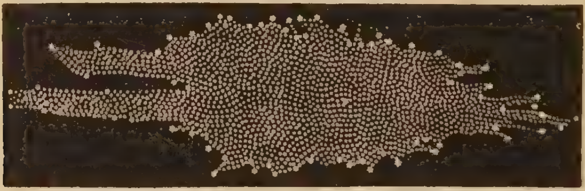


FIG. 179. CROSS-SECTION OF HERSCHEL'S GALACTIC SYSTEM

Recent Views Concerning the Construction of the Galactic System.—Elaborate investigations based on the proper motions of stars and on counts of stars of different magnitudes, together with the known distances and luminosities of the nearer stars, were made by Kapteyn, and were unfinished at the time of his death in 1922. The resulting picture, which, though incomplete, was intended to represent in a general way the form and size of the Galactic System (without taking star clouds, star clusters, and opaque matter into account), is sometimes called the **Kapteyn universe.** Its form is that of an oblate spheroid, with equatorial diameter about five times the polar, and with star density diminishing outward from the center so that there is no definite boundary. The layer in which the star density is 1/100 of that in the neighborhood of the Solar System has a thickness of about 11,000 light-years and an equatorial diameter of about 55,000. Kapteyn placed the center of the system (with considerable mistrust) about 2,000 light-years from the Sun in the direction of Cepheus.

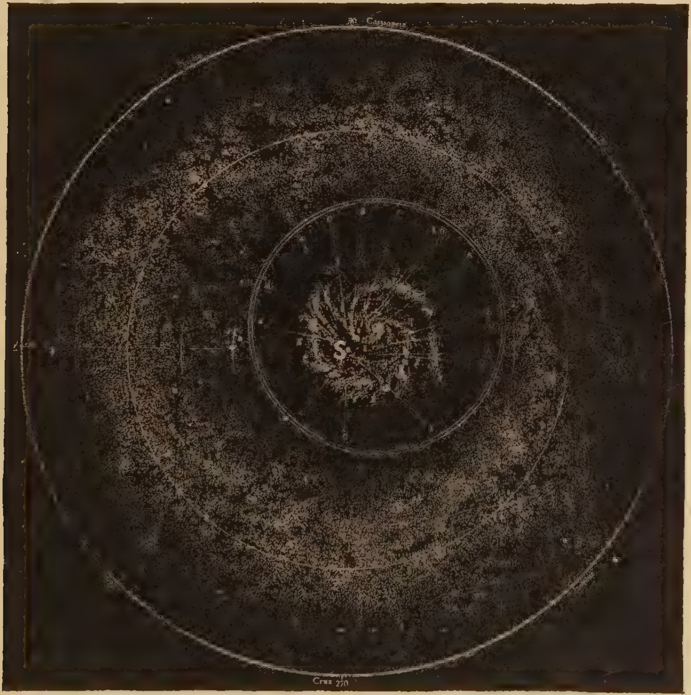


FIG. 180. EASTON'S THEORY OF THE MILKY WAY

The distribution of the globular star clusters is such as to show that they are connected with the Galactic System; they are arranged almost symmetrically on both sides of the galactic plane, and almost all of them are near the Milky Way, in the hemisphere between galactic longitudes 225° and 45°. Their positions in space, as revealed by Shapley's determination of their distances (page 388), clearly connect them with the Milky Way system, and Shapley holds the view—which is rapidly winning the support of other astronomers—that these star clusters mark the approximate boundary of the Galactic System itself. What thus appears to be the **main Galactic System** has, according to Shapley, a diameter of over 200,000 light-years, and its center lies in the direction of the star clouds of Sagittarius (galactic longitude 327°), at a distance of about 50,000 light-years from the Sun. This region of the sky contains much obscuring material, and there is a possibility that, in addition to the visible aggregations, it includes vast, dense clouds of stars and nebulæ that are hidden from our view.

The studies of Shapley, Charlier and others indicate the existence of what Shapley calls the **local system** or local cluster, occupying a flat region within the main Galactic System about 3,000 light-years in diameter and 800 in thickness, and containing nearly all the bright B-type stars, a considerable fraction of the brighter A stars, and many open clusters and galactic nebulæ. Probably the majority of all stars which are visible to the naked eye belong to it. The principal plane of this local system makes an angle of about 14° with that of the Milky Way and coincides closely with that of Gould's belt of bright stars (page 385). The center of the local system seems to be situated about 300 light-years distant from the Sun in galactic longitude 240° in the constellation Argo—a direction almost at right angles to that of the main Galactic System. The interpenetration of the local system with the main Galactic System is responsible for much of the uncertainty now attaching to the results of statistical investigations of the stars, such as Kapteyn's.

Rotation of the Galaxy.—The probability of a rotation of the Galactic System around its shortest axis is suggested by its flattened form and by the analogy of those extra-galactic nebulæ which are shown by the inclination of their spectral lines to be rotating. It is easy to show on gravitational principles that, if the material of the system were distributed uniformly, the attraction on any star would vary directly as its distance from the center and that all parts of the system would have the same period of revolution, the rotation of the whole being like that of a wheel. Such a rotation would be impossible of detection by the observation of stars within the system because it would involve no *relative* motion of the observer and other parts of the system. If, on the other hand, practically all the mass is concentrated near the center as in the Solar System, the attraction will vary inversely as the square of the distance and the revolution of different parts will be in accordance with Kepler's third law, the inner stars revolving more rapidly than the outer. Such motion, if sufficiently rapid, should become apparent in both the proper motions and the radial velocities of distant stars.

Since 1926, the researches of Lindblad and Oort, and especially the radial velocity observations of J. S. Plaskett on the distant O-type stars, have given support to the view that the galaxy is rotating in a manner approximating the second, or planetary, type, around a massive nucleus lying in exactly the same direction as that determined by Shapley for the center of the main Galactic System. Observations by Gerasimovich and Otto Struve on the sharp K lines in the spectra of stars (page 400) give evidence that the interstellar calcium cloud shares in this rotation. The period of that zone of the system which contains the Sun can be estimated only in hundreds of millions of years.

The Galactic System Among Other Galaxies.—It is evident that, seen from a distance of a million light-years or so, our Galactic System would have the appearance of a nebula, or perhaps of a group of nebulæ. If seen edgewise, it would show a spindle form, and if it is a spiral such as Easton and others have imagined, it might resemble N. G. C. 4565, Plate 60, with a longitudinal dark lane due to a peripheral band of dark nebulosity and a central bulge due to the great nuclear star clouds of Sagittarius. Its spectrum would be of stellar type and would have inclined lines due to rotation. If seen in plan, it might exhibit a form like that of M 33, Plate 60, with a great central nucleus and numerous condensations. The larger of the condensations would represent star clouds, and among these would be the local system which now surrounds us. Tiny, star-like condensations would represent the star clusters. Possibly a few of the brightest giant stars would be visible as faintly luminous points. Of the Solar System there would appear no trace. Not far from the main aggregation there would be seen two small companion nebulæ—the Magellanic clouds. In the background would appear the myriads of kindred bodies already known to us as the extra-galactic nebulæ. In respect to size, our system would be prominent among these, with a diameter several times that of any of its neighbors—a continent among island universes.

Recent studies of the extra-galactic nebulæ at the Harvard Observatory have led Shapley to the view that the main Galactic System is a group of galaxies, or **super-galaxy,** rather than a continuous aggregation. Harvard photographs show forty or fifty clusters of extra-galactic nebulæ, some containing but half a dozen objects, and others (like the group shown in Plate 61) containing several hundred. One of the most conspicuous of these groups lies in Coma Berenices and Virgo, at an estimated distance of approximately ten million light-years. It contains some three hundred members of all forms (spindle, oval, spiral, irregular), and extends throughout an irregular region of space about two million light-years in diameter. Comparable clouds of galaxies are found in the same direction but lying far beyond the bright Coma-Virgo group. At least twenty of the clouds of galaxies recognized by Shapley appear to be more distant than one hundred million light-years.

According to Shapley's estimates, the diameter of the average extra-galactic nebula is between five and seven thousand light-years, with a range from less than two thousand to more than twenty thousand. These dimensions are comparable with those of the local system and the other star clouds of the Milky Way. The Andromeda Nebula, a giant among external galaxies, is apparently not larger than the great star cloud in Sagittarius and Scorpius which appears to form the nucleus of our Milky Way. On the other hand, Shapley estimates that the total mass involved in each of the Coma-Virgo clouds and similar groups is much like that of our whole Galactic System, and considers it reasonable to suppose that the latter is like the Coma-Virgo clusters—a super-galaxy. It is more flattened, smaller, and more compact than the cloud of galaxies in Coma and Virgo, but so are other known groups; for example, a highly elongated rich cloud of galaxies, four hundred or more in number, which has just (1930) been discovered on Harvard plates in Centaurus, and which appears to be intermediate between our discoidal Galactic System and the roughly spherical Coma-Virgo groups. Shapley predicts that future investigations will probably show that clouds of galaxies are occasionally grouped into systems of still higher order, and states that already a few *pairs* of super-galaxies have been noted. He finds, however, no indication of the existence of a single all-inclusive super-system.

The Radial Velocities of Distant Nebulæ and De Sitter's Cosmology.—Photographs of the spectra of apparently small extra-galactic nebulæ, made at Mount Wilson in 1928 and 1929, show redward displacements of the lines which, if interpreted as Doppler-Fizeau effects, indicate amazingly great velocities of recession up to about 8000 kilometers a second. Large positive displacements in the spectra of these bodies is the rule; the only exceptions are a few nebulæ such as the great spiral in Andromeda (M 31) and its companion elliptic nebula (M 32), which are believed to be the least remote. According to Hubble, the more distant a nebula is, the greater is its apparent velocity of recession; but some doubt is cast on this generalization by the fact that of three nebulæ in the Coma group, which might reasonably be supposed to be about equally remote, one has a spectral shift corresponding to a velocity of +3800 km./sec., and the others, +7300 and +7800 respectively.

According to the theory of relativity (page 238), the four-dimensional universe of space and time in which we dwell is *finite*, being curved in a fifth dimension so that a train of light-waves traveling forever in the same direction must return after long ages to the starting point. It has even been suggested that some of the extra-galactic nebulæ appearing on photographs are identical with others at the opposite points of the celestial sphere, seen the long way around the universe. This relativistic world is analogous to the world of a two-dimensional being who is confined to the surface of a sphere; his two-dimensional space is curved in a third dimension, and if he travels forever in the same direction (*i.e.*, along a great circle), he must return to the starting point. Though unbounded, his world is finite.

In the cosmology of De Sitter, which is based on the theory of relativity, very distant objects possess an inherent velocity of further separation which would produce a positive Doppler effect, and there would be an additional redward displacement of their spectral lines due to a slowing up of the light-vibrations during their long journey through space. There is a temptation to regard the observations referred to above as confirming this astounding theory, though the data are yet meager. If true, it means, for one thing, that larger and larger telescopes will not continue indefinitely to reveal more and more remote galaxies—that the universe is finite.

Jeans's Cosmogony.—An inclusive theory of cosmic evolution has been based by Sir James Jeans upon twentieth-century developments in astronomical physics (which include his own mathematical treatment of the general problem of a rotating and gravitating fluid body), and upon the now generally accepted theory (page 365) that the source of the energy of the stars is the transformation of their mass into radiation.

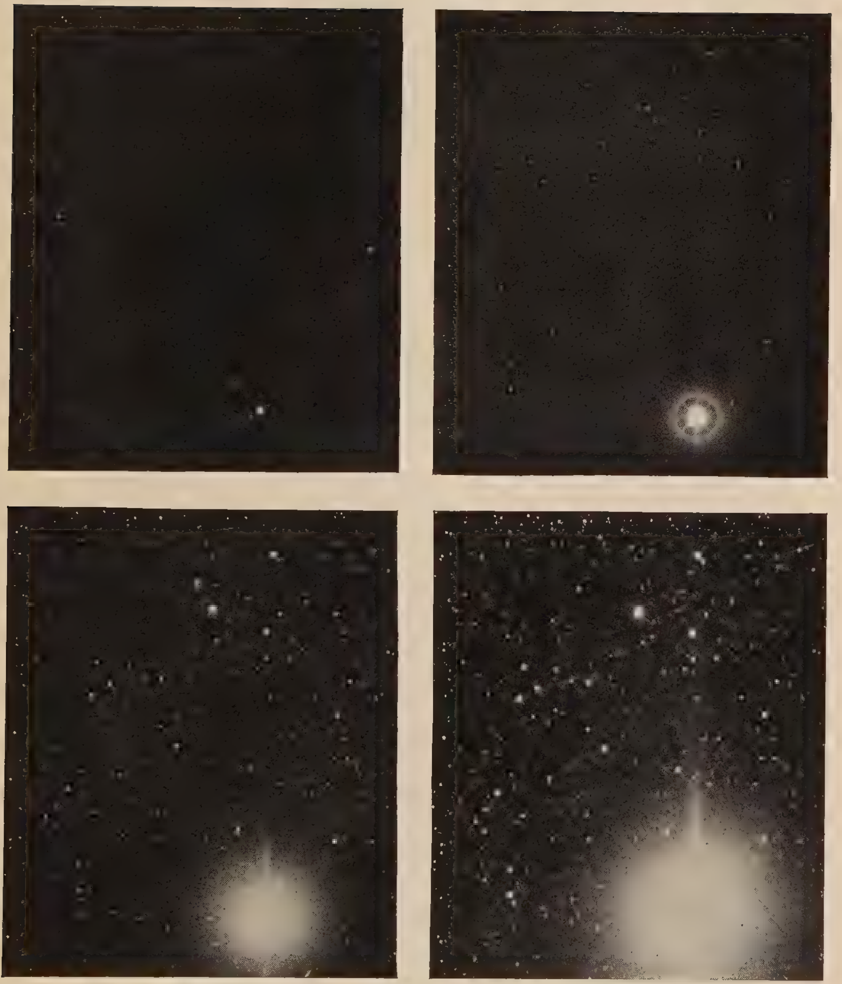
We have seen that, on this theory, the Sun can radiate at its present rate 15 millions of millions of years longer before its mass is consumed; but, as pointed out by Eddington (page 359), the luminosities of very massive stars are greater than those of less massive ones; as the Sun loses mass, its rate of radiation should therefore diminish. As a star grows older, its mass and luminosity decrease together, but at an ever-diminishing rate. Jeans estimates that it will require about 200 millions of millions of years for the Sun to reach the low state of luminescence represented by Krüger 60; that about 6 millions of millions of years ago it may have been as massive and as luminous as Sirius; and that for a star to descend from the state of the most massive of all stars to the condition of Sirius requires only about 1.2 millions of millions of years.

Looking forward, Jeans sees the extinction of the stars in their transformation into intangible radiation. Looking backward, he is constrained, like other cosmogonists, to assume the origin of the stars in the nebulæ, and it is here that his mathematical investigations of the configurations of a rotating fluid are significant.

If a nebula exists alone in infinite space without rotation, gravitation must cause it to assume and retain forever a spherical form. If it rotates, it will take at first the form of an oblate spheroid which will grow flatter as the nebula contracts and its angular velocity increases. Jeans shows, however, that there will come a time when its cross-section is no longer an ellipse, but a pointed figure; four stages of the evolution of the nebula up to this point are illustrated in Plate 63. Further contraction would, if the nebula were absolutely alone, cause a ring of matter to be abandoned as in the Laplace hypothesis; but every nebula has neighbors and their attraction has some influence of a tidal nature; the equator of the rotating mass of gas or dust will not be a circle, but will have two points of high tide and two of low tide. Jeans conjectures (his rigorous mathematics breaks down at this point) that, instead of forming a ring, the peripheral matter will flow out in long filaments at the two opposite points of high tide and so will form a spiral nebula. Again mathematical theory becomes useful by its application to these gaseous filaments; he shows that, if they are massive enough to cohere, they must break up into condensations such as are seen in the whorls of the spiral nebulæ M 33 and M 51 (Plate 60); if the filaments are not so massive as this, the gas composing them must be scattered into space. The nebula here considered is of a size appropriate to the genesis of the whole Galactic System or at least of a cluster containing hundreds of thousands of stars; so far, the hypothesis does not apply to so insignificant an affair as the Solar System. From his mathematical theory Jeans calculates the masses of the condensations in the spiral arms; they turn out to be about equal to those of the most massive of known giant stars.

Such is Jeans's conception of the birth of a great stellar system. Tracing the further history of each massive condensation, he finds that, as it loses mass by radiation, it passes through the sequence from giant to dwarf. If it rotates slowly or not at all,

PLATE 64



FIELD AROUND η AURIGÆ, photographed with increasing exposure-times by Seares with 60-inch reflector to show rapid increase in number of stars with decreasing brightness. The magnitudes of the faintest stars appearing on the four plates are, respectively, 12, 15, 18, and 20

PLATE 65. Photographs, by Seares, of two star fields of the same size, both showing stars to the 18th magnitude. The lower field is in the Milky Way and the upper field is in galactic latitude 80°. These photographs illustrate the great concentration of faint stars in low galactic latitudes

it will go through all its life history as a solitary body. If it rotates rapidly, it will become oblate, as did its parent nebula, but, because of its smaller mass, its subsequent development will be different: it will pass through a pear-shaped stage and, after an era of convulsive pulsations during which it may be a Cepheid variable, it will separate into two stars and so form a binary. The more massive of the pair will radiate the faster and so lose mass the faster, and the two will tend toward equality both in mass and in luminosity.

In all this process there is no tendency toward the formation of a planetary system. To explain this occurrence Jeans resorts, as we have seen (page 298), like Chamberlin and Moulton, to the rare coincidence of a close approach of a visiting body which abruptly transforms the star by tidal action into a diminutive spiral nebula; but instead of being composed of planetesimals, this nebula has gaseous whorls which condense in knots to form the planets.

The Question of Man's Place in the Universe.—Not so very long ago, man regarded himself as the center and crowning glory of the universe. "For him did his high Sun flame, and his river billowing ran." The purpose of the Sun was to give him light by day and that of the Moon to light his way by night, and the stars were mentioned as having been "made also." Copernicus inaugurated an important change in this attitude when he showed that the Earth was but one of a number of little balls which revolved around the Sun. Modern astronomy has gone much farther and shown that the Solar System is but a tiny speck in the great Galactic System and that the latter is probably only one of myriads of galaxies.

Man has lost his exalted position not only in space but also in time. It appears that the stars existed millions of millions of years before the Earth was born; that it was thousands of millions of years later that terrestrial life began at the shore of the primordial sea; that hundreds of millions more elapsed before man appeared; and that during all but a small fraction of his few hundred thousand years of existence he has been but little superior to other animals. The characterization of human history as "a brief and discreditable episode in the life of one of the meaner planets" is not indefensible.

Reacting from the orthodox views of a few centuries ago, many have adopted the doctrine of the plurality of inhabited worlds, imagining that other planets circle other suns and that each is peopled by varied beings. It is now fairly certain that most of the visible universe, so far as it concerns man or any other form of life which we know, is sterile. Mars and Venus seem to be the only planets of the Solar System, except the Earth, which could possibly support life. Probably the vast majority of stars are planetless, for the dynamic encounter by which a planetary system is believed to be engendered can have happened to but few. A quarter or a third of known stars are binary. Of stars which possibly have planets, some are variable and subject their surroundings to intolerable extremes of heat and cold. Even of possibly existing planets few are likely to be in the critical condition of temperature, surface gravity, and atmosphere necessary to the delicate thing called life. Yet, if but one star in a million possesses one habitable planet, there are in the Galactic System alone many thousand such abodes, and among them may dwell beings of a far higher order than ourselves.

"Think you this mould of hopes and fears
Could find no statelier than his peers
In yonder hundred million spheres?"

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